Noname manuscript No.
(will be inserted by the editor)

Sadataka Furui

Entropy of Chaotic Oscillations of Currents in the Chua Circuit and its HMM Analysis

Received: date / Accepted: date

Abstract We study entropy of chaotic oscillation of electric currents in the Chua circuit controlled by triggering a pulse that brings the orbit that goes onto an unstable branch back to a stable branch. A numerical simulation of the voltage of the two capacitors and the current that flows on a coil of the Chua circuit reveals various oscillation patterns as the conductance that is connected between the two capacitors and directly connected to a coil is varied. At small conductance, the Lissajous graph of the voltage of the two capacitors show a spiral, while at high conductance a double scroll pattern appears. The entropy of the current that flows on the coil is local minimum in the spiral state which is in the steady state, while it is local maximum in the stable double scroll state. The stable double scroll samples are analyzed by using the Hidden Markov Model (HMM) and the eigenvectors of transition matrix of long time series are found to be strictly positive but those of unstable short time series have negative components.

Keywords Minimum Entropy production rule, Hidden Markov Model, Chaos, Perron-Frobenius theorem

1 Introduction

Feynman said in the Lectures on Physics [1], "If currents are made to go through a piece of material obeying Ohm's low, the currents distribute themselves inside the piece so that the rate at which energy is generated is as little as possible. Also we can say (if things are kept isothermal) that the rate at which energy is generated is a minimum."

School of Science and Engineering and Graduate School of Teikyo University
1-1 Toyosatodai, Utsunomiya 320-8551, Japan
Tel.: +81-28-6277147
Fax: +81-28-6277184
E-mail: furui@umb.teikyo-u.ac.jp

In the statistical physics, internal energy E, entropy S and the volume V satisfy the relation

$$dE = TdS + pdV \tag{1}$$

and at the constant pressure, $dS = \frac{1}{T}dE$.

When an electric circuit is attached to a battery and the heat $\mathcal{Q}$ and the work $\mathcal{W}$ are supplied to a system, the sum of the two makes the internal energy $\mathcal{U}$ and the heat divided by the absolute temperature T, $\dfrac{d\mathcal{Q}}{T} = dS$ is called the entropy change. When local equilibrium exists in the system, the entropy change consists of that due to exchange of energy and/or matter with external system dS_e, and that due to irreversible processes in the system dS_i: $dS = dS_e + dS_i$.

In the textbook on statistical physics of Landau and Lifshitz [2] it is claimed that in the stable equiliblium states there are stable and metastable states and that the stable state produces the local maximum of the entropy. In non-equilibrium systems, Prigogine [3] showed that in the linearly interacting systems, the rate at which a stable, steady state, non-equilibrium system produces entropy internally is a minimum (Minimum Entropy Production rule)[6]. One could imagine that, when the temperature of subsystems are the same, minimum energy generation corresponds to the minimum entropy generation.

We studied in [4] the chaotic oscillation of the Chua circuit[5]. The Chua circuit consists of two capacitors, a coil, two diodes, a variable resistence and an opeamp, that produces piecewise-linear but bending resistance. When the variable resistance that is located between the two capacitors is decreased, the Lissajous graph of the voltage of the capacitor 1 and that of the capacitor 2 changes from a spiral to a double scroll. The oscillation is chaotic but we showed in [4] that the chaotic oscillation of currents can be controlled by triggering a pulse when the voltage of a capacitor in the Chua circuit passes a certain voltage from the larger absolute value to the lower absolute value. The orbit on the unstable branch is kicked by the pulse to one on a stable branch, and we could generate various oscillation patterns of the time series.

In the system of Chua's circuit, we consider the energy E as $VI = RI^2$ and I^2 is measured by using the Fourier transform and the Parcseval's formula. From the Feynman's conjecture and Prigogine's theorem, the entropy of the stable spiral steady current is expected to be a local minimum.

The entropy creation of the electric circuit system is given by

$$\begin{aligned} I_r &= L_R \frac{V_R}{T} \\ I_c &= L_C \frac{V_c}{T} = -\frac{L_C}{T}\frac{Q}{C} \\ L\frac{dI}{dt} &= -L_L \frac{I}{T} \end{aligned} \tag{2}$$

In the case of capacitors, the entropy creation rate is [3]

$$\frac{d_i S}{dt} = \frac{V_c I}{T} = \frac{V_c}{T} dQdt = -\frac{C}{T} V_c \frac{dV_c}{dt}$$

$$= -\frac{1}{T}\frac{d}{dt}(\frac{C{V_c}^2}{2}) = -\frac{1}{T}\frac{d}{dt}(\frac{Q^2}{2C}) > 0. \quad (3)$$

In the case of inductor,

$$\frac{d_i S}{dt} = -\frac{1}{T}\frac{d}{dt}(\frac{LI^2}{2}) = -\frac{LI}{T}\frac{dI}{dt} = \frac{V_L I}{T} > 0 \quad (4)$$

In speech recognitions and pattern recognitions of time series data, the Hidden Markov Model (HMM) which is an application of the Bayes statistical theory is well known. In this method, parameters that define the time series of the oscillation pattern in the equilibrium are derived by iteration [7,8]. We apply the HMM in the analysis of the time series of the voltage of the capacitor 1 (v_{C1}), that of the capacitor 2 (v_{C2}) and the current on the coil (i_L). We record the data sets of $\{v_{C1}, v_{C2}, i_L\}$, and consider the Poincaré surface specified by $i_L = \pm GF$, where G is the conductance and F is defined from the fixed point of the double scroll orbits $(\bar{v}_{C1}, \bar{v}_{C2}) = (\pm F, 0)$. We define the distance between the point at t_i on the orbit and the fixed point on the Poincaré surface $x(t_i) = \sqrt{(F - |v_{C1}|)^2 + v_{C2}^2}$ where t_i is the time when the orbit crosses the Poincaré surface. We also measure the binary (0,1) sequence $b_1, b_2, \cdots$ whether the sign of v_{C1} is positive or negative between t_i and t_{i+1}.

The contents of this paper is as follows. In sect.2, we present the differential equation used in analyzing the Chua circuit and in sect.3, the result of entropy analysis. In sect.4, the HMM applied to the double scroll system is explained and in sect.5, the result of HMM is summarized. Discussion and conclusion are given in sect.6.

2 The differential equation of the Chua circuit

Chua's circuit consists of a autonomous circuit which contains three-segment piecewise-linear resistor, two capacitors, one inductor and a variable resistor. The equation of the circuit is described by

$$\begin{cases} C_1 \frac{d}{dt} v_{C1} = G(v_{C2} - v_{C1}) - \tilde{g}(v_{C1}, m_0, m_1) \\ C_2 \frac{d}{dt} v_{C2} = G(v_{C1} - v_{C2}) + i_L \\ L \frac{d}{dt} i_L = -v_{C2} \end{cases} \quad (5)$$

where v_{C1} and v_{C2} are the voltages of the two capacitors (in V), i_L is the current that flows in the inductor (in A), C_1 and C_2 are capacitance (in F), L is inductance (in H) and G is the conductance of the variable resistor (in Ω^{-1}). The three-segment piecewise-linear resistor which constitutes the non-linear element is characterized by,

$$\tilde{g}(v_{C1}, m_0, m_1) = (m_1 - m_0)(|v_{C1} + B_p| - |v_{C1} - B_p|)/2 + m_0 v_{C1},$$

where B_p is chosen to be 1V, m_0 is the slope (mA/V) outside $|v_{C1}| > B_p$ and m_1 is the slope inside $-B_p < v_{C1} < B_p$. The function $\tilde{g}(v_{C1}, m_0, m_1)$ can be regarded as an active resistor.

As the conductance between the two capacitors is increased, the spiral changes to double scroll. An example of the orbit, we take $G = 0.7429$ which is close to the transition point of spiral to double scroll, and $G = 0.7052$ which is in the double scroll area. The orbits in $\{v_{C1}, v_{C2}, i_L\}$ space are shown in Fig.1 and in Fig.2 respectively for $G = 0.7052$ and for $G = 0.7429$.

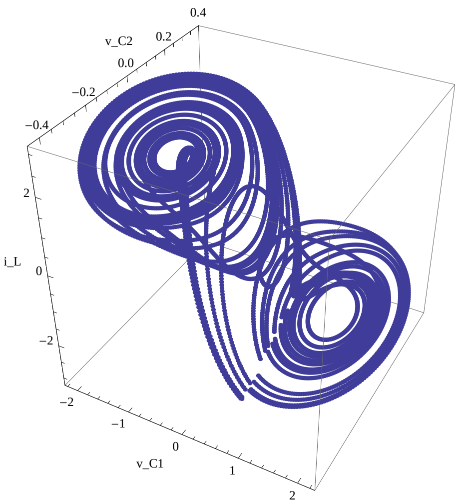

Fig. 1 An example of the orbit in $\{v_{C1}, v_{C2}, i_L\}$ space. ($G = 0.7052$)

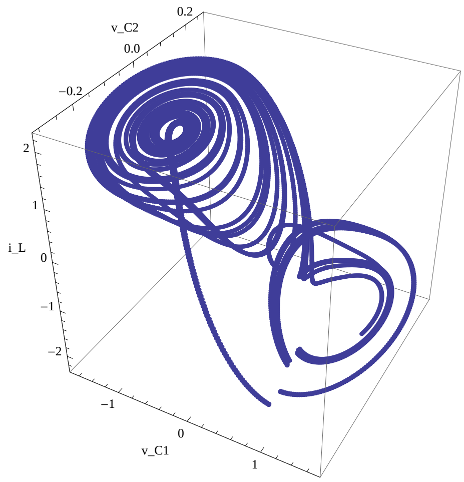

Fig. 2 An example of the orbit in $\{v_{C1}, v_{C2}, i_L\}$ space. ($G = 0.7429$)

3 The entropy analysis of the electric current

For an oscillation of a period T, the expansion of a time series data $f(t)$ is given as

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k \cos 2\pi kt + b_k \sin 2\pi kt)$$

$$a_k = \frac{2}{T}\int_0^T f(t)\cos 2\pi kt dt$$

$$b_k = \frac{2}{T}\int_0^T f(t)\sin 2\pi kt dt$$

and by applying the Parseval's formula, we obtain

$$\int_0^T f^2(t)dt = T\frac{a_0^2}{4} + \frac{T}{2}\sum_{k=1}^{\infty}(a_k^2 + b_k^2) \tag{6}$$

The integral over a period T becomes $\int_0^T \frac{L}{2}I(t)^2 dt$, $\int_0^T \frac{C}{2}V(t)^2 dt$, $\int_0^T \frac{R}{2}I(t)^2 dt$. We plot the zero-mode subtracted average

$$S_C = \frac{1}{T}\int_0^T V(t)^2 - \frac{a_0^2}{4} = \frac{1}{2}\sum_{k=1}^{\infty}(a_k^2 + b_k^2), \text{ etc.}$$

We measure the average entropy of the electric current between the periods when the pulses are triggered. The zero-mode subtracted entropy as a function of the length of the period of $G = 0.7052$ is shown in Fig. 3, and that of $G = 0.7429$ is shown in Fig.4.

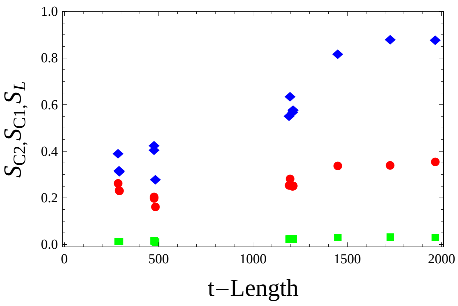

Fig. 3 The zero-mode subtracted entropy of the $G = 0.7052$ system as a function of the length of the time series. The diamond is the entropy of the current S_L, the disk is that of the capacitor 1 S_{C1} and square is that of the capacitor 2 S_{C2}. The stable double scroll shows a local maximum of the entropy.

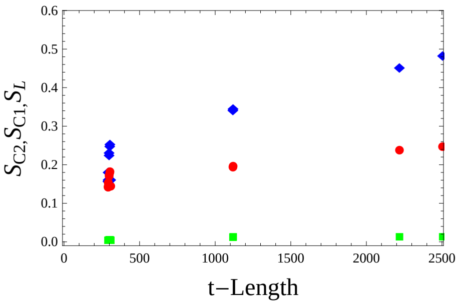

Fig. 4 The zero-mode subtracted entropy of the $G = 0.7429$ system as a function of the length of the time series. The meanings of the symbols are the same as Fig.**??**.

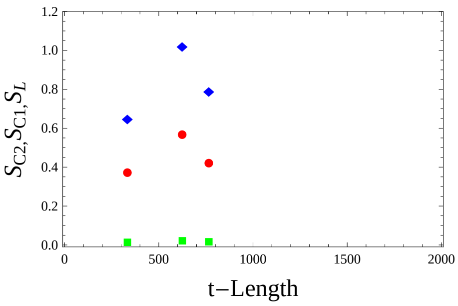

Fig. 5 The zero-mode subtracted entropy of exceptional samples of the $G = 0.7052$ system as a function of the length of the time series.

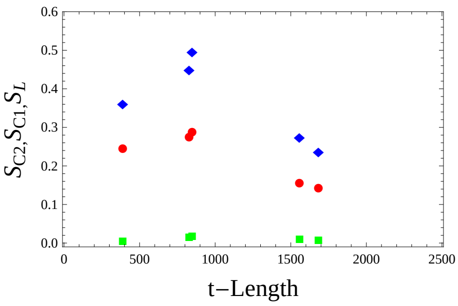

Fig. 6 The zero-mode subtracted entropy of exceptional samples of the $G = 0.7429$ system as a function of the length of the time series.

The zero-mode contribution to S_L is larger in the case of $G = 0.7429$ than in the case of $G = 0.7052$. When the length of the double scroll is long, its entropy increases as the length becomes long.

In addition to these monotonically increasing branch, there is an exceptional branch whose entropy reaches a maximum and then decreases as shown in Fig.5 and in Fig.6. It would be due to the transition onto the stable spiral orbits due to the kick by the pulse.

In order to extract parameters that characterize the oscillation patterns, we assemble samples whose period T is longer than 1000. An example of the ordinary orbit of $G = 0.7052$ is shown in Fig.7 and that of $G = 0.7429$ is Fig.8. .

When T is less than 1000 for $G = 0.7052$, there are ordinary orbits and exceptional orbits. When $G = 0.7429$, there are exceptional orbits Fig.9 and Fig.10.

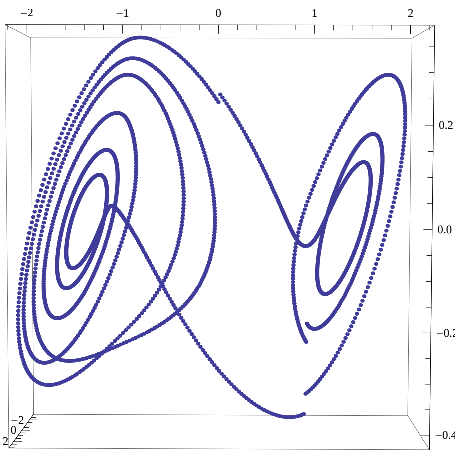

Fig. 7 The orbit of an ordinary sample of the $G = 0.7052$ system.

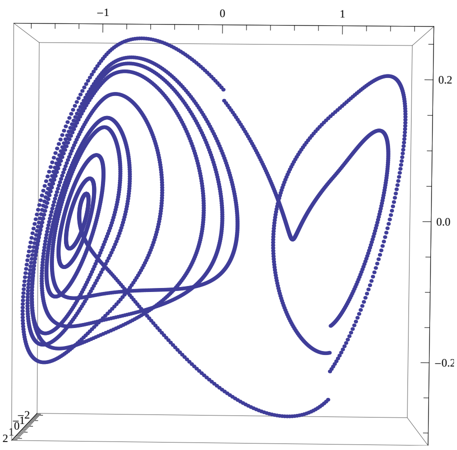

Fig. 8 The orbit of an ordinary sample of the $G = 0.7429$ system.

4 The HMM analysis of the electric current

In general, a sequence of events is a Markov process when the time series is defined by the state of one step before. When one restricts chaotic oscillation of the voltage of capacitors to the stable double scroll, and by taking into account the pulse that kicks the current on the unstable orbit to a stable orbit of the double scroll, one could make the system satisfy the detailed balance, and the time series of the stable double scroll can be regarded as a Markov chain. In this paper, we restrict the analysis of samples whose period T is longer than 1000. All the orbits of this sample are assumed to be absorbed in a limit orbit.

We identify the position of the orbit at time t following Hayes, Grebogi and Ott[9], in which the distance of the position on the orbit from the fixed point is calculated as

$$x(t_j) = \sqrt{(F - |v_{C1}(t_j)|)^2 + v_{C2}(t_j)^2} \tag{7}$$

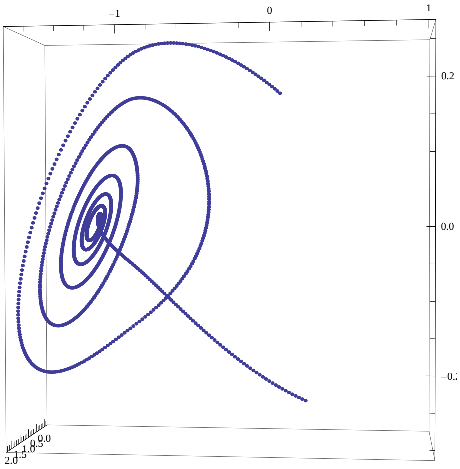

Fig. 9 The long orbit of exceptional sample of the $G = 0.7429$ system(4c).

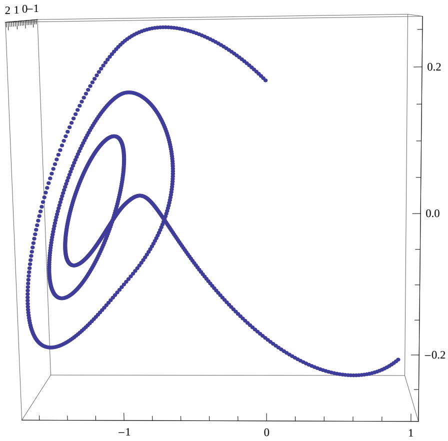

Fig. 10 The short orbit of exceptional sample of the $G = 0.7429$ system(1c).

and specify around which point it circulates by a coding function $r(x)$ which is defined as follows.

We define the binary time series $b_1, b_2, b_3, \cdots$ assigned at each time whien the orbit crosses the Poincaré surface of $i_L = \pm GF$, according to the voltage of the capacitor v_{C1} is positive(1) or negative(0). From the data of b_n, we define

$$r = \sum_{n=1}^{\infty} b_n 2^{-n}. \tag{8}$$

The symbols that occur in earlier times are given greater weight.

Using the list $R = r(1), r(2), \cdots, r(T)$ and the list $X = x_1, x_2, \cdots, x_T$, we denote by w_i the pair of list $(r(t), x(t))$ and the probability that the system is in this state as

$$P(w_i|S) = \frac{P(S|w_i)P(w_i)}{P(S)}. \tag{9}$$

We call the state at $t = t_1$ when the orbit passes the Poincaré surface and the measurement of the oscillation states as i and the state at $t = t_2$ when the orbit passes the Poincaré surface at the next time as j. The transition matrices a_{ij} for arbitrary j can be derived from the sample average when samples of the similar length T are assembled.

We define a model M that the system makes a transition from a state 1 to a state 2 and output the data x_1, then makes a transition from 2 to a state 3 as giving an output x_2, and then output x_3, and the probability

$$P(X, S|M) = a_{12} b_2(x_1) a_{22} b_2(x_2) a_{23} b_3(x_3) \cdots. \tag{10}$$

We try to maximize the probability

$$P(X|M) = \sum_R a_{r(0)r(1)} \prod_{t=1}^{T} b_{r(t)}(x_t) a_{r(t)r(t+1)}. \tag{11}$$

We define the distribution of the observation symbol $b_j(k)$ as the probability that the system in the state j gives the outout y_k, which is calculated by integrating the distribution given by a list vector $b_j(x_t)$.

4.1 Forward probability

When the number of states of the model M is N, the forward probability

$$\alpha_t(j) = P(x_1, \cdots, x_t, r(t) = j|M) \tag{12}$$

can be evaluated from the recursion

$$\alpha_{t+1}(j) = [\sum_{i=1}^{N} \alpha_t(i)a_{ij}]b_j(x_{t+1}) \tag{13}$$

The initial condition is such that the probability that the system is in the state i is π_i: $\alpha_1(i) = \pi_i b_i(x_1)$.

In the calculation α_{t+1}, $b_j(x_t)$ is a sum of the components of a list vector given from the distribution of x at time t.

The a_{ij} and $b_j(k)$ are updated in the backward probability calculation.

4.2 Backward probability

The backward probability $\beta_t(j)$ is defined as

$$\beta_t(j) = P(y_{t+1}, \cdots, y_T|s(t) = j, M) \tag{14}$$

The initial condition $\beta_T(i) = b_T(i)$ is defined from the sample average at $t = T$. $\beta_t(i)$ is calculated by the recursion

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij}b_j(x_{t+1})\beta_{t+1}(j) \tag{15}$$

$b_j(x_{t+1})$ is a list vector, and $\beta_{t-1}(i)$ is the sum of the components of the list vector.

When the model M is specified by $\lambda = (R, X, \pi)$, the probability the data x is output is

$$Pr(x|R, X, \pi) = \sum_{i=1}^{N}\sum_{i=1}^{N} \alpha_t(i)a_{ij}b_j(x_{t+1})\beta_{t+1}(j) \tag{16}$$

We define

$$\xi_t(i,j) = \frac{\alpha_t(i)a_{ij}b_j(x_{t+1})\beta_{t+1}(j)}{Pr(x|R, X, \pi)}, \tag{17}$$

and $\gamma_t(i) = \sum_{j=1}^{N} \xi_t(i,j)$. The probability $Pr(x|R, X, \pi)$ is a sum of the probabilities such that the components of the list vector $b_j(x_{t+1})$ is summed up.

The transition matrix a_{ij} is updated as

$$a_{ij} = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \gamma_t(i)} \tag{18}$$

The distribution $b_j(k)$ is updated as

$$b_j(k) = \frac{\sum_{t=1, y_t=k}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \gamma_t(i)} \tag{19}$$

5 The results of the HMM analysis

Among samples with the same conductance but different initial conditions we pickup about 10 samples of the data of $G = 0.7052$, whose length of the period T is larger than 1000.

The states of the time series are assigned by r which runs from 0(state 1), 1/32(state 2), 3/32(state 3), 7/32(state 4), 15/32(state 5), 1/2(state 6), 31/32(state 7) and the final state (state 8). Since the initial state has $r = \frac{1}{2}$, the first state is assigned as the 6th state, and since the next state has $r = 0$, the second state is assigned as the 1st state.

The state in the 6th state is transfered to the 1st state with the probability of 100%. Then in the next step it stays in the 1st state with the probability of 96% and goes to the 2nd state with the probability of 4%. This kind of information is contained in the matrix a_{ij}.

The transition matrix a_{ij} obtained from the long T samples is shown in Table1.

i\ j	1	2	3	4	5	6	7	8
1	0.955882	0.0441176	0.	0.	0.	0.	0.	0.
2	0.	0.	0.857143	0.	0.	0.	0.	0.142857
3	0.	0.	0.	1.	0.	0.	0.	0.
4	0.	0.	0.	0.	1.	0.	0.	0.
5	0.	0.	0.	0.	0.	0.	1.	0.
6	1.	0.	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.	0.4	0.6
8	0.	0.	0.	0.	0.	0.	0.	1.

Table 1 The a_{ij} matrix of the $G = 0.7052$ long T samples.

When the samples are restricted to short T samples, we obtained the a_{ij} matrix is shown in Table 2.

We define the probability $b_t(i)$ such that when the orbit passes a point x_t on the Poincaré surfaces at the time 't', the state is in the 'i'. The sequence dependent state probability $b_t(i)$ of $G = 0.7052$, long T samples is given in Table3. The corresponding data of short T samples are given in Table4.

In the HMM, the values of a_{ij} and the $b_t(i)$ in the equilibrium are obtained by iteration. The Perron-Frobenius theorem says that a dynamical system

i\ j	1	2	3	4	5	6	7	8
1	0.849673	0.150327	0.	0.	0.	0.	0.	0.
2	0.	0.	0.913043	0.	0.	0.	0.	0.0869565
3	0.	0.	0.	0.952381	0.	0.	0.	0.047619
4	0.	0.	0.	0.	0.95	0.	0.	0.05
5	0.	0.	0.	0.	0.	0.	0.947368	0.0526316
6	1.	0.	0.	0.	0.	0.	0.	0.
7	0.	0.	0.	0.	0.	0.	0.217391	0.782609
8	0.	0.	0.	0.	0.	0.	0.	1.

Table 2 The a_{ij} matrix of the $G = 0.7052$ short T samples.

t\ i	1	2	3	4	5	6	7	8
1	0.	0.	0.	0.	0.	1.	0.	0.
2	1.	0.	0.	0.	0.	0.	0.	0.
3	1.	0.	0.	0.	0.	0.	0.	0.
4	1.	0.	0.	0.	0.	0.	0.	0.
5	1.	0.	0.	0.	0.	0.	0.	0.
6	1.	0.	0.	0.	0.	0.	0.	0.
7	1.	0.	0.	0.	0.	0.	0.	0.
8	1.	0.	0.	0.	0.	0.	0.	0.
9	1.	0.	0.	0.	0.	0.	0.	0.
10	1.	0.	0.	0.	0.	0.	0.	0.
11	1.	0.	0.	0.	0.	0.	0.	0.
12	1.	0.	0.	0.	0.	0.	0.	0.
13	1.	0.	0.	0.	0.	0.	0.	0.
14	1.	0.	0.	0.	0.	0.	0.	0.
15	1.	0.	0.	0.	0.	0.	0.	0.
16	1.	0.	0.	0.	0.	0.	0.	0.
17	1.	0.	0.	0.	0.	0.	0.	0.
18	0.8	0.2	0.	0.	0.	0.	0.	0.
19	0.8	0.	0.2	0.	0.	0.	0.	0.
20	0.7	0.1	0.	0.2	0.	0.	0.	0.
21	0.7	0.	0.1	0.	0.2	0.	0.	0.
22	0.4	0.3	0.	0.1	0.	0.	0.2	0.
23	0.4	0.	0.3	0.	0.1	0.	0.2	0.
24	0.3	0.1	0.	0.3	0.	0.	0.1	0.2
25	0.3	0.	0.	0.	0.3	0.	0.	0.4

Table 3 The sequence dependent state probability $b_t(i)$ of $G = 0.7052$ long samples.

with a finite number of states can be guaranteed to converge to an equilibrium distribution ρ^*, if the computer algorithm is Markovian, is ergodic and satisfies detailed balance[10].

Before iteration, the a_{ij} of the long T samples has three non-zero eigenvalues which are 1, 0.96 and 0.4. After one iteration $a_{12} = 0.044$ and $a_{78} = 0.6$ changes to 0 and nonzero eigenvalues become 1,1 and 0.18. Although an eigenvector of a_{ij} before iteration has negative components, after an iteration, all the eigenvectors have positive components. It is a general property that a Markov chain should satisfy due to the Perron-Frobenius theorem[10].

In the case of short T samples, non-zero eigenvalues before iteration are 1, 0.850 and 0.217. Main difference from the long T samples is that the sign

t\ i	1	2	3	4	5	6	7	8
1	0.	0.	0.	0.	0.	1.	0.	0.
2	1.	0.	0.	0.	0.	0.	0.	0.
3	1.	0.	0.	0.	0.	0.	0.	0.
4	1.	0.	0.	0.	0.	0.	0.	0.
5	1.	0.	0.	0.	0.	0.	0.	0.
6	0.142857	0.857143	0.	0.	0.	0.	0.	0.
7	0.142857	0.	0.857143	0.	0.	0.	0.	0.
8	0.0357143	0.107143	0.	0.857143	0.	0.	0.	0.
9	0.0357143	0.	0.107143	0.	0.857143	0.	0.	0.
10	0.	0.0357143	0.	0.107143	0.	0.	0.857143	0.
11	0.	0.	0.	0.	0.107143	0.	0.0714286	0.821429
12	0.	0.	0.	0.	0.	0.	0.	1.
13	0.	0.	0.	0.	0.	0.	0.	1.
14	0.	0.	0.	0.	0.	0.	0.	1.

Table 4 The sequence dependent state probability $b_t(i)$ of $G = 0.7052$ short samples.

of two components of an eigenvector becomes negative, which suggests that the system is unstable.

6 Discussion and Conclusion

We studied the entropy of the electric current and the parameters of the HMM of the Chua circuit. We observed that the entropy of the steady spiral is a local minimum and that of the stable double scroll is a local maximum. There appears samples which change their double scroll orbits to spiral-like orbits which makes also a local maximum of the entropy. Restricting samples to those of stable double scroll which have the period T longer than 1000 in the case of $G = 0.7052$, we could make the time series satisfy the Markov chain condition and obtain the transition matrix a_{ij} whose largest eigen value is 1 and the components of the eigenvector are positive.

The Perron-Frobenius theorem [11,12] says that for a square nonnegative matrix some power of which is positive, there is a simple root λ of the characteristic polynomial which is strictly greater than the modulus of any other roots, and λ has strictly positive eigenvectors. In the proof of this theorem, Brouwer's fixed point theorem is used.

In the case of transition matrices of the time series of $G = 0.7052$ short T samples, flows to the equilibrium stable spiral state and non-equiliblium stable double scroll occur and we find that the eigenvectors of a_{ij} of short T sample is not strictly positive. The topology of the underlying dynamical system invalidates the application of the Brower's fixed point theorem in this case and introduction of branched manifold and Markov decomposition of the manifold[11] would be necessary. Analyses of the short T data of $G = 0.7052$ and those of $G = 0.7429$ are under way.

A Appendix

In this appendix, we show an example of the transition from spiral to double scroll observed experimentally on the Chua circuit[4], and the entropy of the capacitors 1, 2 and the inductor measured by the simulation[13].

A.1 The transition from spiral to double scroll

When conductance is small, the transition from an orbit around one fixed point to an orbit around another fixed point does not occur and the spairal pattern appears as shown in Table.5. The number of cycles increases as the conductance increases and transition to double scroll occurs. The number of circles around left fixed point and around the right fixed point becomes unequal at some conductance. The symbol $L3R4$ means 3 circulations around left fixed point and 4 circulation around right fixed point.

type	resistance ($R[k\Omega]$)	No. of periods	conductance ($1/R$)
spiral	1.614	1	0.620
spiral	1.543	2	0.648
spiral	1.536	4	0.651
spiral	1.525	3	0.656
double scroll	1.435	6	0.697
double scroll	1.418	5	0.705
double scroll	1.396	4	0.716
double scroll	1.390	L 3,R 4	0.719
double scroll	1.368	3	0.731
double scroll	1.355	L 3,R 2	0.738
double scroll	1.346	2	0.743
double scroll	1.331	2	0.751
double scroll	1.322	L 2,R 1	0.756
double scroll	1.315	L 1,R 2	0.761
double scroll	1.314	L 2,R 1	0.761

Table 5 An example of the transition from spiral to double scroll oscillation pattern of the Chua circuit.

A.2 The entropy produced in $G = 0.7052$ system

The entropy of $G = 0.7052$, 7 time series simulation data are shown in Table6. The symbols S_{C1}, S_{C2}, S_L are the zeromode subtracted Fourier amplitude squared of the capacitor 1, capacitor 2 and the inductor. The symbols e.g. 1a,1b,1c correspond to the period between the pulses triggered in the sequence 1.

A.3 The entropy produced in $G = 0.7429$ system

The entropy of $G = 0.7429$, 9 time series simulation data are shown in Table7. The symbols S_{C1}, S_{C2}, S_L are the zeromode subtracted Fourier amplitude squared of the capacitor 1, capacitor 2 and the inductor. The symbols e.g. 1a,1b,1c correspond to the period between the pulses triggered in the sequence 1.

sequence		period of t	length of the period	S_{C1}	S_{C2}	S_L
1		19.44-38.51	1908	1.8617	0.02805	2.39917
	1a	19.44-24.19	476	0.2067	0.01916	0.42803
	1b	24.20-26.40	221	0.39587	0.05309	1.50065
	1c	26.41-38.51	1211	0.2522	0.02681	0.57082
2		38.52-59.79	2128	1.78011	0.02951	2.34956
	2a	38.52-43.34	483	0.1635	0.01399	0.2815
	2b	43.35-45.29	195	0.22624	0.04507	1.12899
	2c	45.30-59.79	1450	0.33889	0.03246	0.82155
3		59.80-76.57	1678	1.65727	0.02843	2.19666
	3a	59.80-62.64	285	0.26386	0.01654	0.39284
	3b	62.65-64.60	196	0.233	0.0455	1.14973
	3c	64.61-76.57	1197	0.2843	0.02828	0.63801
4		76.58-101.76	2519	1.853	0.02753	2.37791
	4a	76.58-79.48	291	0.23456	0.01512	0.32095
	4b	79.49-81.33	185	0.16841	0.03863	0.93344
	4c	81.34-87.58	625	0.56976	0.02359	1.02295
	4d	87.59-89.84	226	0.41493	0.05281	1.50896
	4e	89.85-101.76	1192	0.2547	0.02589	0.55382
5		101.77-125.97	2421	1.64963	0.03277	2.29042
	5a	101.77-106.52	476	0.20028	0.01854	0.40951
	5b	106.53-108.69	217	0.37711	0.05305	1.48203
	5c	108.70-125.97	1728	0.34053	0.03405	0.88238
6		125.98-160.12	3415	1.8443	0.02943	2.41251
	6a	125.98-128.89	292	0.23172	0.01499	0.31587
	6b	128.90-130.72	183	0.16209	0.03752	0.91628
	6c	130.73-138.38	766	0.42249	0.01906	0.79026
	6d	138.39-140.45	207	0.30744	0.05128	1.34918
	6e	140.46-160.12	1967	0.35706	0.03242	0.88141
7		160.13-177.61	1749	1.55823	0.0279	2.0895
	7a	160.13-163.46	334	0.37432	0.01719	0.65062
	7b	163.47-165.48	202	0.27258	0.04889	1.26092
	7c	165.49-177.61	1213	0.25457	0.02718	0.58073

Table 6 The entropy of various oscillation pattern of $G = 0.7052$ samples

Acknowledgements The author thanks Mr. N. Hoshi for preparing figures and tables, and Dr. K. Moriya for helpful discussions.

sequence		period of t	length of the period	S_{C1}	S_{C2}	S_L
1		19.25-32.54	1329	1.24631	0.01487	1.50291
	1a	19.25-22.19	294	0.14618	0.006	0.16314
	1b	22.20-24.26	207	0.09467	0.01756	0.42977
	1c	24.27-32.54	828	0.02763	0.01711	0.4507
2		32.55-53.68	2114	1.00221	0.01189	1.20997
	2a	32.55-35.68	309	0.18756	0.00814	0.26958
	2b	35.69-38.10	247	0.21155	0.02241	0.65649
	2c	38.11-53.68	1558	0.15711	0.01091	0.2747
3		53.69-67.16	1347	1.22	0.01578	1.49536
	3a	53.69-56.62	293	0.1428	0.00587	0.15884
	3b	56.63-58.68	206	0.09219	0.01729	0.42278
	3c	58.69-67.16	848	0.28869	0.01861	0.49672
4		67.17-89.08	2192	1.0097	0.00992	1.17949
	4a	67.17-70.10	294	0.15461	0.00625	0.18185
	4b	70.11-72.25	215	0.11608	0.0193	0.48424
	4c	72.26-89.08	1683	0.14373	0.00926	0.2377
5		89.09-105.61	1653	1.15233	0.01411	1.39947
	5a	89.09-92.08	300	0.17417	0.00694	0.23253
	5b	92.09-94.43	235	0.17418	0.02234	0.60252
	5c	94.44-105.61	1118	0.19562	0.01412	0.34302
6		105.62-133.28	2767	0.85158	0.01427	1.10939
	6a	105.62-108.66	305	0.18279	0.00724	0.25467
	6b	108.67-111.08	242	0.19716	0.02276	0.63776
	6c	111.09-133.28	2220	0.23936	0.01424	0.45321
7		133.29-150.71	1743	1.06588	0.01376	1.30729
	7a	133.29-137.17	389	0.24625	0.00644	0.3621
	7b	137.18-139.52	235	0.17805	0.02235	0.61051
	7c	139.53-150.71	1119	0.19759	0.01435	0.34891
8		150.72-167.21	1650	1.15656	0.01408	1.40273
	8a	150.72-153.70	299	0.17213	0.00687	0.22618
	8b	153.70-156.02	232	0.16718	0.022	0.59081
	8c	156.02-167.21	1119	0.19775	0.0142	0.34696
9		167.22-197.70	3049	0.80684	0.01508	1.07902
	9a	167.22-170.25	304	0.18018	0.0072	0.24926
	9b	170.26-172.65	240	0.1925	0.02269	0.63215
	9c	172.66-197.70	2505	0.24864	0.01524	0.48473

Table 7 The entropy of various oscillation pattern of $G = 0.7429$ samples.

References

1. Feynman R.P. , Leighton R.B., and Sands M, *The Feynman Lectures on Physics*, Addison-Wesley: vol II p.19-14 (1965)
2. Landau L.D. and Lifshitz E.M. C*Statistical Physics*C(Translated into Japanese by Kobayashi et al.)CIwanami-shoten (1966).
3. Prigogine I. and Kondepty D. , *Modern Thermodynamics*, (Translated into Japanese by Senoo and Iwamoto) Asakura-shoten, (2001).
4. Furui S. and Niiya S. , Shil'nikov Chaos control using Homoclinic orbits and the Newhouse region,@Chaos, Solitons & Fractals , **34**,966(2007), arXiv:nlinCD/0604014.
5. Chua L.O., Komuro M. and Matsumoto M., The double scroll family, IEEE Trans. Circ. Syst. ,CAS ,**33** 1073(1986).
6. Kay J.J., About some slipups in applying Prigogine's minimum entropy production principle to living systems, http://www.jameskay.ca/musings/mep.pdf
7. Baum L.E. , Petrie T. , Soules G. and Weiss N. , A Maximization Technique Occuring in the Statistical Analysis of Probabilistic Function of Markov Chains, Ann. Math. Stat. **41**,164 (1970).

8. Welch L.R. , Hidden Markov Models and the Baum-Welch Algorithm, IEEE Information Theory Society Newsletter, 2003; vol.53, 9 .
9. Hayes S., Grebogi C. and Ott E., Communicating with Chaos, Phys. Rev. Lett. **70**,3031(1993).
10. Sethna J.P. , *Statistical Mechanics: Entropy, Order Parameters, and Complexity*, Oxford Univ. Press , (2006).
11. Robinson C., *Dynamical Systems: Stability, Symbolic Dynamics, and Chaos*, CRC Press, (1998), Translation into Japanese by Kokubu et al., Springer Verlag, (2001).
12. Boyle M. , Notes on the Perron-Frobenius Theory of Nonnegative Matrices, http://www.math.umb.edu/∼mmb.
13. Hoshi N. , About Chaos genarated by Electric Circuit, Master Thesis, Graduate School of Teikyo University, (2010).