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Bilinear effect in complex systems

Lui Lam[1], David C. Bellavia[1], Xiao-Pu Han[2], Chih-Hui Alston Liu[1], Chang-Qing Shu[3], Zhengjin Wei[4], Tao Zhou[2,5] and Jichen Zhu[6]

[1] *Department of Physics and Astronomy, San Jose State University, San Jose, CA 95192-0106, U.S.A.*
[2] *Department of Modern Physics, University of Science and Technology of China, Hefei 230026, China*
[3] *ADACEL Systems Incorporation, 5945 Hazeltine National Drive, Orlando, FL 32822, U.S.A.*
[4] *Nanjing Municipal Museum, 4 Chao Tian Gong, Nanjing 210004, China*
[5] *Web Sciences Center, University of Electronic Science and Technology of China, Chengdu 610054, China*
[6] *School of Literature, Communication, and Culture, Georgia Institute of Technology, 686 Cherry St., Atlanta, GA 30332-0165, U.S.A.*

PACS 89.75.-k – First pacs description
PACS 89.75.Hc – Second pacs description
PACS 89.75.Fb – Third pacs description

Abstract. - A new, generic type of Zipf-plot behavior in complex systems has been discovered. This Zipf plot does not obey the power law or stretched-exponent distribution but appears as two monotonically decreasing straight lines intersecting each other. This is called the Bilinear Effect. Examples from real systems (including lifetime distributions of Chinese dynasties) are presented. A 3-layer network model that leads to the bilinear effect is given.

Introduction. – A common way to characterize and classify complex systems is through the Zipf plots [1]. Given a sequence of numbers, the corresponding Zipf plot is obtained in four steps: (i) The sequence is rearranged in a decreasing order. (ii) For numbers of the same magnitude, retain only one of them in the sequence. (iii) The largest number is assigned rank 1, the second largest rank 2, etc. (iv) The Zipf plot is the curve of number vs. rank (R). Note that as a result of the decreasing order, the Zipf plot so defined is always a monotonically decreasing curve. Since the Zipf plot can give an inverse function of the cumulative distribution from original data [2], it is widely used in the statistical analysis of small samples [3, 4].

There are two well-known types of Zipf plots: power laws [1, 2] and stretched exponents [5]. The Power law distribution has been widely observed in a large number of self-organizing systems. In the last several decades the power law distribution has attracted the attention of many scientists. It is at the center of complex systems research because of its special mathematical and dynamical properties [6–8], and physical implications [9]. In the last decade, the rise of research in the network sciences makes the power law more prominent [10, 11].

The stretched exponent distribution generally can be viewed as an intermediate form between a scaling form (power law) and homogenous types of distributions (such as Poisson distribution). It has also been widely observed in many social and material systems [12–15]. The typical forms of both the power law and stretched exponent is that of a continuous curve in a Zipf plot.

Here we present a new type of Zipf plot, in which the data fall on two straight lines (called the Bilinear Effect). Different than the two types above, the two straight lines intersect at a transition point; and most importantly, the slope of the curve is not continuous in the bilinear effect.

Bilinear effect in the lifetime of dynasties. – The first example of the bilinear effect is the distribution of lifetimes of the Chinese dynasties. The Zipf plots of these are presented in Fig. 1 (A) and (B). Both of the two sets of data range from Qin to Qing dynasty (221 B.C. to 1912 A.D.). The data for Fig. 1 (A) is obtained from Ref. [16], which includes 31 main Chinese dynasties. The data for Fig. 1 (B) is from Cihai [17], a Chinese encyclopedia; which not only includes the 31 main dynasties, but also many local powers and provisional governments, resulting in a total of 74 dynasties. These two sets of data depict similar behavior —– the bilinear effect. The transition point in these two Zipf plots is $\tau = 57 \pm 2$ years. This implies that if a Chinese dynasty survives longer than 57 years, it will have a greater chance of surviving longer,

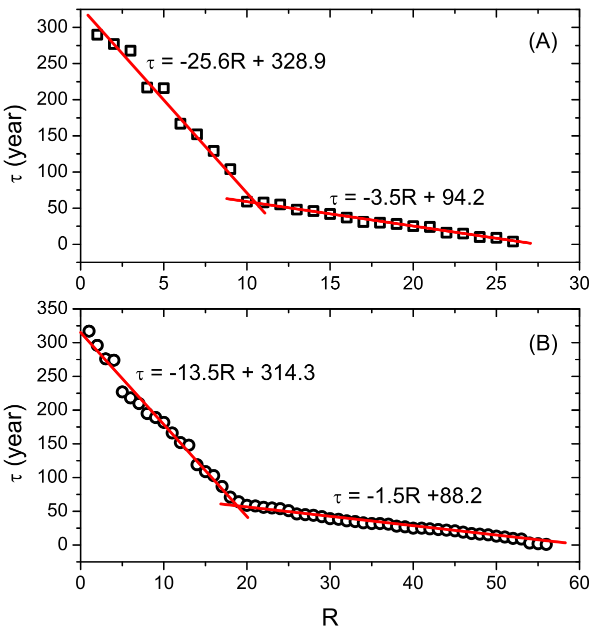

Fig. 1: (Color online) Lifetime distribution of Chinese dynasties in Zipf plot. (A) data from Ref. [16]; (B) data from Ref. [17].

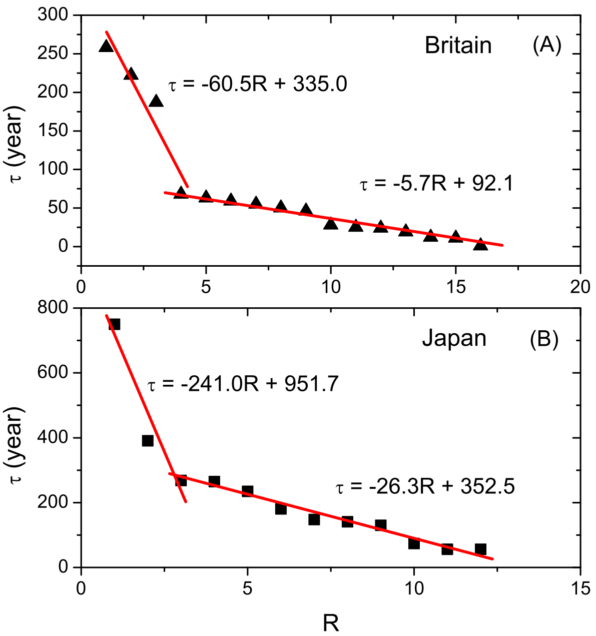

Fig. 2: (Color online) Lifetime distributions of dynasties of Britain (A) and Japan (B). Data from Ref. [16].

and the chance that it will be destroyed is sharply reduced. For example, Fig. 1(A) implies that a dynasty can survive 3.5 ± 0.1 years if it lasts 57 ± 2 years or less; beyond that, every 25.6 ± 0.1 years. In other words, the distribution of the lifetimes of Chinese dynasties is discrete, or "quantized". Moreover, this is the phenomenon that a human entity, a dynasty in this case, becomes stronger or more stable after existing for a period of time. The mere fact of survival reinforces its strength, through adaptive learning, restructuring, or other means.

In less certain terms, similar bilinear effects also can be observed in the lifetime distribution of dynasties of some other countries; Fig. 2 shows two examples. Comparatively, the number of data points for these countries are less than those of China and this is why the bilinear effect is less certain in these systems.

In the following discussion, a governmental structure giving rise to the bilinear effect is introduced.

The 3-layer network model. – Roughly speaking, the government structure of a Chinese regime in the last two thousand or so years since the Qin dynasty consists of three layers: the emperor court (the central government), the provinces, and the cities/villages. They are represented schematically by layer A, B and C, respectively, in Fig. 3 here.

Every year, the cities/villages submit part of their income, in the form of "taxes" to the upper layer, the provincial governments. And the provincial governments in turn submit a certain amount of their revenues to the emperor court, the top layer. At the same time, the emperor court maintains its control by allocating funds/resources to the governments in the middle layer as it pleases. But there will be no downward flow of resources from layer B to layer C. The communication between local governments in each layer is indirect and will be ignored in our model.

In our 3-layer network model, the upper layer A has one node. Node A is connected to all the nodes B_i ($i = 1, \cdots, N_B$) in the second layer (layer B). And each node B_i is connected to nodes C_{ij} ($j = 1, \cdots, N_C$) in the third layer (layer C). So the number of nodes on layer B and layer C, respectively, is N_B and N_C. There is no connection between the nodes in the same layer.

Let $F(B_i, t)$ be the amount of resources (or fitness) possessed by node B_i at time t; similarly $F(A, t)$ for node A, and $F(C_{ij}, t)$ for node C_{ij}. Each node will transfer part of its own resource to the other nodes that are connected to it, according to the following rules.

(i) From A to B_i: At time t, a node in layer B (k, say) is random picked and an amount $T_A(t)$ is transferred from node A to node B_k such that

$$T_A(t) = aF(B_k, t-1) \tag{1}$$

(ii) From B_i to A: An amount T_{BA} is transferred from node B_i to node A such that

$$T_{BA}(B_i, A, t) = bF(B_i, t-1); \tag{2}$$

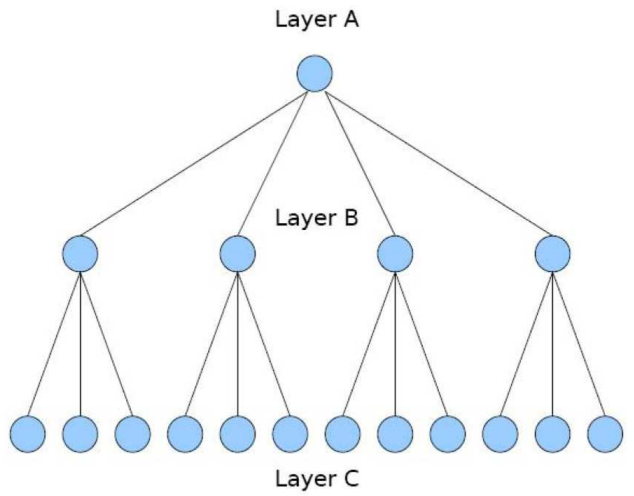

Fig. 3: (Color online) The structure of the 3-layer network model ($N_B = 4$ and $N_C = 3$).

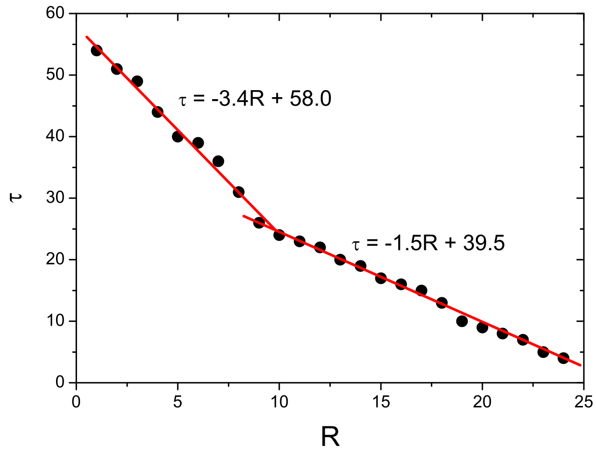

Fig. 4: (Color online) Zipf plot results from a particular pick generated by the 3-layer network model. The parameters used are: $N_B = 4$ and $N_C = 3$; $a = 0.5$, $b = 0.2$, $c = 0.3$ and $e = 0.8$. The initial conditions are: $F(A, 0) = 100$, $F(B_i, 0) = 50$ and $F(C_{ij}, 0) = 30$. It shows the bilinear effect similar to Fig. 1(A).

(iii) From C_{ij} to B_i: An amount T_{CB} is transferred from node C_{ij} to node B_i such that

$$T_{CB}(C_{ij}, B_i, t) = cF(C_{ij}, t-1). \tag{3}$$

where a, b and c are constants (each one is less than 1).

To keep itself running, node A does consume its own resources; the amount is denoted by $eF(A, t)$ with $e(< 1)$ being a constant. It follows that the time evolution of the fitness at each node is given by

(i) For node A:

$$\begin{aligned} &F(A,t) - F(A,t-1) \\ &= -T_A(t) + \textstyle\sum_i T_{BA}(B_i, A, t) - eF(A, t-1) \end{aligned} \tag{4}$$

(ii) For node B_i:

$$\begin{aligned} &F(B_i,t) - F(B_i,t-1) \\ &= T_A(t)\delta_{ik} + \textstyle\sum_j T_{CB}(C_{ij}, B_i, t) - T_{BA}(B_i, A, t) \end{aligned} \tag{5}$$

(iii) For node C_{ij}:

$$F(C_{ij}, t) - F(C_{ij}, t-1) = -T_{CB}(C_{ij}, B_i, t). \tag{6}$$

In this model, given N_B and N_C, there are four parameters (a, b, c and e).

Starting with initial fitness for the nodes, the computer run is stopped when $F(A, t) \leq 0$ for the first time (at time $t = \tau$, say), which mimics the exhaustion of the resources of the central government. The lifetime of the regime is taken to be τ.

For a set of given parameters, many computational runs of this model are performed, giving rise to the normalized probability function $p(\tau)$ —— such that $p(\tau)$ is the probability that τ is found among all the runs. A sequence of numbers, $\{\tau_i\}$ with $i = 1, 2, \cdots, N_p$, are picked according to this $p(\tau)$. The Zipf plot derived from a particular sequence so picked is depicted in Fig. 4. It obviously shows bilinear effect, indicating that the bilinear effect can indeed emerge from the government resource assignment process.

Other examples of bilinear effect. – The distribution of lifetimes of dynasties is not the only instance of the bilinear effect in complex systems. Here are two other interesting examples.

One example is the number of online votes for Chinese *Xiaopin* actors (*Xiaopin* is a popular form of short drama performed by a cast of usually two actors in China; the data is available from http://ent.sin.com.cn/2004-09-30/1050521359.html (Oct. 7, 2004)). In this vote, each voter can choose their favorite actor from a list of 30. The more number of votes an actor gets, the higher the popularity. The bilinear effect shows obviously in the plot in Fig. 5 (A). This implies these actors can be divided into two groups. In other words, the social reputation of actors could be dichotomous: if an actor can pass a critical popularity, he/she will achieve greater popularity more easily. This result contradicts the common understanding that the social reputation of people is continuously distributed.

Another example is the 2004 airline quality ratings (data available from http://www.aqr.aero/aqrreports/2005aqr.pdf), as shown in Fig. 5 (B). These examples imply that that bilinear effect could be widespread in some complex systems, especially in social systems.

Conclusions. – The bilinear effect is a new phenomenon in complex systems. The most interesting characteristic of the bilinear effect is that the samples are di-

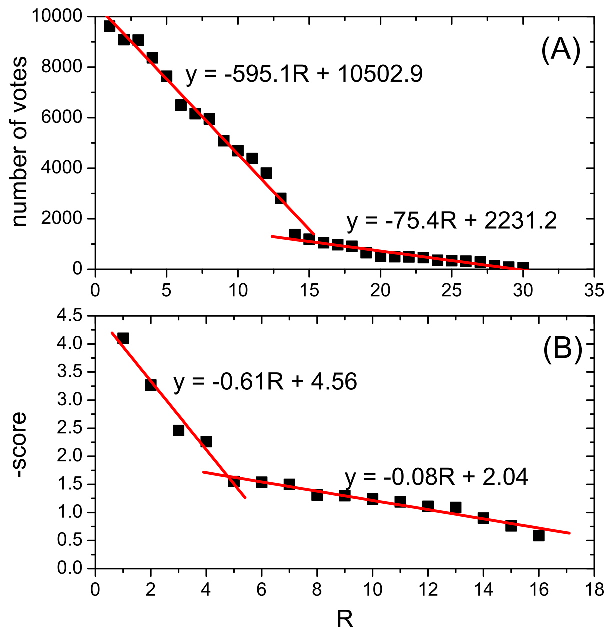

Fig. 5: (Color online) Two examples of bilinear effect. A. Online popular votes for *xiaopin* actors. B. Airline quality data. In both plots "y" in the equation corresponds to the quantity on the vertical axis.

vided into two distinct linearly distributed groups which are connected by a sharp transition point. It indicates that the statistical properties of some complex systems could be discrete. What is the meaning of the transition point? It could be an indication of a "phase transition". However, this supposition needs more empirical evidence and theoretical understanding.

Our research of the bilinear effect covers several different realms, including the lifetimes of dynasties of several countries, online votes of actors, and airline quality ratings. These empirical results imply that the bilinear effect could be a new law for complex systems with universal significance. We propose a 3-layer network model to investigate the underlying mechanisms of the bilinear effect in the lifetimes of dynasties, but it cannot explain the bilinear effect in other systems. A more sophisticated model is needed in these cases. The fact that there could be more than one mechanism in producing the bilinear effect is not that surprising. A similar case exists in the case of power laws in Zipf plots [2].

In summary, we report a new phenomenon called the bilinear effect in complex systems: Although this letter investigates a few examples, the research is just beginning. There are still many open questions that will require insightful research to understand.

* * *

XPH and TZ acknowledge the support of 973 program (2006CB705500), and the National Natural Science Foundation of China (10532060, 10635040 and 70871082).

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