

Nonlinear and chaotic ice ages: data vs speculations

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It is shown that, the wavelet regression detrended fluctuations of the reconstructed temperature for the past 400,000 years (Antarctic ice cores data) are completely dominated by one-third subharmonic resonance, presumably related to Earth precession effect on the energy that the intertropical regions receive from the Sun. Effects of Galactic turbulence on the temperature fluctuations are also discussed. Direct evidence of chaotic response of the atmospheric CO_2 dynamics to obliquity periodic forcing has been found in a reconstruction of atmospheric CO_2 data (deep sea proxies), for the past 650,000 years.

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INTRODUCTION

Forcing of nonlinear systems does not always have the results that one might expect. Subharmonic and chaotic resonances are prominent examples of such response. Recent paleoclimate reconstructions provide indications of nonlinear properties of Earth climate at the late Pleistocene [1],[2] (the period from 0.8 Myr to present). Long term decrease in atmospheric CO_2 , which could result in a change in the internal response of the global carbon cycle to the obliquity forcing, has been mentioned as one of the principal reasons for this phenomenon (see, for instance, [3]-[5]). At present time one can recognize at least three problems of the nonlinear paleoclimate, which we will address in present paper using recent data and speculations.

A. Reconstructed air temperature on millennial time scales are known to be strongly fluctuating. See, for instance figures 1 and 2. While the nature of the trend is widely discussed (in relation to the glaciation cycles) the nature of these strong fluctuations is still quite obscure. The problem has also a technical aspect: detrending is a difficult task for such strong fluctuations. In order to solve this problem a wavelet regression detrending method was used in present investigation. Then a spectral analysis of the detrended data reveals rather surprising nature of the strong temperature fluctuations. Namely, the detrended fluctuations of the reconstructed temperature are completely dominated by so-called one-third subharmonic resonance, presumably related to Earth precession effect on equatorial insolation.

B. Influence of Galactic turbulent processes on the Earth climate can be very significant for time-scales less than 2.5 kyr.

C. Nonlinearity of the Earth climate can also results in a chaotic response to an external periodic forcing. Already pioneering studies of the effect of external periodic forcing on the first Lorenz model of the chaotic climate (convection) revealed very interesting properties of chaotic response (see, for instance, [6],[7],[8]). The cli-

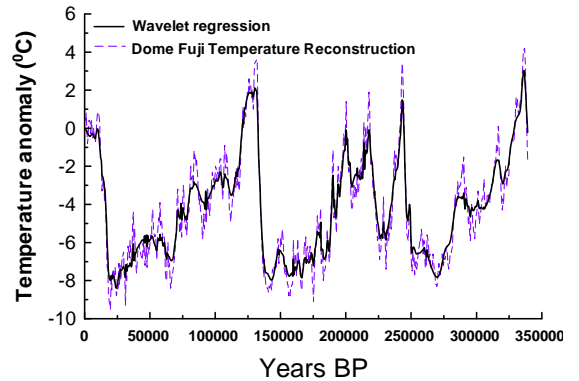


FIG. 1: The reconstructed air temperature data (dashed line) for the period 0-340 kyr. The data were taken from Ref. [11] (see also Ref. [12]). The solid curve (trend) corresponds to a wavelet (symmlet) regression of the data.

mate, where the chaotic behavior was discovered for the first time, is still one of the most challenging areas for the chaotic response theory. One should discriminate between chaotic weather (time scales up to several weeks) and a more long-term climate variation. The weather chaotic behavior usually can be directly related to chaotic convection (as it was done for the first time by Lorenz), while appearance of the chaotic properties for the long-term climate events is a non-trivial and challenging phenomenon. It was suggested that such properties can play a significant role for glaciation cycles at multi-millennium time scales [2],[9],[10].

Cyclic forcing, due to astronomical modulations of the solar input, rightfully plays a central role in the long-term climate models. Paradoxically, it is a very non-trivial task to find imprints of this forcing in the long-term climate data. It will be shown in present paper that just unusual properties of nonlinear and chaotic response are the main source of this problem.

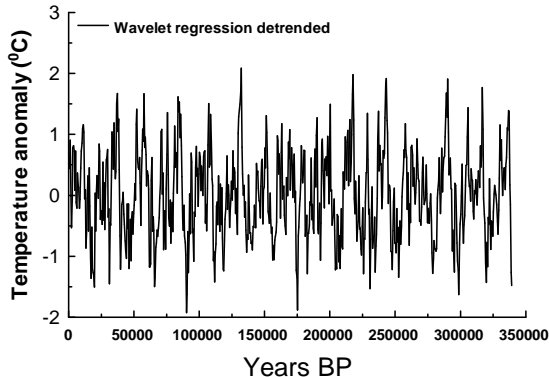


FIG. 2: The wavelet regression detrended fluctuations from the data shown in Fig. 1.

SUBHARMONIC RESONANCE

Figure 1 shows reconstructed air temperature data (dashed line) for the period 0-340 kyr as presented at the site [11] (Antarctic ice cores data, see also Ref. [12]). The solid curve (trend) in the figure corresponds to a wavelet (symmlet) regression of the data (cf Ref. [13]). Figure 2 shows corresponding detrended fluctuations, which produce a statistically stationary set of data. Most of the regression methods are linear in responses. At the nonlinear nonparametric wavelet regression one chooses a relatively small number of wavelet coefficients to represent the underlying regression function. A threshold method is used to keep or kill the wavelet coefficients. In this case, in particular, the Universal (VisuShrink) thresholding rule with a soft thresholding function was used. At the wavelet regression the demands to smoothness of the function being estimated are relaxed considerably in comparison to the traditional methods. Figure 3 shows a spectrum of the wavelet regression detrended data calculated using the maximum entropy method (because it provides an optimal spectral resolution even for small data sets). One can see in this figure a small peak corresponding to period ~ 5 kyr and a huge well defined peak corresponding to period ~ 15 kyr. We also obtained analogous results (approximately 10% larger) from the "Vostok" ice core data for period 0-420kyr (for the data description see Refs. [14],[15]).

In order to understand underlying physics of the very characteristic picture shown in the Fig. 3 (cf. Ref. [16]) let us imagine a forced excitable system with a large amount of loosely coupled degrees of freedom schematically represented by Duffing oscillators (which has become a classic model for analysis of nonlinear phenomena and can exhibit both deterministic and chaotic behavior [17]-[20] depending on the parameters range) with a wide range of the natural frequencies ω_0 :

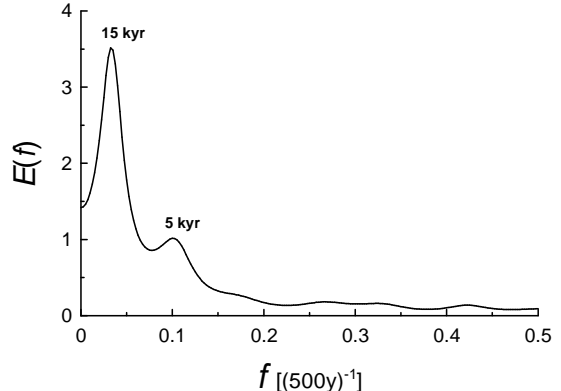


FIG. 3: Spectrum of the wavelet regression detrended fluctuations shown in Fig. 2.

$$\ddot{x} + \omega_0^2 x + \gamma \dot{x} + \beta x^3 = F \sin \omega t \quad (1)$$

where \dot{x} denotes the temporal derivative of x , β is the strength of nonlinearity, and F and ω are characteristic of a driving force. It is known (see for instance Ref. [20]) that when $\omega \approx 3\omega_0$ and $\beta \ll 1$ the equation (1) has a resonant solution

$$x(t) \approx a \cos\left(\frac{\omega}{3}t + \varphi\right) + \frac{F}{(\omega^2 - \omega_0^2)} \cos \omega t \quad (2)$$

where the amplitude a and the phase φ are certain constants. This is so-called one-third subharmonic resonance with the driving frequency ω corresponding approximately to 5kyr period (the huge peak in Fig. 3 correspond to the first term in the right-hand side of the Eq. (2)). For the considered system of the oscillators an effect of synchronization can take place and, as a consequence of this synchronization, the characteristic peaks in the spectra of partial oscillations coincide [21]. It can be useful to note, for the climate modeling, that the odd-term subharmonic resonance is a consequence of the reflection symmetry of the natural nonlinear oscillators (invariance to the transformation $x \rightarrow -x$).

Origin of the periodic energy input with the period ~ 5 kyr can be related to dynamics of the energy that the intertropical regions receive from the Sun (equatorial insolation). Indeed, it is found in Ref. [22] that a clear and significant 5kyr period is present in this dynamic over last 1 Myr. The amplitude of the 5kyr cycle in the insolation decreases rapidly when getting away from the equator. Using the fact that double insolation maximum and minimum arise in the tropical regions in the course of one year, the authors of the Ref. [22] speculated that this period in seasonal amplitude of equatorial insolation is determined by fourth harmonic of the Earth precession cycle. It should be noted, that the idea of a significant

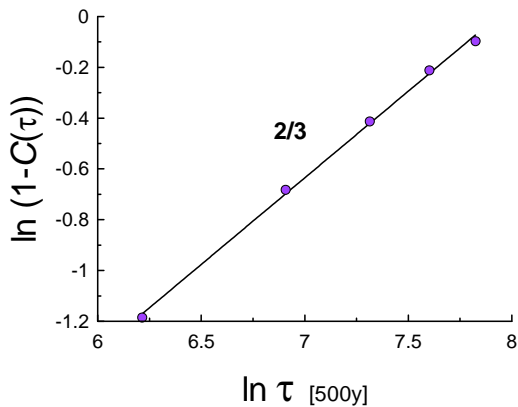


FIG. 4: A small-time-scales part of the autocorrelation function defect of the wavelet regression detrended fluctuations from the data shown in Fig. 2. The straight line indicates the Kolmogorov's '2/3' power law for the structure function in ln-ln scales.

role of tropics in generating long-term climatic variations is rather a new one (see Ref. [22] for relevant references). In Ref. [23], for instance, the authors speculated that the high frequency climate variability (in the millennial time scales) could be related to high sensitivity of the tropics to summer time insolation. Then, the oceanic advective transport could transmit an amplified response of tropical precipitation and temperature to high latitudes. Physical mechanism of this amplification is still not clear and the above discussed one-third subharmonic resonance can be a plausible possibility (in this respect it is significant, that we used the Antarctic data).

GALACTIC TURBULENCE AND THE TEMPERATURE FLUCTUATIONS

Since the high frequency part of the spectrum is corrupted by strong fluctuations (the Nyquist frequency equals $0.5 [(500y)^{-1}]$), it is interesting to look at corresponding autocorrelation function $C(\tau)$ in order to understand what happens on the millennial time scales. Figure 4 shows a relatively small-times part of the correlation function defect. The ln-ln scales have been used in this figure in order to show a power law (the straight line) for structure function: $\langle (v(t+\tau) - v(t))^2 \rangle$:

$$1 - C(\tau) \propto \langle (v(t+\tau) - v(t))^2 \rangle \propto \tau^{2/3} \quad (3)$$

This power law: '2/3', for structure function (by virtue of the Taylor hypothesis transforming the time scaling into the space one [24],[25]) is known for fully developed turbulence as Kolmogorov's power law.

Although, the scaling interval is short, the value of the exponent is rather intriguing. This exponent is well known in the theory of fluid (plasma) turbulence and corresponds to so-called Kolmogorov's cascade process. This process is very universal for turbulent fluids and plasmas [26],[27]. For turbulent processes on Earth and in Heliosphere the Kolmogorov-like scaling with such large time scales certainly cannot exist. Therefore, one should think about a Galactic origin of Kolmogorov turbulence with such large time scales (let us recall that diameter of the Galaxy is approximately 100,000 light years). This is not surprising if we recall possible role of the galactic cosmic rays for Earth climate (see, for instance, [10],[28]-[30]). In this respect, it should be also noted that the '2/3' scaling law can reflect not the velocity field of the galactic interstellar media but the turbulent electron density field (presumably produced by supernova) according to the Obukhov-Corrsin turbulent mixing process (see, for instance, Ref. [31],[32] and references therein).

ATMOSPHERIC CO_2 DYNAMICS AT MULTI-MILLENNIUM TIME SCALES

The angle between Earth's rotational axis and the normal to the plane of its orbit (known as *obliquity*) varies periodically between 22.1 degrees and 24.5 degrees on about 41,000-year cycle. Such multi-millennium timescale changes in orientation change the amount of solar radiation reaching the Earth in different latitudes. In high latitudes the annual mean insolation (incident solar radiation) decreases with obliquity, while it increases in lower latitudes. Obliquity forcing effect is maximum at the poles and comparatively small in the tropics. Milanković theory suggests that lower obliquity, leading to reduction in summer insolation and in the mean insolation in high latitudes, favors gradual accumulation of ice and snow leading to formation of an ice sheet. The obliquity forcing on Earth climate is considered as the primary driving force for the cycles of glaciation (see for a recent review [1]). Observations show that glacial changes from -1.5 to -2.5 Myr (early Pleistocene) were dominated by 41 kyr cycle [9],[33],[34], whereas the period from 0.8 Myr to present (late Pleistocene) is characterized by approximately 100 kyr glacial cycles [35],[36]. While the 41 kyr cycle of early Pleistocene glaciation is readily related to the 41 kyr period of Earth's obliquity variations the 100 kyr period of the glacial cycles in late Pleistocene still presents a serious problem. Influence of the obliquity variations on global climate started amplifying around 2.5 Myr, and became nonlinear at the late Pleistocene. Long term decrease in atmospheric CO_2 , which could result in a change in the internal response of the global carbon cycle to the obliquity forcing, has been mentioned as one of the principal reasons for this phenomenon (see, for instance, [3]-[5]). Therefore, investigation of the his-

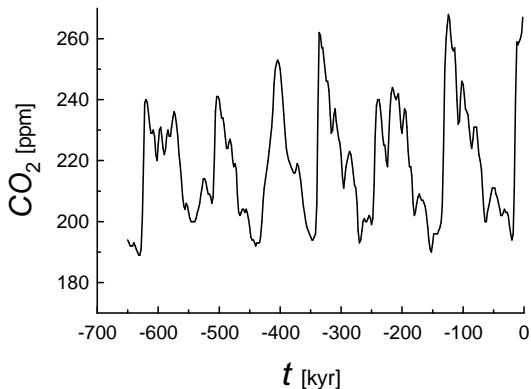


FIG. 5: A reconstruction of atmospheric CO_2 based on deep-sea proxies, for the past 650kyr. The data were taken from [37].

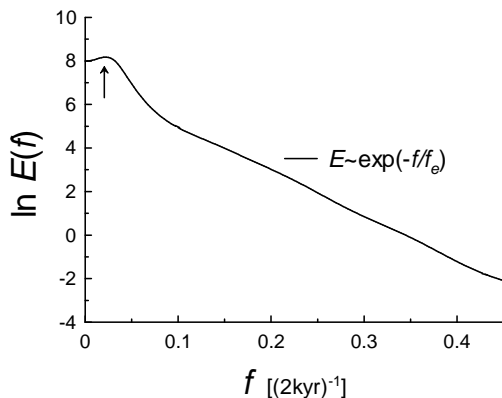


FIG. 6: Spectrum of atmospheric CO_2 fluctuations for the data shown in Fig. 5

toric variability in atmospheric CO_2 can be crucial for understanding the global climate changes at millennial timescales.

Figure 5 shows a reconstruction of atmospheric CO_2 based on deep-sea proxies, for the past 650kyr (the data taken from [37]). Resolution of the data set is 2kyr. Fluctuations with time-scales less than 2kyr could be rather large (statistically up to 308ppm [37]), but they are smoothed by the resolution. Figure 6 shows a power spectrum of the data set calculated using the maximum entropy method. The spectrum exhibits a peak indicating a periodic component (the arrow in the Fig. 6 indicates a 100kyr period) and a broad-band part with

exponential decay

$$E(f) \sim e^{-f/f_e} \quad (4)$$

A semi-logarithmic plot was used in Fig. 6 in order to show the exponential decay more clearly (at this plot the exponential decay corresponds to a straight line). Both stochastic and deterministic processes can result in the broad-band part of the spectrum, but the decay in the spectral power is different for the two cases. The exponential decay indicates that the broad-band spectrum for these data arises from a deterministic rather than a stochastic process. For a wide class of deterministic systems a broad-band spectrum with exponential decay is a generic feature of their chaotic solutions Refs. [38]-[43]. Let us recall that the discussed above Duffing oscillators can exhibit both deterministic and chaotic behavior [17]-[20],[38] depending on the parameters range.

Nature of the exponential decay of the power spectra of the chaotic systems is still an unsolved mathematical problem. A progress in solution of this problem has been achieved by the use of the analytical continuation of the equations in the complex domain (see, for instance, [43]). In this approach the exponential decay of chaotic spectrum is related to a singularity in the plane of complex time, which lies nearest to the real axis. Distance between this singularity and the real axis determines the rate of the exponential decay. For many interesting cases chaotic solutions are analytic in a finite strip around the real time axis. This takes place, for instance for attractors bounded in the real domain (the Lorentz attractor, for instance). In this case the radius of convergence of the Taylor series is also bounded (uniformly) at any real time. If parameters of the dynamical system fluctuate periodically around their mean values with period T_e , then the restriction of the Taylor series convergence (at certain conditions) is determined by the period of the parametric modulation, and the width of the analytic strip around real time axis equals $T_e/2\pi$ [10]. Let us consider, for simplicity, solution $u(t)$ with simple poles only, and to define the Fourier transform as follows

$$u(\omega) = (2\pi)^{-1/2} \int_{-T_e/2}^{T_e/2} dt e^{-i\omega t} u(t) \quad (5)$$

Then using the theorem of residues

$$u(\omega) = i(2\pi)^{1/2} \sum_j R_j \exp(i\omega x_j - |\omega y_j|) \quad (6)$$

where R_j are the poles residue and $x_j + iy_j$ are their location in the relevant half plane, one obtains asymptotic behavior of the spectrum $E(\omega) = |u(\omega)|^2$ at large ω

$$E(\omega) \sim \exp(-2|\omega y_{min}|) \quad (7)$$

where y_{min} is the imaginary part of the location of the pole which lies nearest to the real axis. Therefore, exponential decay rate of the broad-band part of the system

spectrum, Eq. (4), equals the period of the parametric forcing.

The chaotic spectrum provides two different characteristic time-scales for the system: a period corresponding to fundamental frequency of the system, T_{fun} , and a period corresponding to the exponential decay rate, $T_e = 1/f_e$ (cf. Eq. (4)). The fundamental period T_{fun} can be estimated using position of the low-frequency peak, while the exponential decay rate period $T_e = 1/f_e$ can be estimated using the slope of the straight line of the broad-band part of the spectrum in the semi-logarithmic representation (Fig. 6). From Fig. 6 we obtain $T_{fun} \simeq 95 \pm 8$ kyr (the peak is quite broad due to small data set) and $T_e \simeq 41 \pm 1$ kyr (the estimated errors are statistical ones).

Thus, the obliquity period of 41 kyr is still a dominant factor in the chaotic CO_2 fluctuations, although it is hidden for linear interpretation of the power spectrum. In the nonlinear interpretation the additional period $T_{fun} \simeq 100$ kyr may correspond to the fundamental frequency of the underlying nonlinear dynamical system and it determines the apparent 100 kyr 'periodicity' of the glaciation cycles for the last 650 kyr (cf Refs. [2],[4],[44] and references therein).

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