Large-volatility dynamics in financial markets

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Abstract. - We investigate the large-volatility dynamics in financial markets, based on the minutely and daily data of the Chinese Indices and German DAX. The dynamic relaxation both before and after large volatilities is characterized by a power law, and the exponents $p_\pm$ usually vary with the strength of the large volatilities. The large-volatility dynamics is time-reversal symmetric at the minutely time scale, while asymmetric at the daily time scale. Careful analysis reveals that the time-reversal asymmetry is mainly induced by exogenous events. It is also the exogenous events which drive the financial dynamics to a non-stationary state. In general, the Chinese Indices and German DAX are in different universality classes. An interacting herding model without and with exogenous driving forces could qualitatively describe the large-volatility dynamics.

Financial markets are complex systems which share common features with those in traditional physics. In recent years, it has been piled up large amount of financial data. This allows an analysis of the fine structure and interaction of the financial dynamics, and many empirical results have been documented [1–10]. Although the price return of a financial index is short-range correlated in time, the volatility exhibits a long-range temporal correlation [2, 3]. The dynamic behavior of volatilities is an important topic in econophysics [2, 3, 11, 12].

In usual cases, one assumes that the financial market is in the stationary state, and analyzes the statistical properties of the financial dynamics. For a comprehensive understanding of the financial markets, however, it is also important to investigate the non-stationary dynamic properties. A typical example is the so-called financial crash [6,13]. Lillo and Mantegna study three huge crashes of the stock market, and find that the rate of volatilities larger than a given threshold after such market crashes decreases by a power law with certain corrections in shorter times [14]. This dynamic behavior is analogous to the classical Omori law, which describes the aftershocks following a large earthquake [15]. Selcuk analyzes the daily data of the financial indices from 10 emerging stock markets and also observed the Omori law after the two largest crashes [16]. Recently, Weber et al. demonstrate that the Omori law holds also after "intermediate shocks", and the memory of volatilities is mainly related to such relaxation processes [17].

Stimulated by these works, we systematically analyze the large-volatility dynamics in financial markets, based on the minutely and daily data of the Chinese Indices and German DAX. In our study, a large volatility is so selected that it is sufficiently large compared with the average volatility, but maybe not yet a real financial crash or rally. The purpose of this paper is multi-folds. We investigate the dynamic relaxation both *before* and *after* large volatilities. We focus on the time-reversal symmetry or asymmetry at the *minutely* and *daily* time scales. To achieve more reliable results, we introduce the remanent and anti-remanet volatilities to describe the large-volatility dynamics. In particular, we examine the dynamic behavior of different categories of large volatilities, and search for the origin of the time-reversal asymmetry at the daily time scale. We reveal how the dynamic system is driven to a non-stationary state by exogenous events. We compare the results of the mature German market and the emerging Chinese market. Finally we present a multi-agent model to simulate the large-volatility dynamics.

In this paper, we have collected the daily data of the German DAX from 1959 to 2009 with 12407 data points, and the minutely data from 1993 to 1997 with 360000 data points. The daily data of the Shanghai Index are from 1990 to 2009 with 4482 data points, and the minutely data are from 1998 to 2006 with 95856 data points. The daily data of the Shenzhen Index are from 1991 to 2009

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with 4435 data points, and the minutely data are from 1998 to 2003 with 50064 data points. The minutely data are recorded every minute in the German market, while every 5 minutes in the Chinese market. A working day is about 450 minutes in Germany while exactly 240 minutes in China. In our terminology, the results of the so-called "Chinese Indices" are the averages of the Shanghai Index and Shenzhen Index.

Denoting a financial index at time t as $P(t)$, the return and volatility are defines as $R(t) \equiv \ln P(t+1) - \ln P(t)$ and $|R(t)|$ respectively. Naturally, the dynamic behavior of volatilities may depend on the time scale. To study the dynamic relaxation after and before large volatilities, we introduce the remanent and anti-remanent volatilities $v_+(t)$ and $v_-(t)$,

$$v_\pm(t) = [< |R(t' \pm t)| >_c - \sigma]/Z, \qquad (1)$$

where $Z =< |R(t')| >_c - \sigma$, σ is the average volatility, and $< \cdots >_c$ represents the average over those t' with specified large volatilities. In our analysis, the large volatilities are selected by the condition $|R(t')| > \zeta$, and the given threshold ζ is well above σ. When ζ is sufficiently large, it corresponds to the financial crashes and rallies. $v_+(t)$ describes how the system relaxes from a large volatility to the stationary state, while $v_-(t)$ depicts how the system approaches a large volatility.

Large shocks in volatilities are usually followed by a series of aftershocks. Thus we assume that both $v_+(t)$ and $v_-(t)$ obey a power law,

$$v_\pm(t) \sim (t + \tau_\pm)^{-p_\pm}, \qquad (2)$$

where $p_\pm$ are the exponents and $\tau_\pm$ are positive constants. For reducing the fluctuations, we integrate Eq. (2) from 0 to t. Thus the cumulative function of $v_\pm(t)$ is written as

$$V_\pm(t) \sim [(t + \tau_\pm)^{1-p_\pm} - \tau_\pm^{1-p_\pm}] \qquad (3)$$

for $p_\pm \neq 1$. The physical origin for the power-law behavior of $v_\pm(t)$ is clear. It just represents the long-range temporal correlation of volatilities. Such a power-law behavior has been well understood in dynamic critical phenomena, even in the case far from equilibrium [18, 19].

We first analyze the minutely data of the Chinese Indices and German DAX. Now $|R(t)|$ is calculated in the unit of five minutes for the Chinese Indices and one minute for the German DAX. To select the large volatilities, we set the threshold $\zeta = 2\sigma$, 4σ, 6σ, and 8σ. For the minutely data, a large volatility may not indicate a real macroscopic crash or rally, and it possibly brings the dynamic system to a *microscopic* non-stationary state. In Fig. 1 (a), $V_\pm(t)$ of the Chinese Indices are displayed on a log-log scale. Because of the intra-day pattern [3,8,20,21], the curves periodically fluctuate at a working day, i.e., $t \sim 240$ minutes. We remove this intra-day pattern, e.g., with the procedure in Ref. [3, 22] and recalculate $V_\pm(t)$. This is shown in Fig. 1 (b). Now an almost perfect power-law behavior is observed for both $V_-(t)$ and $V_+(t)$, starting from $t \sim 5$ minutes. The curves of $V_\pm(t)$ of the German DAX look very similar to those of the Chinese Indices, with the intra-day pattern around $t \sim 450$ minutes.

Fitting the curves of $V_\pm(t)$ of the minutely data to Eq. (3), we obtain the exponents $p_\pm$ summarized in the first and second sectors of Table 1. For both the Chinese Indices and German DAX, both p_+ and p_- increase with the threshold ζ. Especially, p_+ and p_- of every ζ are equal within statistical errors. In other words, the dynamic behavior at the microscopic time scale, typically in minutes, is symmetric before and after large volatilities. However, the exponents $p_\pm$ of the German DAX are larger than those of the Chinese Indices.

To further understand the large-volatility dynamics of the financial markets, we have also calculated $V_\pm(t)$ with the daily data of the Chinese Indices and German DAX. As shown in Fig. 2, the dynamic behavior of $V_\pm(t)$ can also be described by Eq. (3), although the curves look somewhat fluctuating, compared with those of the minutely data. From the fitting, we obtain $\tau_\pm \approx 0$ for the Chinese Indices, while $\tau_\pm \neq 0$ for the German DAX. The exponents $p_\pm$ are listed in the fourth and fifth sectors of Table 1. $p_\pm$ of the daily data also depend on ζ, similar to those of the minutely data. However, the ζ-dependence of p_+ becomes obviously weaker, i.e., $p_- \neq p_+$. In other words, the time-reversal symmetry before and after large volatilities is violated at the daily time scale. These results are true for both the Chinese Indices and German DAX. But again $p_\pm$ of the German DAX are larger than those of the Chinese Indices. The dynamic relaxation of the Chinese Indices is slower. As a representative of emerging markets, the Chinese market shares common features with the western markets in basic statistical properties [22], meanwhile exhibits its own characteristics in the return-volatility correlation and "spatial" structure [9, 10].

In Refs. [14, 23], the dynamic relaxation after a financial crash, which is an event corresponding to an extremely large ζ and with $R(t') < 0$ in our terminology, has been investigated. The observable $N_+(t)$, which is the number of times that the volatility exceeds a certain threshold ζ_1 in the time t after the financial crash, decays by a power law. For comparison, we have also performed such an analysis. To reduce the fluctuations, we choose a large but not extremely large threshold $\zeta = 12\sigma$ to gain some samples for average. Additionally we extend the calculations to both $N_+(t)$ and $N_-(t)$. For the minutely data, we observe that $N_\pm(t)$ of $\zeta_1 = 2\sigma$ to 5σ could be fitted by Eq. (3). The exponents $p_\pm$ are weak ζ_1-dependent, and with $p_- = p_+$, i.e., similar to those for $V_\pm(t)$ in Table 1. For the daily data, the fluctuations are large, although the asymmetric behavior between $N_+(t)$ and $N_-(t)$ could be qualitatively observed. The weak point of this analysis is that there are two thresholds ζ and ζ_1.

Up to here, we always average over all the selected large volatilities in computing $V_\pm(t)$ (or $N_\pm(t)$). However, the large volatilities could originate differently, and the dynamic relaxation may depend on the category of the large volatilities. Especially, it is puzzling *how the time-reversal asymmetry arises at the daily time scale?* Our first thought is to classify the large volatilities $|R(t')|$ by $R(t') < 0$ and $R(t') > 0$, i.e., the so-called "crashes" and "rallies". We compute $V_\pm(t)$ for the "crashes" and "rallies" separately, and fit the curves to Eq. (3). At the minutely time scale, the exponents $p^c_\pm$ of the crashes are equal to $p^r_\pm$ of the rallies within statistical errors. At the daily time scale, however, $p^c_\pm \neq p^r_\pm$. As shown in Table 2, such an asymmetry behavior between the "crashes" and "rallies" is especially prominent for the German DAX, usually $p^c_\pm > p^r_\pm$. This result sounds reasonable, for the "crashes" should generally be more radical than the "rallies". In addition, the time-reversal asymmetry remains for both the "crashes" and "rallies".

The large volatilities at the daily time scale could be also classified into endogenous events and exogenous events [6, 24]. An exogenous event is associated with the market's response to external forces, and an endogenous event is generated by the dynamic system itself. Looking carefully at the history of the Shanghai stock market, for example, we find that there are 9 exogenous events among the 16 large volatilities selected by the threshold $\zeta = 8\sigma$. For the large volatilities corresponding to the thresholds such as $\zeta = 2\sigma$ and 4σ, it is not so meaningful to naively identify the external forces. In Fig. 3, $V_\pm(t)$ of $\zeta = 6\sigma$ and 8σ are displayed for the endogenous and exogenous events of the Shanghai Index. Obviously, the endogenous and exogenous events exhibit different dynamic behaviors. The dynamic relaxation of the exogenous events is faster. For the Shenzhen Index, we obtain qualitatively the same results but with somewhat larger fluctuations. For the German DAX, we are not able to directly identify the external forces, due to our unfamiliarity of the history of the German stock market.

In Fig. 3, we observe that the dynamic relaxation of the exogenous events is time-reversal asymmetric, i.e., $p_- \neq p_+$, while that of the endogenous events is approximately time-reversal symmetric, i.e., $p_- \approx p_+$. Based on this observation, we introduce an asymmetric factor r to characterize the large volatilities,

$$r = \frac{s_+ - s_-}{s_+ + s_-}, \tag{4}$$

where $s_\pm$ are the areas bounded by the curves of $m_\pm(t)$ and two coordinate axes. Careful analysis of the Shanghai Index shows that $r > 0.25$ and $r < 0.25$ reasonably classify the exogenous and endogenous events respectively. With such a classification, we are able to compute $V_\pm(t)$ for the exogenous and endogenous events respectively, for both the Chinese Indices and German DAX. These calculations confirm that the time-reversal asymmetry at the daily time scale is mainly induced by the exogenous events. In particular, $p_\pm(t)$ of the endogenous events are almost independent of the threshold ζ.

In financial markets, a large volatility does not necessarily indicate that the dynamic system already jumps to a non-stationary state, for the probability distribution of volatilities is with a power-law tail. Since $p_\pm(t)$ of the endogenous events are almost independent of ζ, the dynamic system probably remains still in the stationary state. However, exogenous events may drive the dynamic system to a non-stationary state.

Although there have been many activities devoted to the macroscopic description of the financial crashes, it remains a great challenge to simulate the dynamic relaxation of the large volatilities or financial crashes and rallies at the microscopic level. A Gaussian process or a usual minority game fails to explain the large-volatility dynamics. In this paper, we present an interacting herding model, which may qualitatively reproduce the dynamic behavior of the large volatilities. The model consists of N agents, which form clusters during dynamic evolution. Initially, each agent is a cluster. The dynamics evolves in the following way.

(1) At a time step t', an agent i (and thus its cluster) is selected at random.

(2) With probability a, i becomes active and decides buying or selling, and all agents in the cluster follow. After that, this cluster is broken into a state that each agent is a separate cluster. The size of this cluster is then recorded as $s(t')$.

(3) With probability $1 - a$, i remains inactive. Another agent j is randomly selected. If i and j are in different clusters, combine the two clusters into a bigger one.

The parameter a apparently controls the dynamic evolution. In fact, $1/a$ is the rate of transmission of information [25]. Practically, one assumes that $s(t')$ is proportional to the volatility at time t'. If a is a constant, the model does not produce the long-range temporal correlation of volatilities. To mimic the real markets, a should not be a constant. For example, a may interact with the volatility in a form like [26, 27]

$$a(t') = b + c/s(t' - 1). \tag{5}$$

Here b and c are positive constants. Such an interaction may generate a long-range temporal correlation of volatilities. We have performed the simulations with $N = 40000$ and $b = 0.00025$. The critical value of c is estimated to be 0.6. In Fig. 4 (a), $V_\pm(t)$ is displayed. Fitting the curves to Eq. (3), we obtain the exponents $p_\pm(t)$, as shown in the third sector of Table 1. Obviously, $p_\pm(t)$ are ζ-dependent, and the difference of p_- and p_+ is small. Therefore, this model could be compared with the minutely data of the financial markets, although its exponents $p_\pm(t)$ are larger. Actually, one may define the volatility $|R| = s^\alpha$ with $\alpha \neq 1$ to change the values of $p_\pm(t)$.

To simulate the dynamic relaxation before and after large volatilities at the daily time scale, we may introduce exogenous driving forces to the interacting herding model. Here we present a simple scheme. At a certain time, we randomly select several pairs of clusters, and combine each pair into a bigger cluster. As shown in Fig. 4 (b) and the sixth sector of Table 1, such a simple scheme does enhance the time-reversal asymmetry in the dynamic relaxation before and after large volatilities. To achieve an accurate comparison with the financial markets, one needs more sophisticated exogenous driving schemes, e.g., to modify the dynamic rules for the exogenous events.

In summary, we have investigated the large-volatility dynamics in financial markets, based on the minutely and daily data of the Chinese Indices and German DAX. The dynamic relaxation before and after large volatilities is characterized by the power law in Eq. (3). At the minutely time scale, the exponents $p_\pm$ increase with the threshold ζ, and the large-volatility dynamics is time-reversal symmetric, i.e., $p_- = p_+$. At the daily time scale, the ζ-dependence of p_+ is weaker, and the large-volatility dynamics is time-reversal asymmetric, i.e., $p_- \neq p_+$. Careful analysis reveals that not only the time-reversal asymmetry but also the ζ-dependence of $p_\pm$ are mainly induced by exogenous events. In this sense, the exogenous events may drive the financial dynamics to a non-stationary state, while the endogenous events may not. Quantitatively, the exponents $p_\pm$ of the Chinese Indices and German DAX belong to different universality classes. The dynamic relaxation of the Chinese Indices is slower. An interacting herding model without and with exogenous driving forces could qualitatively describe the large-volatility dynamics.

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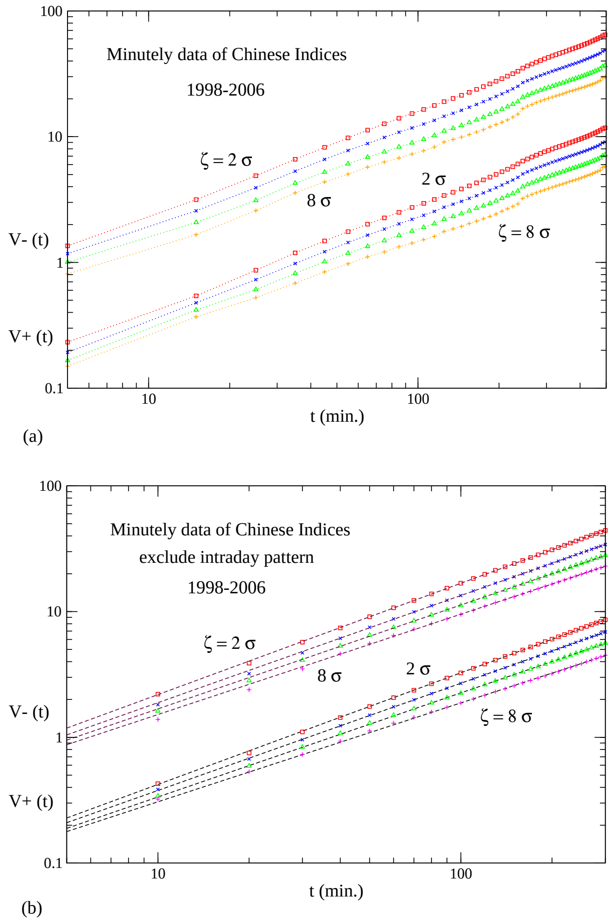


Fig. 1: (a) $V_{\pm}(t)$ for the minutely data of the Chinese Indices. From above, the threshold is $\zeta = 2\sigma$, 4σ, 6σ and 8σ respectively. (b) The same as (a), but the intra-day pattern has been removed. Dashed lines show the power-law fits with Eq. (3). The ζ-dependent exponents $p_- = p_+$, and $\tau_\pm = 0$.

ζ	2σ	4σ	6σ	8σ
	CHN(min)			
p_-	0.11(1)	0.15(1)	0.17(1)	0.20(1)
p_+	0.11(1)	0.15(1)	0.18(1)	0.22(1)
	DAX(min)			
p_-	0.16(1)	0.23(1)	0.27(1)	0.29(1)
p_+	0.16(1)	0.22(1)	0.26(1)	0.29(1)
	Model 1			
τ_-	1.58	2.51	3.07	3.24
p_-	0.46(1)	0.58(2)	0.65(1)	0.70(2)
τ_+	2.67	2.13	1.94	1.71
p_+	0.44(2)	0.52(1)	0.58(2)	0.62(2)
	CHN(day)			
p_-	0.27(3)	0.31(4)	0.36(4)	0.51(6)
p_+	0.26(2)	0.32(3)	0.33(4)	0.36(5)
	DAX(day)			
τ_-	13.11	9.06	4.07	3.78
p_-	0.41(3)	0.47(4)	0.60(5)	0.77(7)
τ_+	10.66	9.23	7.28	3.98
p_+	0.40(2)	0.42(3)	0.45(5)	0.46(5)
	Model 2			
τ_-	1.33	2.08	2.88	3.99
p_-	0.55(3)	0.68(4)	0.77(3)	0.86(5)
τ_+	2.07	2.39	2.05	1.96
p_+	0.46(3)	0.57(3)	0.63(3)	0.68(3)

Table 1: $p_\pm$ are measured with the minutely and daily data of the Chinese Indices (CHN) and German DAX, in comparison with those of the interacting herding model (model 1) and its variant with exogenous driving forces (model 2). For CHN(min), DAX(min) and CHN(day), $\tau_\pm = 0$.

ζ	2σ	4σ	6σ	8σ
	DAX(day)			
p^c_-	0.46(4)	0.47(5)	0.69(6)	0.99(13)
p^r_-	0.34(3)	0.35(4)	0.49(5)	0.55(7)
p^c_+	0.44(2)	0.46(4)	0.52(6)	0.66(7)
p^r_+	0.31(3)	0.30(3)	0.33(5)	0.30(5)
	CHN(day)			
p^c_-	0.21(3)	0.30(4)	0.35(5)	0.53(7)
p^r_-	0.31(3)	0.32(4)	0.39(5)	0.47(6)
p^c_+	0.23(2)	0.34(4)	0.36(6)	0.41(6)
p^r_+	0.30(2)	0.32(4)	0.33(5)	0.35(6)

Table 2: $p^c_\pm$ for the "crashes" and $p^r_\pm$ for the "rallies" measured with the daily data of the Chinese Indices and German DAX.

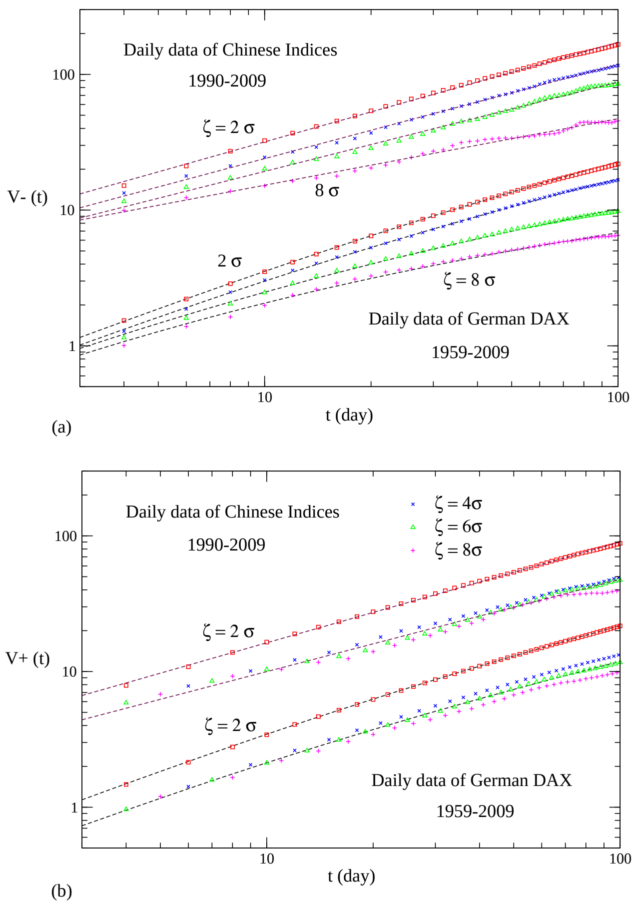


Fig. 2: $V_{\pm}(t)$ for the daily data of the Chinese Indices and German DAX. From above, the threshold is $\zeta = 2\sigma$, 4σ, 6σ and 8σ respectively. Dashed lines show the power-law fits with Eq. (3). $\tau_{\pm} = 0$ for the Chinese Indices and $\tau_{\pm} \neq 0$ for the German DAX. (a) p_{-} depends on ζ. (b) the ζ-dependence of p_{+} is weak. For clarity, some curves have been shifted downwards or upwards.

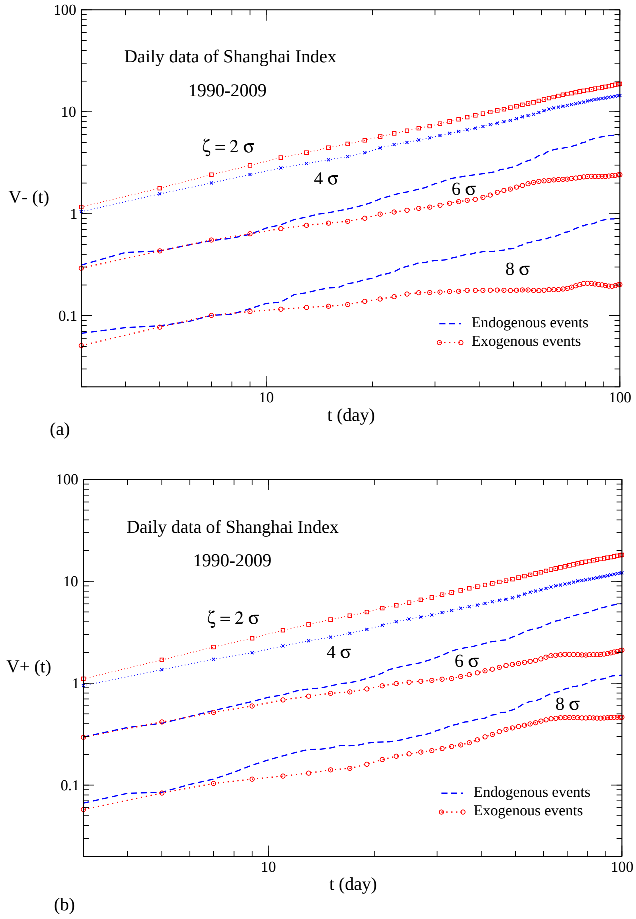
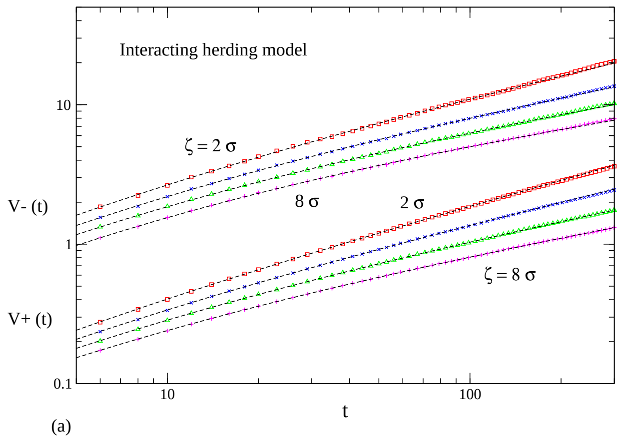


Fig. 3: $V_{\pm}(t)$ for the daily data of the Shanghai Index. For $\zeta = 6\sigma$ and 8σ, $V_{\pm}(t)$ are displayed for endogenous and exogenous events separately.



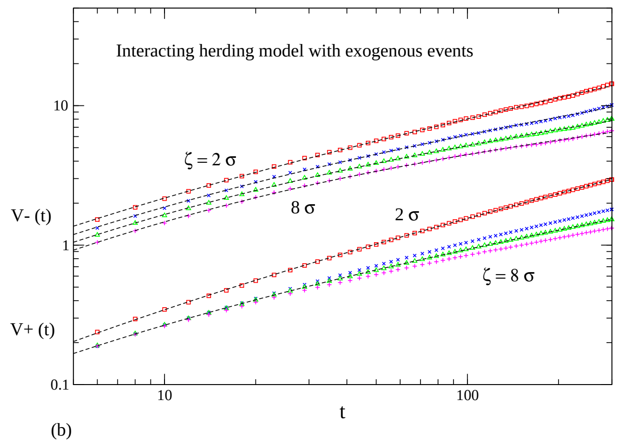


Fig. 4: $V_{\pm}(t)$ for the interacting herding model and its variant with exogenous driving forces. From above, the threshold is $\zeta = 2\sigma$, 4σ, 6σ and 8σ respectively.