Gauge Boson Theory of Quantum State Reduction

Vladimir S. MASHKEVICH[1]

Physics Department
Queens College
The City University of New York
65-30 Kissena Boulevard
Flushing
New York 11367-1519

Abstract

A theory of quantum state reduction is advanced. It is based on two principles: (1) Gauge decomposition; (2) Maximum entropy. To wit: (1) The reduction decomposition of a state vector is the Schmidt decomposition with respect to the states of a set of (dressed) gauge boson modes; (2) The reduction instant is that of the maximum entropy of a resulting mixed state. The theory determines states undergoing the reduction, its instant, resulting pure states and their probabilities. Applications: (Polarized) photon absorption and transmission, emission, particle detection, reduction of a superposition of states, nonintegral photon states, photon and matter-photon entanglement, processes with weak bosons, and the role of gluons.

[1]E-mail: Vladimir.Mashkevich100@qc.cuny.edu

Introduction and motivation

Conventionally, the quantum state reduction problem is considered in the light of the measurement problem [1-9] and reduces to the problem of reduction of a superposition of (macroscopically different) states of an apparatus. It appears, however, that it is the concept of state reduction that is primary rather than that of measurement. (In fact, this was Bell's point of view [10,11].) Thus, state reduction should be treated in its own right.

The challenge of constructing a theory of reduction involves the problems of determining (1) decomposition of states undergoing reduction, (2) its instant, (3) resulting states and (4) their probabilities; the solution to the latter two problems amounts to that of the former two.

A starting point is this. A dynamical theory of state reduction should be based on an actual interaction, which is assumed to involve gauge bosons. Following Dirac [12], consider a photon passing through a crystal of tourmaline. In the case of the photon polarized obliquely to the optic axis, unitary time evolution results in a superposition of the two states: $|\text{no photons}\rangle \otimes |\text{crystal excited}\rangle$ and $|\text{one photon polarized perpendicular to the axis}\rangle \otimes |\text{crystal unchanged}\rangle$. Thus the photon mode is entangled with the crystal, i.e., the rest of the system. The state vector has the form of a Schmidt decomposition. But in an actual experiment, the entanglement does not last: there occurs a quantum jump, which results in a disentangled state.

In general, we may assume that a gauge interaction gives rise to an entanglement of a set of gauge boson modes with the rest of the universe, and reduction causes the disentanglement. This leads us to the solution of the decomposition problem.

Had there been no disentanglement, the world would be a complete mess. It is gauge bosons that engender entanglement, so let them play a crucial role in disentanglement.

In the simplest case, like the one above, the state of a boson mode is integral (i.e., with an integral number of bosons), but nonintegral states are possible as well (for example, a coherent state of a laser mode).

Now turn to the problem of reduction instant. The reduction of a pure state results in an increase of entropy. Therefore it is natural to define the reduction instant as that of the maximum entropy increase.

1 Two basic principles

1.1 The principle of gauge decomposition

The gauge decomposition principle determines the form of the decomposition of a state vector undergoing reduction. It reads:

The reduction decomposition of a state vector is the Schmidt decomposition with respect to the states of a set of (dressed) gauge boson modes.

The decomposition is of the form

$$|\,\rangle = \sum_j c_j |\mathrm{G}j\rangle \otimes |\mathrm{R}j\rangle := \sum_j c_j |\mathrm{Gauge}j\rangle \otimes |\mathrm{Rest}j\rangle \tag{1.1.1}$$

$$\langle \mathrm{G}j\,'|\mathrm{G}j\rangle = \delta_{j\,'j} \qquad \langle \mathrm{R}j\,'|\mathrm{R}j\rangle = \delta_{j\,'j} \qquad \sum_j |c_j|^2 = 1 \tag{1.1.2}$$

The reduction of the above pure state at an instant $t = t_{\mathrm{red}}$ is this:

$$\hat{\rho}(t_{\mathrm{red}} - 0) \xrightarrow{\mathrm{red}} \hat{\rho}(t_{\mathrm{red}}) \tag{1.1.3}$$

$$\hat{\rho}(t_{\mathrm{red}} - 0) = |\ \rangle\langle\ | \tag{1.1.4}$$

$$\hat{\rho}(t_{\mathrm{red}}) = \sum_j w_j[\,|\mathrm{G}j\rangle\langle\mathrm{G}j|\,] \otimes [\,|\mathrm{R}j\rangle\langle\mathrm{R}j|\,] \qquad w_j = |c_j|^2 \tag{1.1.5}$$

i.e.,

$$|t_{\mathrm{red}} - 0\rangle \xrightarrow{\mathrm{red}} |t_{\mathrm{red}}\rangle \tag{1.1.6}$$

$$|t_{\mathrm{red}} - 0\rangle = \sum_j c_j |\mathrm{G}j\rangle \otimes |\mathrm{R}j\rangle \tag{1.1.7}$$

$$|t_{\mathrm{red}}\rangle = |\mathrm{G}j\rangle \otimes |\mathrm{R}j\rangle \qquad \text{with probability } w_j = |c_j|^2 \tag{1.1.8}$$

The reduction of a mixed state of the form

$$\hat{\rho}(t_{\mathrm{red}} - 0) = \sum_k p_k |k\rangle\langle k| \qquad \langle k|k\rangle = 1 \qquad |\langle k\,'|k\rangle| \le 1 \qquad \sum_k p_k = 1 \tag{1.1.9}$$

is determined in the following way. The Schmidt decomposition is

$$|k\rangle = \sum_{j_k} c_{kj_k} |kj_k\rangle \qquad |kj_k\rangle = |\mathrm{G}kj_k\rangle \otimes |\mathrm{R}kj_k\rangle \tag{1.1.10}$$

$$\langle \mathrm{G/R}kj\,'_k|\mathrm{G/R}kj_k\rangle = \delta_{j_k\,'j_k} \qquad |\langle \mathrm{G/R}k\,'j\,'_{k\,'}|\mathrm{G/R}kj_k\rangle| \le 1 \tag{1.1.11}$$

Now

$$\hat{\rho}(t_{\mathrm{red}} - 0) \xrightarrow{\mathrm{red}} \hat{\rho}(t_{\mathrm{red}}) = \sum_k p_k \sum_{j_k} w_{kj_k} |kj_k\rangle\langle kj_k| \qquad w_{kj_k} = |c_{kj_k}|^2 \tag{1.1.12}$$

i.e.,

$$\hat{\rho}(t_{\mathrm{red}} - 0) \xrightarrow{\mathrm{red}} |kj_k\rangle\langle kj_k| \Leftrightarrow |kj_k\rangle \qquad \text{with probability } p_k w_{kj_k} \tag{1.1.13}$$

and

$$\sum_k \sum_{j_k} p_k w_{kj_k} = \sum_k p_k = 1 \tag{1.1.14}$$

Notice that the result of the reduction is a pure state (1.1.8) or (1.1.13), respectively.

The representation (1.1.9) of the mixed state is not uniquely defined; it will be fixed in the following Subsection.

1.2 The principle of maximum entropy

The maximum entropy principle determines the reduction instant, t_{red}, and, by the same token, actual resulting states and their probabilities. It reads:

The reduction instant is that of the maximum entropy of the resulting mixed state.

Introduce

$$\hat{\rho}_{\text{red}}(t) := \sum_j w_j |j\rangle\langle j| \quad \text{or} \quad \sum_k p_k \sum_{j_k} w_{kj_k} |kj_k\rangle\langle kj_k| \tag{1.2.1}$$

in the case of the reduction of a pure (1.1.6) or mixed (1.1.13) state, respectively. The entropy

$$\sigma_{\text{red}}(t) = -\text{Tr}\{\hat{\rho}_{\text{red}}(t) \ln \hat{\rho}_{\text{red}}(t)\} \tag{1.2.2}$$

In the pure case

$$\sigma_{\text{red}}(t) = -\sum_j w_j \ln w_j \qquad w_j = w_j(t) \tag{1.2.3}$$

and t_{red} is determined by

$$\max_t \sigma_{\text{red}}(t) = \sigma_{\text{red}}(t_{\text{red}}) \qquad \frac{\mathrm{d}\sigma_{\text{red}}}{\mathrm{d}t} = 0 \tag{1.2.4}$$

In the mixed case, we introduce

$$\sigma_{\text{red max}}(t) = \max_{\{|k\rangle\}} \sigma_{\text{red}}(t) \tag{1.2.5}$$

where $\{|k\rangle\}$ is the set of vectors $|k\rangle$ in the representation of the mixed state (1.1.9); this fixes the representation. Now t_{red} is determined by

$$\max_t \sigma_{\text{red max}}(t) = \sigma_{\text{red max}}(t_{\text{red}}) \qquad \frac{\mathrm{d}\sigma_{\text{red max}}}{\mathrm{d}t} = 0 \tag{1.2.6}$$

Let us consider the pure case in more detail. We have

$$\frac{\mathrm{d}\sigma_{\text{red}}}{\mathrm{d}t} = \sum_j \frac{\partial \sigma_{\text{red}}}{\partial w_j} \frac{\mathrm{d}w_j}{\mathrm{d}t} = \sum_j (-\ln w_j - 1) \frac{\mathrm{d}w_j}{\mathrm{d}t} = -\sum_j (\ln w_j) \frac{\mathrm{d}w_j}{\mathrm{d}t} \tag{1.2.7}$$

so that

$$\sum_j (\ln w_j) \frac{\mathrm{d}w_j}{\mathrm{d}t} = 0 \qquad \text{with} \sum_j \frac{\mathrm{d}w_j}{\mathrm{d}t} = 0 \tag{1.2.8}$$

Let $j = 1, 2$. Then

$$\left[\ln\left(\frac{1}{w_1} - 1\right)\right] \frac{\mathrm{d}w_1}{\mathrm{d}t} = 0 \tag{1.2.9}$$

whence either

$$w_2 = w_1 = \frac{1}{2} \tag{1.2.10}$$

or

$$\frac{\mathrm{d}w_2}{\mathrm{d}t} = \frac{\mathrm{d}w_1}{\mathrm{d}t} = 0 \tag{1.2.11}$$

2 Some implications

2.1 Cluster noncorrelatedness

There is "a crucial physical requirement, the cluster decomposition principle, which says in effect that distant experiments yield uncorrelated results"[13]. Let us show that the principle of maximum entropy provides the noncorrelatedness of reduction in independent systems.

Let 1 and 2 be such systems and

$$| \, \rangle = |1\rangle \otimes |2\rangle \tag{2.1.1}$$

(for the sake of simplicity, we consider pure states). The Schmidt decomposition is

$$|l\rangle = \sum_{j_l} c_{j_l} |\mathrm{G}lj_l\rangle \otimes |\mathrm{R}lj_l\rangle, \; l = 1, 2 \tag{2.1.2}$$

Consider the possibility of reduction of the composite system. We have

$$| \, \rangle = \sum_{j_1 j_2} c_{j_1 j_2} |\mathrm{G}j_1 j_2\rangle \otimes |\mathrm{R}j_1 j_2\rangle \tag{2.1.3}$$

$$| \cdot j_1 j_2\rangle = |1 \cdot j_1\rangle \otimes |2 \cdot j_2\rangle \qquad c_{j_1 j_2} = c_{j_1} c_{j_2} \tag{2.1.4}$$

so that

$$| \, \rangle \xrightarrow{\mathrm{red}} |\mathrm{G}j_1 j_2\rangle \otimes |\mathrm{R}j_1 j_2\rangle \qquad \text{with probability } w_{j_1 j_2} = |c_{j_1 j_2}|^2 = |c_{j_1}|^2 |c_{j_2}|^2 =: w_{j_1} w_{j_2} \tag{2.1.5}$$

Now

$$\hat{\rho}_{\mathrm{red}} = \hat{\rho}_{1\mathrm{red}} \otimes \hat{\rho}_{2\mathrm{red}} \qquad \hat{\rho}_{l\mathrm{red}} = \sum_{j_l} w_{lj_l} |lj_l\rangle\langle lj_l| \qquad |lj_l\rangle = |\mathrm{G}lj_l\rangle \otimes |\mathrm{R}lj_l\rangle \tag{2.1.6}$$

Thus

$$\sigma_{\mathrm{red}} = \sigma_{1\mathrm{red}} + \sigma_{2\mathrm{red}} \tag{2.1.7}$$

From

$$\frac{\mathrm{d}\sigma_{\mathrm{red}}}{\mathrm{d}t} = 0 \quad \text{and} \quad \frac{\mathrm{d}\sigma_{l\mathrm{red}}}{\mathrm{d}t} \neq 0, \quad l = 1, 2, \quad t = t_{\mathrm{red}} \tag{2.1.8}$$

follows

$$\frac{\mathrm{d}\sigma_{1\mathrm{red}}}{\mathrm{d}t} \frac{\mathrm{d}\sigma_{2\mathrm{red}}}{\mathrm{d}t} < 0 \tag{2.1.9}$$

Let

$$\frac{\mathrm{d}\sigma_{1\mathrm{red}}}{\mathrm{d}t} < 0 \qquad \frac{\mathrm{d}\sigma_{2\mathrm{red}}}{\mathrm{d}t} > 0 \qquad t = t_{\mathrm{red}} \tag{2.1.10}$$

Then a reduction in the system 1 should have occurred at $t_{1\mathrm{red}} < t_{\mathrm{red}}$ when

$$\frac{\mathrm{d}\sigma_{1\mathrm{red}}}{\mathrm{d}t} = 0 \qquad t = t_{\mathrm{red}} \tag{2.1.11}$$

Thus we have

$$t_{1\mathrm{red}} < t_{\mathrm{red}} < t_{2\mathrm{red}} \tag{2.1.12}$$

and t_{red} does not correspond to any reduction.

2.2 Reduction, nonlocality, and relativity

Quantum state reduction is a nonlocal phenomenon. As for the relativistic aspect of the reduction, there are two possible points of view. On the one hand, the stated theory may be considered to be nonrelativistic. On the other hand, it is possible to assume that the reduction occurs in the cosmic reference frame, so that t (and $t_{\rm red}$) is cosmic time. It is quantum jumps that click cosmic time.

2.3 The role of fluctuations

In view of possible fluctuations, a state with $d\sigma_{\rm red}/dt \to +0$ may be unstable with respect to reduction.

3 Applications: Integral photon states

3.1 Photon passing through a tourmaline crystal

First of all, let us return to a classic example of reduction: a photon passing through a crystal of tourmaline. The initial state is

$$|t=0\rangle = [c_\perp|\mathrm{M}_\perp 1\rangle \otimes |\mathrm{M}_\| 0\rangle + c_\| |\mathrm{M}_\perp 0\rangle \otimes |\mathrm{M}_\| 1\rangle] \otimes |\mathrm{T}0\rangle \qquad c_\perp = \sin\alpha \quad c_\| = \cos\alpha \tag{3.1.1}$$

where M stands for photon mode, $\perp/\|$ for polarization, and T for tourmaline. A unitary time evolution is of the form

$$\begin{aligned} |t=0\rangle \xrightarrow{\mathrm{U}} |t\rangle &= c_\perp[\,|\mathrm{M}_\perp 1\rangle \otimes |\mathrm{M}_\| 0\rangle] \otimes |\mathrm{T}0\rangle + c_\| |\mathrm{M}_\perp 0\rangle \otimes \{\mu_1^1 |\mathrm{M}_\| 1\rangle \otimes |\mathrm{T}0\rangle + \mu_0^1 |\mathrm{M}_\| 0\rangle \otimes |\mathrm{T}1\rangle\} \\ &= [c_\perp|\mathrm{M}_\perp 1\rangle \otimes |\mathrm{M}_\| 0\rangle + c_\| \mu_1^1 |\mathrm{M}_\perp 0\rangle \otimes |\mathrm{M}_\| 1\rangle] \otimes |\mathrm{T}0\rangle + c_\| \mu_0^1 [\,|\mathrm{M}_\perp 0\rangle \otimes |\mathrm{M}_\| 0\rangle\,] \otimes |\mathrm{T}1\rangle \\ &\qquad \mu = \mu(t) \qquad |\mu_0^1|^2 + |\mu_1^1|^2 = 1 \end{aligned} \tag{3.1.2}$$

Thus for the first reduction,

$$\hat\rho^1_{\rm red}(t) = \sum_j^{0,1} w_j^1 |j\rangle\langle j| \tag{3.1.3}$$

with the states

$$|0\rangle = [\,|\mathrm{M}_\perp 0\rangle \otimes |\mathrm{M}_\| 0\rangle\,] \otimes |\mathrm{T}1\rangle \qquad |1\rangle = \frac{1}{\sqrt{w_1^1}}[c_\perp|\mathrm{M}_\perp 1\rangle \otimes |\mathrm{M}_\| 0\rangle + c_\| \mu_1^1 |\mathrm{M}_\perp 0\rangle \otimes |\mathrm{M}_\| 1\rangle\,] \otimes |\mathrm{T}0\rangle \tag{3.1.4}$$

and probabilities

$$\text{no photons}: \ w_0^1 = |c_\||^2 |\mu_0^1|^2 \qquad \text{one photon}: \ w_1^1 = |c_\perp|^2 + |c_\||^2 |\mu_1^1|^2 \tag{3.1.5}$$

Now let $t = t_{\rm red}$ and use (1.2.10), (1.2.11). If $|c_\|| \leq 1/2$, then

$$|\mu_0^1| = 1 \qquad \mu_1^1 = 0 \qquad w_0^1 = |c_\||^2 \qquad w_1^1 = |c_\perp|^2 \tag{3.1.6}$$

$$|1\rangle = [\,|\mathrm{M}_\perp 1\rangle \otimes |\mathrm{M}_\| 0\rangle\,] \otimes |\mathrm{T}0\rangle \tag{3.1.7}$$

and the first reduction is the only one.

Let now $|c_\parallel|^2 > 1/2$. In this case,

$$w_0^1 = w_1^1 = \frac{1}{2} \qquad |\mu_0^1|^2 = \frac{1}{2|c_\parallel|^2} < 1 \tag{3.1.8}$$

and the resulting one-photon state is

$$|1\rangle = \sqrt{2}[c_\perp|\mathrm{M}_\perp 1\rangle \otimes |\mathrm{M}_\parallel 0\rangle + c_\parallel \mu_1^1 |\mathrm{M}_\perp 0\rangle \otimes |\mathrm{M}_\parallel 1\rangle\,] \otimes |\mathrm{T}0\rangle \tag{3.1.9}$$

Now consider the U evolution of the one-photon state (3.1.9). If the resulting state is

$$\sqrt{2}c_\perp[\,|\mathrm{M}_\perp 1\rangle \otimes |\mathrm{M}_\parallel 0\rangle\,] \otimes |\mathrm{T}0\rangle + (1 - 2|c_\perp|^2)^{1/2}[\,|\mathrm{M}_\perp 0\rangle \otimes |\mathrm{M}_\parallel 0\rangle\,] \otimes |\mathrm{T}1\rangle \tag{3.1.10}$$

so that under the second reduction

$$w_1^2 = 2|c_\perp|^2 \tag{3.1.11}$$

then the second reduction is the last one and the total probabilities are

$$W_1 = w_1^1 w_1^2 = \frac{1}{2} \times 2|c_\perp|^2 = |c_\perp|^2 \qquad W_0 = 1 - |c_\perp|^2 = |c_\parallel|^2 \tag{3.1.12}$$

Otherwise we proceed in the same way. The final result is this:

$$\frac{1}{2^n} \le |c_\perp|^2 < \frac{1}{2^{n-1}} \qquad n \text{ reductions} \tag{3.1.13}$$

$$W_1 = w_1^1 w_1^2 \cdots w_1^n = \left(\frac{1}{2}\right)^{n-1} \left[\left(\sqrt{2}\right)^{n-1}\right]^2 |c_\perp|^2 = |c_\perp|^2 \qquad W_0 = |c_\parallel|^2 \tag{3.1.14}$$

3.2 Absorption and transmission

Let absorption and transmission factors be p_{abs} and p_{trans}, respectively. The initial state is

$$|t = 0\rangle = |\mathrm{M}1\rangle \otimes |\mathrm{A}0\rangle \tag{3.2.1}$$

where A stands for an absorbing medium. Now

$$|\mathrm{M}1\rangle \otimes |\mathrm{A}0\rangle \xrightarrow{\mathrm{U}} \mu_1^1 |\mathrm{M}1\rangle \otimes |\mathrm{A}0\rangle + \mu_0^1 |\mathrm{M}0\rangle \otimes |\mathrm{A}1\rangle \qquad |\mu_1^1|^2 + |\mu_0^1|^2 = 1 \qquad \mu = \mu(t) \tag{3.2.2}$$

so that

$$\hat{\rho}_{\mathrm{red}}^1(t) = \sum_j^{0,1} w_j^1 |j\rangle\langle j| \tag{3.2.3}$$

with the states

$$|0\rangle = |\mathrm{M}0\rangle \otimes |\mathrm{A}1\rangle \qquad |1\rangle = |\mathrm{M}1\rangle \otimes |\mathrm{A}0\rangle \tag{3.2.4}$$

and probabilities

$$w_j^1 = |\mu_j^1|^2\,,\ j = 0, 1 \tag{3.2.5}$$

The reduction is determined by (1.2.10), (1.2.11). If $p_{\rm abs} \leq 1/2$, then

$$(w_1^1)_{\max} = p_{\rm abs} \tag{3.2.6}$$

so that there occurs only one reduction, with

$$w_0^1 = p_{\rm abs} \qquad w_1^1 = p_{\rm trans} \tag{3.2.7}$$

If $p_{\rm abs} > 1/2$, then

$$w_0^1 = \frac{1}{2} \qquad w_1^1 = \frac{1}{2} \tag{3.2.8}$$

and

$$|1\rangle = |{\rm M1}\rangle \otimes |{\rm A0}\rangle \xrightarrow{\rm U} \mu_1^2 |{\rm M1}\rangle \otimes |{\rm A0}\rangle + \mu_0^2 |{\rm M0}\rangle \otimes |{\rm A1}\rangle \tag{3.2.9}$$

The final result:

$$n \text{ reductions} \qquad W_1 = \left(\frac{1}{2}\right)^{n-1} |\mu_1^n|^2 = p_{\rm trans} \qquad W_0 = p_{\rm abs} \tag{3.2.10}$$

3.3 Emission

The initial state is

$$|t=0\rangle = |{\rm M0}\rangle \otimes |{\rm Atom1}\rangle \tag{3.3.1}$$

and the U evolution is

$$|{\rm M0}\rangle \otimes |{\rm Atom1}\rangle \xrightarrow{\rm U} \mu_0^1 |{\rm M0}\rangle \otimes |{\rm Atom1}\rangle + \mu_1^1 |{\rm M1}\rangle \otimes |{\rm Atom0}\rangle \tag{3.3.2}$$

so that

$$\hat{\rho}_{\rm red}^1(t) = \sum_j^{0,1} |\mu_j^1|^2 |j\rangle\langle j| \tag{3.3.3}$$

$$|0\rangle = |{\rm M0}\rangle \otimes |{\rm Atom1}\rangle \qquad |1\rangle = |{\rm M1}\rangle \otimes |{\rm Atom0}\rangle \tag{3.3.4}$$

We have

$$\mu_0^1(t) \to 0 \qquad \mu_1^1(t) \to 1 \qquad \text{for } \ t \to \infty \tag{3.3.5}$$

From (1.2.10), (1.2.11) follows

$$W_0^n = w_0^1 w_0^2 \cdots w_0^n = \left(\frac{1}{2}\right)^n \qquad W_1^n = 1 - \left(\frac{1}{2}\right)^n \qquad n = 1, 2, \cdots \tag{3.3.6}$$

Let the U evolution be such that

$$|\mu_0^1|^2(t) = {\rm e}^{-t/\tau} \tag{3.3.7}$$

Then

$${\rm e}^{-t_{\rm red}^1/\tau} = \frac{1}{2} \qquad t_{\rm red}^1 = (\ln 2)\tau \tag{3.3.8}$$

and under the evolution with the reductions

$$W_0(t) = {\rm e}^{-t/\tau} \qquad 0 \leq t < \infty \tag{3.3.9}$$

3.4 Particle detection

Consider the detection of a particle (electron, photon, atom) via a (secondary) photon emission. The initial state is

$$|t=0\rangle = \left[\bigotimes_{s=1}^{N} |\mathrm{M}_s 0\rangle\right] \otimes \left[\sum_{s=1}^{N} c_s |\mathrm{DPs1}\rangle\right] \tag{3.4.1}$$

where DP stands for detector+particle. The unitary evolution is this:

$$\begin{aligned}|t=0\rangle &\xrightarrow{\mathrm{U}} \sum_{s=1}^{N} c_s \bigotimes_{s'}^{s'\neq s} |\mathrm{M}_{s'} 0\rangle \otimes \{\mu^1_{s0}|\mathrm{M}_s 0\rangle \otimes |\mathrm{DP}s1\rangle + \mu^1_{s1}|\mathrm{M}_s 1\rangle \otimes |\mathrm{DP}s0\rangle\} \\ &= \left[\bigotimes_{s=1}^{N} |\mathrm{M}_s 0\rangle\right] \otimes \left[\sum_{s=1}^{N} c_s \mu^1_{s0} |\mathrm{DPs1}\rangle\right] + \sum_{s=1}^{N} c_s \mu^1_{s1} \left[|\mathrm{M}_s 1\rangle \bigotimes_{s'}^{s'\neq s} |\mathrm{M}_{s'} 0\rangle\right] \otimes |\mathrm{DP}s0\rangle\end{aligned} \tag{3.4.2}$$

Thus

$$\hat{\rho}^1_{\mathrm{red}}(t) = \sum_{j=0}^{N} w^1_j |j\rangle\langle j| \tag{3.4.3}$$

with the states

$$|0\rangle = \frac{1}{\sqrt{w^1_0}} \left[\bigotimes_{s=1}^{N} |\mathrm{M}_s 0\rangle\right] \otimes \left[\sum_{s=1}^{N} c_s \mu^1_{s0} |\mathrm{DPs1}\rangle\right] \qquad |s\rangle = \left[|\mathrm{M}_s 1\rangle \bigotimes_{s'}^{s'\neq s} |\mathrm{M}_{s'} 0\rangle\right] \otimes |\mathrm{DPs0}\rangle \tag{3.4.4}$$

and the probabilities

$$w^1_0 = \sum_{s=1}^{N} |c_s|^2 |\mu^1_{s0}|^2 \qquad w^1_s = |c_s|^2 |\mu^1_{s1}|^2 \tag{3.4.5}$$

The reduction is determined by (1.2.8):

$$\sum_{j=0}^{N} (\ln w^1_j) \frac{\mathrm{d}w^1_j}{\mathrm{d}t} = 0 \qquad \text{with } \sum_{j=0}^{N} \frac{\mathrm{d}w^1_j}{\mathrm{d}t} = 0 \tag{3.4.6}$$

Consider the simplest case:

$$w^1_1 = w^1_2 = \cdots = w^1_N \qquad w^1_0 = 1 - N w^1_1 \qquad \frac{\mathrm{d}w^1_0}{\mathrm{d}t} = -N \frac{\mathrm{d}w^1_1}{\mathrm{d}t} \tag{3.4.7}$$

Now (3.4.6) boils down to

$$[\ln(1 - N w^1_1)] \left(-N \frac{\mathrm{d}w^1_1}{\mathrm{d}t}\right) + N (\ln w^1_1) \frac{\mathrm{d}w^1_1}{\mathrm{d}t} = 0 \tag{3.4.8}$$

i.e., in view of $\mathrm{d}w^1_1/\mathrm{d}t \neq 0$,

$$\ln\left(\frac{1}{w^1_1} - N\right) = 0 \qquad w^1_1 = \frac{1}{N+1} \tag{3.4.9}$$

Thus

$$w_0^1 = w_s^1 = \frac{1}{N+1}\,, \quad s = 1, 2, \cdots, N \tag{3.4.10}$$

After n reductions

$$W_0^n = \left(\frac{1}{N+1}\right)^n \qquad W_s^1 = \frac{1}{N}\left[1 - \left(\frac{1}{N+1}\right)^n\right] \tag{3.4.11}$$

3.5 The spectral line narrowing effect

It is important to note the following. In the case of emission from one source into one mode, $w_0^1 = 1/2$ (3.3.6); whereas in the case of N sources with related N modes, $w_0^1 = 1/(N+1)$. This results in the narrowing of a spectral line with increasing N.

3.6 Reduction of a superposition

Consider the reduction of a superposition of two states via interaction with a particle resulting in a photon emission. The initial state is

$$|t=0\rangle = |\mathrm{M}_s 0\rangle \otimes \left[|\mathrm{P}s\rangle \otimes \sum_{s'}^{1,2} c_{s'}|\mathrm{S}s'\rangle\right] \tag{3.6.1}$$

where P stands for particle and S for system. The unitary evolution is this:

$$\begin{aligned} |t=0\rangle \xrightarrow{\mathrm{U}} & \, c_{\bar{s}}|\mathrm{M}_s 0\rangle \otimes |\mathrm{P}s\rangle \otimes |\mathrm{S}\bar{s}\rangle + c_s\{\mu_0^1|\mathrm{M}_s 0\rangle \otimes |\mathrm{P}s\rangle \otimes |\mathrm{S}s\rangle + \mu_1^1|\mathrm{M}_s 1\rangle \otimes |\mathrm{SP}s\rangle\} \\ &= |\mathrm{M}_s 0\rangle \otimes [c_{\bar{s}}|\mathrm{P}s\rangle \otimes |\mathrm{S}\bar{s}\rangle + c_s\mu_0^1|\mathrm{P}s\rangle \otimes |\mathrm{S}s\rangle] + c_s\mu_1^1|\mathrm{M}_s 1\rangle \otimes |\mathrm{SP}s\rangle \end{aligned} \tag{3.6.2}$$

where $s = 1, 2 \Leftrightarrow \bar{s} = 2, 1$. Thus

$$\hat{\rho}_{\mathrm{red}}^1(t) = \sum_j^{0,1} w_j^1 |j\rangle\langle j| \tag{3.6.3}$$

$$|0\rangle = \frac{1}{\sqrt{w_0^1}}|\mathrm{M}_s 0\rangle \otimes [c_{\bar{s}}|\mathrm{P}s\rangle \otimes |\mathrm{S}\bar{s}\rangle + c_s\mu_0^1|\mathrm{P}s\rangle \otimes |\mathrm{S}s\rangle] \qquad |1\rangle = |\mathrm{M}_s 1\rangle \otimes |\mathrm{SP}s\rangle \tag{3.6.4}$$

$$w_0^1 = |c_{\bar{s}}|^2 + |c_s|^2|\mu_0^1|^2 \qquad w_1^1 = |c_s|^2|\mu_1^1|^2 \tag{3.6.5}$$

The subsequent treatment is based on (1.2.10), (1.2.11). If $|c_s|^2 \leq 1/2$, then

$$|\mu_1^1|^2(t_{\mathrm{red}}^1) = 1 \qquad |\mu_0^1|^2(t_{\mathrm{red}}^1) = 0 \qquad w_1^1 = |c_s|^2 \qquad w_0^1 = |c_{\bar{s}}|^2 \tag{3.6.6}$$

and there occurs only one reduction, $n_{\max} = 1$. If under the unitary evolution $|\mu_0^1|^2 = \mathrm{e}^{-t/\tau}$, then $t_{\mathrm{red}}^1 = \infty$. But due to fluctuations, $t_{\mathrm{red}}^1 < \infty$.

If $|c_s|^2 > 1/2$, then $n_{\max} > 1$. In any case, after the last reduction, the states are

$$|s\rangle = |1\rangle = |\mathrm{M}_s 1\rangle \otimes |\mathrm{SP}s\rangle \qquad |\bar{s}\rangle = |0\rangle = |\mathrm{M}_s 0\rangle \otimes |\mathrm{P}s\rangle \otimes |\mathrm{S}\bar{s}\rangle \tag{3.6.7}$$

with the probabilities

$$W_s = W_1 = |c_s|^2 \qquad W_{\bar{s}} = W_0 = |c_{\bar{s}}|^2 \tag{3.6.8}$$

We have

$$|\mu_0^1|^2(t_{\mathrm{red}}^1) = \left\{\begin{array}{ll} 0 & \text{for } |c_s|^2 \leq 1/2 \\ 1 - |c_s|^2 & \text{for } |c_s|^2 > 1/2 \end{array}\right\} < \frac{1}{2} \tag{3.6.9}$$

so that the spectral line narrowing effect takes place.

4 Applications: Nonintegral photon states

4.1 One-mode nonintegral states

A familiar example of a nonintegral photon state is that of a laser mode: a coherent state. Consider the simplest case of the formation of one-mode nonintegral states. The initial state is

$$|t=0\rangle = [\alpha_0^0|\mathrm{Atom0}\rangle + \alpha_1^0|\mathrm{Atom1}\rangle]\otimes|\mathrm{M0}\rangle \qquad \alpha_0^0\alpha_1^0 \neq 0 \tag{4.1.1}$$

and the unitary evolution is this:

$$\begin{aligned}|t=0\rangle \xrightarrow{\mathrm{U}} |t\rangle &= \alpha_0^0|\mathrm{Atom0}\rangle\otimes|\mathrm{M0}\rangle + \alpha_1^0\{\mu_0^0|\mathrm{Atom1}\rangle\otimes|\mathrm{M0}\rangle + \mu_1^0|\mathrm{Atom0}\rangle\otimes|\mathrm{M1}\rangle\}\\ &= [\alpha_0^0|\mathrm{Atom0}\rangle + \alpha_1^0\mu_0^0|\mathrm{Atom1}\rangle]\otimes|\mathrm{M0}\rangle + \alpha_1^0\mu_1^0|\mathrm{Atom0}\rangle\otimes|\mathrm{M1}\rangle\end{aligned} \tag{4.1.2}$$

This is not the Schmidt decomposition. The latter is of the form

$$|t\rangle = \sum_{j_1}^{1,2} c_{j_1}^1 \left[\sum_k^{0,1} \alpha_{j_1k}^1|\mathrm{Atom}k\rangle\right]\otimes\left[\sum_n^{0,1}\mu_{j_1n}^1|\mathrm{M}n\rangle\right] \tag{4.1.3}$$

In the case of a unitary evolution, (4.1.3) is valid for all $t \geq 0$ so that the photon mode remains entangled with the atom.

After the first reduction, the state is

$$|j_1t_{\mathrm{red}}^1\rangle = \left[\sum_k^{0,1} \alpha_{j_1k}^1|\mathrm{Atom}k\rangle\right]\otimes\left[\sum_n^{0,1}\mu_{j_1n}^1|\mathrm{M}n\rangle\right] \qquad \text{with probability } \ w_{j_1}^1 = |c_{j_1}^1|^2 \tag{4.1.4}$$

Again

$$\begin{aligned}|\mathrm{Atom0}\rangle\otimes|\mathrm{M}n\rangle &\xrightarrow{\mathrm{U}} |\mathrm{Atom0}\rangle\otimes|\mathrm{M}n\rangle\\ |\mathrm{Atom1}\rangle\otimes|\mathrm{M0}\rangle &\xrightarrow{\mathrm{U}} \mu_0^0|\mathrm{Atom1}\rangle\otimes|\mathrm{M0}\rangle + \mu_1^0|\mathrm{Atom0}\rangle\otimes|\mathrm{M1}\rangle\\ |\mathrm{Atom1}\rangle\otimes|\mathrm{M1}\rangle &\xrightarrow{\mathrm{U}} \mu_1^0|\mathrm{Atom1}\rangle\otimes|\mathrm{M1}\rangle + \mu_2^0|\mathrm{Atom0}\rangle\otimes|\mathrm{M2}\rangle\end{aligned} \tag{4.1.5}$$

so that going over to the Schmidt decomposition we obtain

$$|j_1t_{\mathrm{red}}^1\rangle \xrightarrow{\mathrm{U}} |j_1t > t_{\mathrm{red}}^1\rangle = \sum_{j_2}^{1,2} c_{j_1j_2}^2\left[\sum_k^{0,1}\alpha_{j_1j_2k}^2|\mathrm{Atom}k\rangle\right]\otimes\left[\sum_{n=0}^{2}\mu_{j_1j_2n}^2|\mathrm{M}n\rangle\right] \tag{4.1.6}$$

After the second reduction,

$$|j_1j_2t_{\mathrm{red}}^2\rangle = \left[\sum_k^{0,1}\alpha_{j_1j_2k}^2|\mathrm{Atom}k\rangle\right]\otimes\left[\sum_{n=0}^{2}\mu_{j_1j_2n}^2|\mathrm{M}n\rangle\right] \tag{4.1.7}$$

with the conditional and total probabilities

$$w_{j_1j_2}^2 = |c_{j_1j_2}^2|^2 \qquad W_{j_1j_2}^2 = w_{j_1}^1 w_{j_1j_2}^2 \tag{4.1.8}$$

After the r-th reduction,

$$|j_1j_2\cdots j_rt_{\mathrm{red}}^r\rangle = \left[\sum_k^{0,1}\alpha_{j_1j_2\cdots j_rk}^r|\mathrm{Atom}k\rangle\right]\otimes\left[\sum_{n=0}^{r}\mu_{j_1j_2\cdots j_rn}^r|\mathrm{M}n\rangle\right] \tag{4.1.9}$$

with the probabilities

$$w_{j_1j_2\cdots j_r}^r = |c_{j_1j_2\cdots j_r}^r|^2 \qquad W_{j_1j_2\cdots j_r}^r = w_{j_1}^1 w_{j_1j_2}^2\cdots w_{j_1j_2\cdots j_r}^r \tag{4.1.10}$$

4.2 Entangled pair of photons

Consider the reduction of an entangled pair of photons via absorption. The initial state is

$$|t=0\rangle = \left[\sum_{l}^{1,2} c_l |\mathrm{M}al\rangle \otimes |\mathrm{M}b\bar{l}\rangle\right] \otimes |\mathrm{R}0\rangle \qquad l=1,2 \Leftrightarrow \bar{l}=2,1 \tag{4.2.1}$$

where the energy related to the mode $|\mathrm{M}a/bl\rangle$ with the location a/b is ω_l. The unitary evolution is of the form

$$\begin{aligned}
&|\mathrm{M}al\rangle \otimes |\mathrm{M}b\bar{l}\rangle \otimes |\mathrm{R}0\rangle \xrightarrow{\mathrm{U}} \\
&\mu_{alb\bar{l}} |\mathrm{M}al\rangle \otimes |\mathrm{M}b\bar{l}\rangle \otimes |\mathrm{R}0\rangle \\
&+\mu^1_{al} |\mathrm{M}al\rangle \otimes |\mathrm{M}b0\rangle \otimes |\mathrm{R}b\bar{l}\rangle \\
&+\mu^1_{b\bar{l}} |\mathrm{M}a0\rangle \otimes |\mathrm{M}b\bar{l}\rangle \otimes |\mathrm{R}al\rangle \\
&+\mu^1_{l} |\mathrm{M}a0\rangle \otimes |\mathrm{M}b0\rangle \otimes |\mathrm{R}alb\bar{l}\rangle
\end{aligned} \tag{4.2.2}$$

so that the Schmidt decomposition is this:

$$\begin{aligned}
&|t=0\rangle \xrightarrow{\mathrm{U}} |t\rangle \\
&= \left[\sum_{l}^{1,2} c_l \mu_{alb\bar{l}} |\mathrm{M}al\rangle \otimes |\mathrm{M}b\bar{l}\rangle\right] \otimes |\mathrm{R}0\rangle \\
&+ \sum_{l}^{1,2} c_l \mu^1_{al} [\, |\mathrm{M}al\rangle \otimes |\mathrm{M}b0\rangle \,] \otimes |\mathrm{R}b\bar{l}\rangle \\
&+ \sum_{l}^{1,2} c_l \mu^1_{b\bar{l}} [\, |\mathrm{M}a0\rangle \otimes |\mathrm{M}b\bar{l}\rangle \,] \otimes |\mathrm{R}al\rangle \\
&+ [\, |\mathrm{M}a0\rangle \otimes |\mathrm{M}b0\rangle \,] \otimes \left[\sum_{l}^{1,2} c_l \mu^1_l |\mathrm{R}alb\bar{l}\rangle\right]
\end{aligned} \tag{4.2.3}$$

The states after the first reduction are:

$$|2t^1_{\mathrm{red}}\rangle := \frac{1}{\sqrt{w_2}} \left[\sum_{l}^{1,2} c_l \mu_{alb\bar{l}} |\mathrm{M}al\rangle \otimes |\mathrm{M}b\bar{l}\rangle\right] \otimes |\mathrm{R}0\rangle \tag{4.2.4}$$

$$|alt^1_{\mathrm{red}}\rangle := [\, |\mathrm{M}al\rangle \otimes |\mathrm{M}b0\rangle \,] \otimes |\mathrm{R}b\bar{l}\rangle \,,\ l=1,2 \tag{4.2.5}$$

$$|blt^1_{\mathrm{red}}\rangle := [\, |\mathrm{M}a0\rangle \otimes |\mathrm{M}bl\rangle \,] \otimes |\mathrm{R}a\bar{l}\rangle \,,\ l=1,2 \tag{4.2.6}$$

$$|0t^1_{\mathrm{red}}\rangle := \frac{1}{\sqrt{w_0}} [\, |\mathrm{M}a0\rangle \otimes |\mathrm{M}b0\rangle \,] \otimes \left[\sum_{l}^{1,2} c_l \mu^1_l |\mathrm{R}alb\bar{l}\rangle\right] \tag{4.2.7}$$

with the probabilities

$$w_2 = \sum_{l}^{1,2} |c_l|^2 |\mu^1_{alb\bar{l}}|^2 \qquad w_{al} = |c_l|^2 |\mu^1_{al}|^2 \qquad w_{bl} = |c_{\bar{l}}|^2 |\mu^1_{bl}|^2 \qquad w_0 = \sum_{l}^{1,2} |c_l|^2 |\mu^1_l|^2 \tag{4.2.8}$$

The subsequent treatment presents no special problems.

4.3 Atom-photon entanglement

Consider the reduction of an atom-photon entanglement due to photon absorption. The initial state is

$$|t=0\rangle = \sum_{l}^{1,2} c_l[\,|\mathrm{M}_l 1\rangle \otimes |\mathrm{M}_{\bar{l}}0\rangle\,] \otimes [\,|\mathrm{Atom}l\rangle \otimes |\mathrm{R}0\rangle\,] \qquad l=1,2 \Leftrightarrow \bar{l}=2,1 \tag{4.3.1}$$

We have

$$|\mathrm{M}_l 1\rangle \otimes |\mathrm{R}0\rangle \xrightarrow{\mathrm{U}} \mu^1_{l1}|\mathrm{M}_l 1\rangle \otimes |\mathrm{R}0\rangle + \mu^1_{l0}|\mathrm{M}_l 0\rangle \otimes |\mathrm{R}l\rangle \tag{4.3.2}$$

so that

$$\begin{aligned}|t=0\rangle \xrightarrow{\mathrm{U}} |t\rangle = \sum_{l}^{1,2} c_l\mu^1_{l1}[\,|\mathrm{M}_l 1\rangle \otimes |\mathrm{M}_{\bar{l}}0\rangle\,] \otimes [\,|\mathrm{Atom}l\rangle \otimes |\mathrm{R}0\rangle\,]\\ +[|\mathrm{M}_1 0\rangle \otimes |\mathrm{M}_2 0\rangle] \otimes \left[\sum_{l}^{1,2} c_l\mu^1_{l0}|\mathrm{Atom}l\rangle \otimes |\mathrm{R}l\rangle\right]\end{aligned} \tag{4.3.3}$$

Both (4.3.1) and (4.3.3) are the Schmidt decompositions.

The states after the first reduction are:

$$|lt^1_{\mathrm{red}}\rangle := [\,|\mathrm{M}_l 1\rangle \otimes |\mathrm{M}_{\bar{l}}0\rangle\,] \otimes [\,|\mathrm{Atom}l\rangle \otimes |\mathrm{R}0\rangle\,] \qquad w_l = |c_l|^2|\mu^1_{l1}|^2\,,\ \ l=1,2 \tag{4.3.4}$$

$$|0t^1_{\mathrm{red}}\rangle := \frac{1}{\sqrt{w_0}}[\,|\mathrm{M}_1 0\rangle \otimes |\mathrm{M}_2 0\rangle\,] \otimes \left[\sum_{l}^{1,2} c_l\mu^1_{l0}|\mathrm{Atom}l\rangle \otimes |\mathrm{R}l\rangle\right] \qquad w_0 = \sum_{l}^{1,2}|c_l|^2|\mu^1_{l0}|^2 \tag{4.3.5}$$

We have

$$\sigma(t=0) = -\sum_{l}^{1,2}|c_l|^2\ln|c_l|^2 \tag{4.3.6}$$

and

$$\sigma(t>0) = -\sum_{l}^{1,2} w_l\ln w_l - w_0\ln w_0 \tag{4.3.7}$$

Let

$$|\mu^1_{11}|^2 = |\mu^1_{21}|^2 =: |\mu^1_1|^2 \qquad |\mu^1_{10}|^2 = |\mu^1_{20}|^2 =: |\mu^1_0|^2 \qquad |\mu^1_1|^2 + |\mu^1_0|^2 = 1 \tag{4.3.8}$$

then

$$\sigma(t>0) = (1-|\mu^1_0|^2)\sigma(t=0) + (-|\mu^1_1|^2\ln|\mu^1_1|^2 - |\mu^1_0|^2\ln|\mu^1_0|^2) \tag{4.3.9}$$

From

$$\frac{\mathrm{d}[\sigma(t>0)]}{\mathrm{d}t} = 0 \tag{4.3.10}$$

follows

$$|\mu^1_0|^2(t^1_{\mathrm{red}}) = \frac{1}{1+\mathrm{e}^{\sigma(t>0)}} \qquad |\mu^1_1|^2(t^1_{\mathrm{red}}) = \frac{1}{1+\mathrm{e}^{-\sigma(t>0)}} \tag{4.3.11}$$

Next,

$$|lt^1_{\mathrm{red}}\rangle \xrightarrow{\mathrm{U}} \mu^1_1[\,|\mathrm{M}_l 1\rangle \otimes |\mathrm{M}_{\bar{l}}0\rangle\,] \otimes [\,|\mathrm{Atom}l\rangle \otimes |\mathrm{R}0\rangle\,] + \mu^1_0[\,|\mathrm{M}_1 0\rangle \otimes |\mathrm{M}_2 0\rangle\,] \otimes [\,|\mathrm{Atom}l\rangle \otimes |\mathrm{R}l\rangle\,] \tag{4.3.12}$$

so that after the second reduction

$$|\mu_1^1|^2(t_{\rm red}^2) = |\mu_0^1|^2(t_{\rm red}^2) = \frac{1}{2} \tag{4.3.13}$$

The subsequent treatment is trivial.

5 Applications: Weak bosons and gluons

5.1 A process with a weak boson

Now let us consider a process with an intermediate weak boson. The initial state is

$$|t=0\rangle = |{\rm in}\rangle = |{\rm P_{in}}\rangle \otimes |{\rm W0}\rangle \tag{5.1.1}$$

where P stands for particle and W for weak boson mode. The unitary evolution is this:

$$|t=0\rangle = |{\rm in}\rangle \xrightarrow{\rm U} |t\rangle = \alpha_{\rm in}|{\rm in}\rangle + \alpha_{\rm inter}|{\rm inter}\rangle + \alpha_{\rm out}|{\rm out}\rangle \tag{5.1.2}$$

where

$$|{\rm inter}\rangle = |{\rm P_{out\,1}}\rangle \otimes |{\rm W1}\rangle \qquad |{\rm out}\rangle = |{\rm P_{out\,1}}\rangle \otimes |{\rm P_{out\,2}}\rangle \otimes |{\rm P_{out\,3}}\rangle \otimes |{\rm W0}\rangle \tag{5.1.3}$$

The Schmidt decomposition is

$$|t\rangle = [\alpha_{\rm in}|{\rm in}\rangle + \alpha_{\rm out}|{\rm P_{out\,123}}\rangle] \otimes |{\rm W0}\rangle + \alpha_{\rm inter}|{\rm P_{out\,1}}\rangle \otimes |{\rm W1}\rangle \tag{5.1.4}$$

where

$$|{\rm P_{out\,123}}\rangle := |{\rm P_{out\,1}}\rangle \otimes |{\rm P_{out\,2}}\rangle \otimes |{\rm P_{out\,3}}\rangle \tag{5.1.5}$$

The states after the first reduction are these:

$$|0t_{\rm red}^1\rangle = \frac{1}{\sqrt{w_0}}[\alpha_{\rm in}|{\rm in}\rangle + \alpha_{\rm out}|{\rm P_{out\,123}}\rangle] \otimes |{\rm W0}\rangle \qquad w_0 = |\alpha_{\rm in}|^2 + |\alpha_{\rm out}|^2 =: w_{\rm in} + w_{\rm out} \tag{5.1.6}$$

$$|1t_{\rm red}^1\rangle = |{\rm P_{out\,1}}\rangle \otimes |{\rm W1}\rangle \qquad w_1 = |\alpha_{\rm inter}|^2 \tag{5.1.7}$$

The entropy is

$$\sigma(t) = -w_0 \ln w_0 - w_1 \ln w_1 = -(1-w_1)\ln(1-w_1) \tag{5.1.8}$$

and

$$\frac{{\rm d}\sigma}{{\rm d}t} = \left[\ln\left(\frac{1}{w_1} - 1\right)\right]\frac{{\rm d}w_1}{{\rm d}t} \tag{5.1.9}$$

so that either

$$w_1 = \frac{1}{2} \tag{5.1.10}$$

or

$$\frac{{\rm d}w_1}{{\rm d}t} = 0 \qquad w_1 \le \frac{1}{2} \tag{5.1.11}$$

In the case $|t^1_{\rm red}\rangle = |0t^1_{\rm red}\rangle$ the unitary evolution is determined by (5.1.2), and in the case $|t^1_{\rm red}\rangle = |1t^1_{\rm red}\rangle$ by

$$|{\rm W1}\rangle \xrightarrow{\rm U} \alpha_1|{\rm W1}\rangle + \alpha_0|{\rm P_{out123}}\rangle \otimes |{\rm W0}\rangle \qquad (5.1.12)$$

which is the Schmidt decomposition. The subsequent treatment is standard.

It should be particularly emphasized that it is the reduction that disentangles the weak boson.

Let under the unitary evolution

$$\mathrm{d}w_{\rm in} = -\lambda_{\rm in}w_{\rm in}\mathrm{d}t \qquad \mathrm{d}w_1 = \lambda_{\rm in}w_{\rm in}\mathrm{d}t - \lambda_1 w_1\mathrm{d}t \qquad (5.1.13)$$

whence

$$w_{\rm in} = \mathrm{e}^{-\lambda_{\rm in}t} \qquad w_1 = \frac{\lambda_{\rm in}}{\lambda_1 - \lambda_{\rm in}}\left(\mathrm{e}^{-\lambda_{\rm in}t} - \mathrm{e}^{-\lambda_1 t}\right) \qquad (5.1.14)$$

and

$$w_{\rm out} = 1 - w_{\rm in} - w_1 = 1 - \frac{1}{\lambda_1 - \lambda_{\rm in}}\left(\lambda_1\mathrm{e}^{-\lambda_{\rm in}t} - \lambda_{\rm in}\mathrm{e}^{-\lambda_1 t}\right) \qquad (5.1.15)$$

Now $\mathrm{d}w_1/\mathrm{d}t = 0$ results in

$$t = t_0 := \frac{\ln(\lambda_1/\lambda_{\rm in})}{\lambda_{\rm in}[(\lambda_1/\lambda_{\rm in}) - 1]} \qquad (5.1.16)$$

Introduce

$$\tau = \lambda_{\rm in} \qquad \beta = \lambda_1/\lambda_{\rm in} \qquad (5.1.17)$$

then

$$w_{\rm in} = \mathrm{e}^{-\tau} \qquad w_1 = \frac{\mathrm{e}^{-\tau}}{\beta - 1}\left[1 - \mathrm{e}^{-(\beta-1)\tau}\right] \qquad w_{\rm out} = 1 - \frac{1}{\beta - 1}\left(\beta\mathrm{e}^{-\tau} - \mathrm{e}^{-\beta\tau}\right) \qquad (5.1.18)$$

$$\tau_0 = \frac{\ln\beta}{\beta - 1} \qquad (5.1.19)$$

and

$$w_1(\tau_0) = \frac{1}{\beta}\mathrm{e}^{\ln\beta/(1-\beta)} \qquad (5.1.20)$$

Specifically,

$$\begin{aligned}
&\text{for } \beta \gg 1 \quad w_1(\tau_0) \approx \frac{1}{\beta} \ll 1 \quad \tau^1_{\rm red} = \tau_0 = \frac{\ln\beta}{\beta}\\
&\text{for } \beta \ll 1 \quad w_1(\tau_0) \approx 1 > \frac{1}{2} \quad w_1(\tau^1_{\rm red}) = \frac{1}{2} \quad \tau^1_{\rm red} \approx \ln 2\\
&\text{for } \beta = 1 \quad w_1(\tau_0) = \frac{1}{\mathrm{e}} < \frac{1}{2} \quad \tau^1_{\rm red} = \tau_0 = 1
\end{aligned} \qquad (5.1.21)$$

5.2 On the role of gluons

Finally, let us dwell on the reduction problem in nuclear decay (specifically, alpha decay). In such a problem, gauge bosons are gluons. But as long as there are no free gluons, the only reasonable conclusion is this: A reduction in a nuclear decay is due to a change of the state of gluon degrees of freedom.

Acknowledgments

I would like to thank Alex A. Lisyansky for support and Stefan V. Mashkevich for helpful discussions.

References

[1] Bernard d'Espagnat, Conceptual Foundations of Quantun Mechanics (W.A. Benjamin, Inc., 1976).

[2] Bernard d'Espagnat, In Search of Reality (Springer-Verlag, 1983).

[3] Bernard d'Espagnat, Reality and the Physicist (Cambridge University Press, 1989).

[4] Bernard d'Espagnat, Veiled Reality (Westview Press, 2003).

[5] Bernard d'Espagnat, On Physics and Pilosophy (Princeton University Press, 2006).

[6] Peter Mittelstaedt, The Interpretation of Quantum Mechanics and the Measurement Process (Cambridge University Press, 1998).

[7] P. Bush, P. Lahti, P. Mittelstaedt, The Quantum Theory of Measurement (Springer Verlag, 1996).

[8] Gennaro Auletta, Foundations and Interpretation of Quantum Mechanics (World Scientific, 2001).

[9] Roger Penrose, The Road to Reality (Alfred A. Knoff, 2005).

[10] John S. Bell, Against 'Measurement', Phys. World, 3 (1990) 33–40.

[11] John S. Bell, Are There Quantum Jumps?, Schrödinger: Centenary of a Polymath (Cambridge University Press, 1987).

[12] P.A.M. Dirac, The Principles of Quantum Mechanics (Oxford University Press, 1978).

[13] Steven Weinberg, The Quantum Theory of Fields I (Cambridge University Press, 1995).