Overview of the structural unification of quantum mechanics and relativity using the algebra of quantions

Florin Moldoveanu

*Department of Theoretical Physics, National Institute for Physics
and Nuclear Engineering, PO Box MG-6, Bucharest, Romania* [†][*]

Abstract

The purpose of this contribution is to provide an introduction for a general physics audience to the recent results of Emile Grgin that unifies quantum mechanics and relativity into the same mathematical structure. This structure is the algebra of quantions, a non-division algebra that is the natural framework for electroweak theory on curved space-time. Similar with quaternions, quantions preserve the core features of associativity and complex conjugation while giving up the unnecessarily historically biased property of division. Lack of division makes possible structural unification with relativity (one cannot upgrade the linear Minkowski space to a division algebra due to null light-cone vectors) and demands an adjustment from Born's standard interpretation of the wave function in terms of probability currents. This paper is an overview to the theory of quantions, followed by discussions and conjectures.

I. INTRODUCTION

Unification of quantum mechanics and general relativity is the main challenge of today's physics. Several approaches have been proposed so far with various degrees of success: string theory [1], loop quantum gravity [2], twistor theory [3], and non-commuting geometry [4]. The root cause of the tension between quantum mechanics and relativity stems from the difference in the underlying Lie groups and Lie algebras: unitary groups for quantum mechanics, and orthogonal groups for relativity. This paper will present some of the core results of a relatively new approach towards unification pioneered by Emile Grgin: structural unification of quantum mechanics and relativity based on the algebra of quantions [5]. This is an overview of those results aimed at presenting the material for a general physics audience.

There are several points of view that can illustrate quantions. We will start with the historical account and original justification for quantions. Basic algebraic properties will be presented. Then quantions can be described mathematically as the algebra that removes a degeneracy of the complex numbers. Next, Born's interpretation of the wave function admits an interesting geometric interpretation, and deformations of the geometry were considered in the past as a way to search for relativity and quantum mechanics unification. In quantionic physics, Born's interpretation is naturally generalized and replaced by Zovko's interpretation which leads directly to Dirac's equation and the semi-classical aspect of the electroweak theory[1]. Because electroweak physics follows as a theorem from quantionic properties, quantions are a major step towards the axiomatization of physics. Last, the open problems are considered in an extended discussion section. The author's conjectures about quantions and a possible new physics paradigm

are presented as well.

II. QUANTIONS, THE HISTORICAL PERSPECTIVE

Structural unification of quantum mechanics and relativity started with a collaboration between Emile Grgin and Aage Petersen and was rooted into Bohr's belief that the correspondence principle has more secrets to reveal. Acting on this belief, Bohr's assistant Aage Petersen in collaboration with Emile Grgin started looking at the common elements of classical and quantum mechanics. The idea was that classical and quantum mechanics shared characteristics reveal core physics features that are otherwise obscured by the (non-essential) details related to the realization of those theories in phase and Hilbert space respectively. The resulting mathematical structure called a "quantal algebra" is a unification of a Poisson algebra with a Lie-Jordan algebra, a result also obtained by other authors [6].

Quantal algebra is rooted into two postulates, or observations which can be made about classical and quantum mechanics. The first observation is that classical and quantum mechanics use two products: one symmetric and one anti-symmetric. For example, in the classical case one has the regular multiplication and the Poisson bracket. In the usual formulation of quantum mechanics, one has the anti-commutator (the Jordan product) and the commutator. In phase space, quantum mechanics is described by the cosine and sine Moyal brackets [5]. The second observation was that classical and quantum mechanics obey the so-called composability principle: any two physical systems can interact with each other. When two physical systems interact we need to preserve the original structure, meaning the symmetric and anti-symmetric products. Let us call S_1 and A_1 the symmetric and anti-symmetric products of system 1, S_2 and A_2 the corresponding products of system 2, and S_T and A_T the products of the total system. Then compos-

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† On leave.
[*]Electronic address: fmoldove@gmail.com
[1] Second quantization is not yet researched in this approach.

ability implies:

$$S_T = S_1 S_2 - a A_1 A_2 \qquad (1)$$
$$A_T = A_1 S_2 + S_1 A_2 \qquad (2)$$

where a could be 1, 0, or -1 [7].

Comparing Eqs. 1 and 2 with complex number multiplication, it is easy see that when $a = 1$ one can identify S with the real part, and A with the imaginary part of a complex number. Detail analysis reveals that $a = \hbar^2$ for quantum mechanics and $a = 0$ for classical mechanics. The $a = -1$ case would correspond to a quantum mechanics based on split-complex numbers. This case might be considered unphysical because split complex numbers (which use $j^2 = 1$) do not satisfy the spectral theorem which gives uniqueness to quantum mechanics [8]. In general, in quantum mechanics S is a product in the space of observables $\mathcal{O}$ and A is a product in the space of abstract generators $\mathcal{L}$. Hermitian matrices represent observables, while anti-hermitian matrices represent generators. Since Hermitian and anti-hermitian matrices are in one-to-one correspondence, it is tempting to postulate the equivalence of $\mathcal{O}$ and $\mathcal{L}$, but in fact this is just a natural consequence of the composability principle. In terms of interpretation of quantum mechanics, the origin of complex numbers is a very unintuitive feature of quantum mechanics. From Eqs. 1 and 2 it is easy to understand it as a structure preserving requirement under composability. Also, the naive limit $\hbar \to 0$ that is typically assumed to describing the transition from quantum to classical mechanics is replaced with the correct exact structural transition $\hbar^2 = 0$ to a nilpotent algebra.

Combining classical and quantum mechanics into a unified structure called a quantal algebra (a term coined by Peterson and Grgin), and renaming the symmetric and the anti-symmetric product as σ and α respectively, one has the following requirements:

$$(f\alpha g)\alpha h + (g\alpha h)\alpha f + (h\alpha f)\alpha g = 0 \qquad (3)$$
$$g\alpha(f\sigma h) = (g\alpha f)\sigma h + f\sigma(g\alpha h) \qquad (4)$$
$$(f\sigma g)\sigma h - f\sigma(g\sigma h) = ag\alpha(h\alpha f) \qquad (5)$$

The difference between a quantal algebra and a Lie-Jordan algebra is that a Lie-Jordan algebra has additional properties relating to its spectral properties[6]. Those properties eliminate the need for the split-complex numbers and they are not derived from the composability principle. In the following, unless we explicitly specify it, we will restrict the discussion to only the quantum mechanics case of $a = 1$.

Eq. 3 represents the usual Jacobi identity and captures the Lie part of the quantal algebra. Eq. 4 is the distribution law of the Lie over the Jordan product and can be understood in terms of infinitesimal automorphisms. Suppose that $T = I + \epsilon F\alpha$ is an infinitesimal automorphism. Then infinitesimal motions in the quantal algebra must be compatible with the algebraic product sigma: $T(f\sigma g) = (Tf)\sigma(Tg)$. This simplifies to the Leibniz identity: $F\alpha(f\sigma g) = (F\alpha f)\sigma g + f\sigma(F\alpha g)$

In general, a Jordan algebra is non-associative. Introducing the associator as a measure of non-associativity:

$$[f, g, h] = (f\sigma g)\sigma h - f\sigma(g\sigma h) \qquad (6)$$

then Eq. 5, (proposed to be called "the Petersen's identity" by Emile Grgin), can be written as [5]:

$$[f, g, h]_\sigma = ag\alpha(h\alpha f) \qquad (7)$$

In general, one can construct a mapping J between $\mathcal{O}$ (less the unit element) and $\mathcal{L}$:

$$J : \mathcal{O} \to \mathcal{L} \qquad (8)$$

such that:

$$F = Jf \qquad (9)$$

where $f \in \mathcal{O}$ and $F \in \mathcal{L}$, and:

$$f = -aJF \qquad (10)$$

which for quantum mechanics implies:

$$JJ = -I \qquad (11)$$

If one introduces a new product beta defined as:

$$f\beta g = f\sigma g + if\alpha g \qquad (12)$$

then β is an associative product. There are two ways to introduce the associative product. The typical way, (called external complexification by Grgin), follows the prescription of Eq 12. However, there is another way, (called internal complexification[2] by Grgin [5]). In this case one element of the algebra will play the role of $\sqrt{-1}$. Let us assume that $J = \sqrt{-e}$ in $\mathcal{O}$ exists. If $\mathcal{O}_\mathcal{J}$ is the centralizer of J, i.e. the set of all observables f in $\mathcal{O}$ such that $J\alpha f = 0$, then $\{\mathcal{O}_\mathcal{J}, \sigma, \alpha, e\}$ is a quantal algebra. J may, or may not exist, but if it does, J plays a unique role in the algebra, and will later introduce relativity into the quantal framework. From Eq. 1 it is easy to see that the spectral characteristics are defined only by the symmetric product (due to the choice of complex, or split complex numbers based on a). Quantum mechanics and relativity share Jordan algebra characteristics [9].

At this point, it is useful to review Lie algebras [10]and Lie groups. Lie groups are manifolds endowed with group properties. Lie algebras are associated with the tangent space of the Lie group at the identity element. Different Lie groups can share the same Lie algebra, and there are Lie algebras which do not correspond to any Lie group. There are four infinite families of "classical" simple Lie

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[2] Because internal complexification is the critical idea of quantionic research, I propose to call it the Grgin complexification.

algebras: unitary algebras $su(n + 1)$ (A series), odd orthogonal algebras $so(2n + 1)$ (B series), symplectic algebras $sp(2n)$ (C series), and even orthogonal algebras $so(2n)$ (D series). In addition to those, there are five "exceptional" simple Lie algebras: g_2, f_4, e_6, e_7, and e_8. In terms of normed division number systems over the real numbers, the orthogonal algebras correspond to real numbers $\mathbb{R}$, the unitary algebras correspond to complex numbers $\mathbb{C}$, the symplectic algebras correspond to quaternions $\mathbb{H}$, and the exceptional algebras correspond to the non-associative octonions $\mathbb{O}$, terminating the series.

One way to analyze Lie algebras is by the Cartan classification based on the Jacoby identity (Eq. 3). When one imposes the additional constraints of Eqs. 4 and 5, then one expects a restriction in terms of possible Lie algebras. Grgin identified four cases: the infinite family of unitary algebras $su(n + 1)$ and three "sporadic" orthogonal algebras: $so(3)$, $so(6)$, and $so(2, 4)$ [11, 12, 13, 14]. Since the Lie group $SO(3)$ is isomorphic with the $SU(2)$ group and $SO(6)$ is isomorphic with $SU(4)$, the only case that does not reduce itself to standard non-relativistic quantum mechanics is $so(2, 4)$. The Lie group $SO(2, 4)$ corresponds to the conformal compactification of the Minkowski space, is isomorphic with $SU(2, 2)$, and leads to Penrose's twistor theory [3]. The Lie algebra $so(2, 4)$ leads to the algebra of quantions and is the unique mathematical structure that contains both quantum mechanics (a quantal algebra) and relativity in exactly four dimensions. Since the translation group does not appear in quantionic algebra, the space is intrinsic Riemannian, and quantionic physics structurally unifies quantum mechanics with general relativity. Wolfgang Bertram identified another family of quantal algebra realization, the pseudo-unitary $u(p, q)$ algebras of indefinite signature [15]. He also pointed out that quantal algebras are basically C^* algebras with no positivity condition.

But what is the heuristic reason for using internal complexification in the first place, and why does it lead to relativity? As seen from Cartan's classification, we have only symplectic, unitary, and orthogonal algebras. A quantal algebra contains the symplectic and unitary ingredients by default because it unifies classical and quantum mechanics. Relativity requires orthogonal algebras and non divisibility. If we can obtain a generalization of complex numbers that is not isomorphic with a unitary group (which implies divisibility), then it must contain some form of orthogonal algebra with the hope that maybe relativity will somehow arise from it. For a Hermitian matrix H, one has: $Tr(H^2) > 0$ and $Tr(-I) < 0$ and therefore standard complexification does not contain generalizations of complex numbers. Only internal complexification can lead to non-unitary quantal algebras and $so(2, 4)$ is the only possible orthogonal solution. To obtain relativity, recall that we are looking only at a subset space defined by the constraint: $J\alpha f = 0$. Once J is selected, quantions are defined into a subspace of $so(2, 4)$, the centralizer space $O_J(2, 4)$. The centralizer reduces itself to a complex Minkowski space of dimensionality 8: $M_0(\mathbb{C}) = M_0 \oplus iM_0$ and any element $f \in O_J(2, 4)$ is of the form:

$$f = f_r + J\beta f_i \tag{13}$$

with f_r and f_i real. The linear space $L^{(2,4)}$ on which the group $SO(2, 4)$ acts, is a distinguished unique space, because only in this case one can define uniquely complex conjugation as a reflection that cannot be undone by continuous transformations.

A. Algebraic properties of quantions

Let us explore same basic properties of the quantions. This section will follow closely the quantionic book of Emile Grgin [5]. The first observation is that $J = \sqrt{(-e)}$ is not unique. There are an infinity of solutions of dimensionality 3 which are transitively related by the $SO(1, 3)$ group. The algebraic unit e of quantion algebra $\mathbb{D}$ is a contravariant complex four vector that defines the time direction in the local frame.

In terms of complex numbers, a quantion is a 2×2 matrix $\begin{pmatrix} z & v \\ u & w \end{pmatrix}$ with the following multiplication rule:

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} * \begin{pmatrix} z & v \\ u & w \end{pmatrix} = \begin{pmatrix} az + cu & av + cw \\ bz + du & bv + dw \end{pmatrix} \tag{14}$$

Using the Minkowski scalar product:

$$(u, v) \equiv \eta_{\mu\nu} u^\mu v^\nu \tag{15}$$

where $\eta_{\mu\nu} = diag(1, -1, -1, -1)$ and renaming the unit e as Ω, the product β is:

$$u\beta v = (\Omega, u)v + (\Omega, v)u - (u, v) - i * (\Omega \wedge u \wedge v) \tag{16}$$

where $*$ is the Hodge duality mapping.

In general, one can decompose any arbitrary quantion in the following form:

$$u = U\Omega + \overrightarrow{u} \tag{17}$$

If we introduce Π as the 3-dimensional hyperplane orthogonal to Ω, and choosing a set $\{\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}\}$ of orthonormal vectors in Π, then the multiplication table for β is:

β	Ω	$\overrightarrow{e_1}$	$\overrightarrow{e_2}$	$\overrightarrow{e_3}$
Ω	Ω	$\overrightarrow{e_1}$	$\overrightarrow{e_2}$	$\overrightarrow{e_3}$
$\overrightarrow{e_1}$	$\overrightarrow{e_1}$	Ω	$i\overrightarrow{e_3}$	$-i\overrightarrow{e_2}$
$\overrightarrow{e_2}$	$\overrightarrow{e_2}$	$-i\overrightarrow{e_3}$	Ω	$i\overrightarrow{e_1}$
$\overrightarrow{e_3}$	$\overrightarrow{e_3}$	$i\overrightarrow{e_2}$	$-i\overrightarrow{e_1}$	Ω

$$\tag{18}$$

This multiplication table is identical with the Pauli matrices multiplication table with the following identification: $(\Omega \leftrightarrow \sigma_0, \overrightarrow{e_i} \leftrightarrow \sigma_i)$. Hence, in a fixed tetrad, the

algebra of quantions can be represented by the algebra of 2×2 complex matrices. This is because the Lorenz group is isomorphic with $SL(2, \mathbb{C})$. Expressed in terms of Pauli matrices, a quantion can be written as:

$$q = q_0 I + \overrightarrow{q} . \overrightarrow{\sigma} \qquad (19)$$

This form was first studied by James Edmonds [16] in 1972.

Quaternionic multiplication table is:

$$
\begin{array}{|c||c|c|c|c|}
\hline
. & 1 & \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\
\hline\hline
1 & 1 & \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\
\hline
\overrightarrow{i} & \overrightarrow{i} & -1 & \overrightarrow{k} & -\overrightarrow{j} \\
\hline
\overrightarrow{j} & \overrightarrow{j} & -\overrightarrow{k} & -1 & \overrightarrow{i} \\
\hline
\overrightarrow{k} & \overrightarrow{k} & \overrightarrow{j} & -\overrightarrow{i} & -1 \\
\hline
\end{array}
\qquad (20)
$$

Comparing quaternions to quantions, the transformation rule between the two algebras is:

$$
\begin{aligned}
\Omega &= 1 \\
i\overrightarrow{e_1} &= \overrightarrow{i} \\
i\overrightarrow{e_2} &= \overrightarrow{j} \\
i\overrightarrow{e_3} &= \overrightarrow{k}
\end{aligned}
\qquad (21)
$$

The linear spaces of real quantions and real quaternions are different four-dimensional slices of the algebra of complex quaternions.

Given the tetrad $\{\Omega, \overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3}\}$, let us introduce the null tetrad $\{l, n, m, \overline{m}\}$ by the relations:

$$
\begin{aligned}
l &= \tfrac{1}{2}(\Omega + \overrightarrow{e_3}) \\
n &= \tfrac{1}{2}(\Omega - \overrightarrow{e_3}) \\
m &= \tfrac{1}{2}(\overrightarrow{e_1} + i\overrightarrow{e_2}) \\
\overline{m} &= \tfrac{1}{2}(\overrightarrow{e_1} - i\overrightarrow{e_2})
\end{aligned}
\qquad (22)
$$

Up to the coefficients, those are also the Newman-Penrose null tetrads [17].

The multiplication table for $\{l, n, m, \overline{m}\}$ is:

$$
\begin{array}{|c||c|c|c|c|}
\hline
\beta & l & \overline{m} & m & n \\
\hline\hline
l & l & 0 & m & 0 \\
\hline
\overline{m} & \overline{m} & 0 & n & 0 \\
\hline
m & 0 & l & 0 & m \\
\hline
n & 0 & \overline{m} & 0 & n \\
\hline
\end{array}
\qquad (23)
$$

This multiplication table was first obtained in 1882 by Benjamin Pierce [18] and was named algebra g_4.

B. Quantions: a mixed relativity and quantum mechanics object

In quantum field theory an important theorem is the CPT theorem. This theorem mixes quantum mechanics and relativity concepts. Complex conjugation and charge are properties of the quantum theory, and parity and time are relativity concepts. Since the quantionic algebra $\mathbb{D}$ is the only possible mathematical structure that structurally unifies relativity with quantum mechanics, the CPT theorem arises naturally from it via the group of discrete transformation for quantions.

A real quantion is defined as $p = \begin{pmatrix} r & z^* \\ z & s \end{pmatrix}$ where $r, s \in \mathbb{R}$ and $z \in \mathbb{C}$. Expressing r, s, and z in terms of four real variables: p_0, p_1, p_2, p_3:

$$
\begin{aligned}
r &= p_0 + p_3 \\
s &= p_0 - p_3 \\
z &= p_1 + ip_2
\end{aligned}
\qquad (24)
$$

one has:

$$(p, p) = p_0{}^2 - p_1{}^2 - p_2{}^2 - p_3{}^2 \qquad (25)$$

and

$$\begin{pmatrix} r & z^* \\ z & s \end{pmatrix}^{-1} = \frac{1}{(p, p)} \begin{pmatrix} s & -z \\ -z^* & r \end{pmatrix} \qquad (26)$$

Quantions are not a division algebra, and the real quantions that lack an inverse are the null rays in the Minkowski cone. Having an inverse is not a mandatory property in quantum mechanics. An easy way to see this is the fact that we do not divide by the wavefunctions directly. In the case of perturbation theory, Feynman diagrams, and propagators, one deforms the integration contour to avoid exactly the points where quantions do not have an inverse.

III. QUANTIONS: LIFTING A DEGENERACY OF COMPLEX NUMBERS

Quantionic algebra was originally discovered in 1882, but its properties remained unexplored for a very long time until the quantal algebra research program rediscovered them using a systematic approach. However, there is another road that leads to quantions, this time completely in the realm of mathematics. For a long time, there was a mathematical bias towards division algebras, and the reason for this was an old Hurwitz theorem that states that there are only four normed division algebras: real numbers $\mathbb{R}$, complex numbers $\mathbb{C}$, quaternions $\mathbb{H}$, and octonions $\mathbb{O}$ [19]. Probably the original appeal of the theorem stems from the restriction of the number of such algebras, as opposed to an infinite number of associative non-division algebras. However, as seen earlier, null space-time intervals do not have an inverse in quantionic algebra $\mathbb{D}$, and imposing the unnecessary division property eliminates relativity from $\mathbb{D}$, forcing us back at using complex numbers.

But the complex numbers themselves have an additional property that can be regarded as a "defect": they

have a mathematical degeneracy of algebraic and geometrical concepts which if lifted will lead uniquely to the quantionic algebra. The algebraic norm of complex numbers is defined as:

$$A(z) = zz^* \tag{27}$$

Expanding $A(z)$ in terms of the components $z = x+iy$, one has:

$$A(z) = x^2 + y^2 \tag{28}$$

Now Eq. 28 can be understood as a metric ($M(z) = x^2 + y^2$) and this is a geometric concept. Since complex numbers were introduced for their property of algebraic closure, and since the metric is the trivial Euclidean metric in two dimensions, it takes a bit of effort to see $A(z)$ and $M(z)$ as really separate concepts. However, once the separation is made, straightforward algebraic analysis will lead uniquely to the quantionic algebra $\mathbb{D}$ as the only algebra that is able to lift this degeneracy [5] and has different algebraic and geometric norms. The two norms of quantions also have a remarkable physics interpretation. The algebraic property of quantions is related to standard quantum mechanics, and the geometric property is related to relativity.

In quantionic algebra one can introduce complex conjugation ($*$) and metric dual ($\sharp$) as follows:

$$q^* = \{a^*, c^*, b^*, d^*\} \tag{29}$$
$$q^\sharp = \{d, -b, -c, a\} \tag{30}$$

where $q = \{a, b, c, d\}$.

The quantionic algebraic norm $A(q)$ is defined using standard Hermitian conjugation:

$$A(q) = q^*q = \{a^*a + b^*b, c^*a + d^*b, a^*c + b^*d, c^*c + d^*d\} \tag{31}$$

and the quantionic metric norm $M(q)$ is the determinant of the quantionic matrix:

$$M(q) = ad - bc \tag{32}$$

The inverse of a quantion is:

$$q^{-1} = \frac{q^\sharp}{M(q)} \tag{33}$$

Since $M(q)$ may be zero, quantions are not a division algebra.

Not only $A(q) \neq M(q)$ in general, but as functions they reduce an eight-dimensional quantion to a four, and a two dimensional object respectively. $M(q)$ is obviously a complex number and $A(q)$ is a real quantion because:

$$(A(q))^* = (q^*q)^* = q^*q^{**} = q^*q = A(q) \tag{34}$$

$M(q)$ maps quantions to complex numbers and non-relativistic quantum mechanics, while $A(q)$ maps quantions into Minkowski four vectors, thus extracting relativity.

By removing the algebraic-geometric degeneracy of complex numbers, quantions are the next number system in the sequence: natural numbers, real numbers, and complex numbers. Quantionic physics does not deform the Hilbert space; it only replaces complex numbers with a new number system. The unnecessary division property of complex numbers was the main hindrance in uncovering the relativity structure. Due to their uniqueness, quantions are nature's number system where a lot of physics will follow straight as mathematical theorems with no external ad-hoc justification. Another reason of calling quantions a number system is the existence of a hyperquantionic sequence. For real numbers, the Cayley-Dickson construction combines two real numbers into a complex number, four real numbers into a quaternion number, eight real numbers into an octonion number, and so forth using the powers of two. In the hyperquantionic sequence one starts with complex numbers and constructs groups of complex numbers using the powers of four.

IV. BORN AND ZOVKO INTERPRETATION OF THE WAVE FUNCTION

Standard quantum mechanics based on complex numbers consists of several parts. First, we have the Hilbert space. Then, we need to postulate space and time as concepts outside Hilbert space. Finally, we need to add Born's interpretation of the wave function and the Schrödinger equation. Generalizations of quantum mechanics were attempted to solve the unification problem. One approach is to uncover first the geometrical formulation of quantum mechanics [20]. Hilbert space is understood as a Kähler space endowed with a symplectic and a metric structure. The starting point is the Hermitian inner product decomposition into real and imaginary parts:

$$< \Phi, \Psi > = \frac{1}{2\hbar}G(\Phi, \Psi) + \frac{i}{2\hbar}\Omega(\Phi, \Psi) \tag{35}$$

with $G(\Phi, \Psi) = \Omega(\Phi, J\Psi)$, $J = G^{-1}\Omega$, and $J^2 = -1$. The space of physical states is the projective Hilbert space $CP(n) = U(n+1)/U(n) \times U(1)$ and the Schrödinger equation describes a Killing Hamiltonian flow along $CP(n)$.

A complex number $z = x + iy$ can be represented as $z = xG + y\Omega$ where $G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. We can see that from Born's interpretation, complex numbers occurs naturally in quantum mechanics but the interpretation of G and Ω have completely different meaning when compared with the complex numbers introduced as a consequence of the composability principle. This geometric approach stems from the usual quantization procedure of replacing the Poisson brackets with commutators. What this does is to augment a symplectic structure with a metric structure resulting into a Kähler

space. Born's interpretation of the function $\rho = \psi^*\psi$ as a probability density implies a positive norm which in turn guarantees a division algebra. Since quantions are not a division algebra, if one is to find deformations of quantum mechanics to obtain (structural) unification with relativity, the staring point must be the replacement of Born's interpretation with something else. In 2002, Nikola Zovko proposed a generalization of Born's interpretation. In quantionic algebra, Zovko's interpretation uses a current probability density $j = q^\dagger q$ with j being a future oriented time-like Minkowski vector. Combining quantions with Zovko's interpretation leads to Dirac and Schrödinger equations. Moreover, the Minkowski metric is fully contained within quantions and does not need to be postulated as an outside component.

So far we have discussed the main algebraic properties of quantions. As a 2×2 matrix, quantions has only the symmetries of the Lorenz group. To have equations of motions, we need to introduce additional degrees of freedom and the new structure requires the Riemannian space. Only in the flat case, derivations generate the Abelian group of translations and therefore the Poincaré group. The unique way to generalize quantions is using a sub-algebra of the 4×4 complex matrices in the following block diagonal form [21]:

$$Q = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} \tag{36}$$

where

$$A = \begin{pmatrix} z & v \\ u & w \end{pmatrix} \tag{37}$$

is a regular 2×2 quantion. This representation appears naturally from the complex number degeneracy elimination problem.

Up to a similarity transformation, Q are unique generalizations of the 2×2 quantions. The extension, called the left algebra of quantions, allows derivation, limited analyticity properties, quantion-spinor complementarity, and Dirac equation [21]. The algebra of matrices Q is a representation in terms of matrices the quantionic algebra, acts on ket column vectors, and has the $SU(2) \times U(1)$ electroweak symmetry. Associated with the left representation is a right representation which acts on bra row vectors and the left and right representations commute. The commutation property is equivalent with the associative property of quantions.

Those advanced topics are outside the scope of this introductory paper and interested readers should consult the *Structural Unification of Quantum Mechanics and Relativity* book by Emile Grgin[21].

V. DISCUSSIONS AND OPEN PROBLEMS

The author believes that structural unification of relativity and quantum mechanics is a major milestone in understanding nature because it holds the potential to support a new physics paradigm centered on the old question of physics axiomatization. Although quantionic physics is still in the early development stages with many critical questions not yet researched, quantionic physics may put the phenomenological postulates of the Standard Model on a solid axiomatic foundation and might one day become the backbone of an ultimate theory of everything challenging string theory's aspirations in this area. While Emile Grgin refrains from speculations about the future and prefers to follow the math wherever it may lead, in this section the author is free to use the glimpses and insights learned from this new research area to provide discussions, conjectures and speculations. As such, math rigor will be replaced mostly by heuristic and philosophical arguments. Existing results will be presented in a way that will support the new paradigm, and although this paradigm is inspired in part by quantionic research, it is independent from it, or from the original intention of the other cited results.

One of the major successes of quantionic physics is the fact that structural unification is only possible for a four dimensional space time obeying the Minkowski metric. Without a complete unification theory, the proof of the space-time dimensionality is incomplete, but quantionic research is a big step forward. No other unification approaches (string theory included) can claim any credible success in this area. (Outside unification approaches, the four dimensionality is singled out as the only case where Yang-Mills theories are renormalizable. Also, from the geometrical point of view, one can construct uncountably many inequivalent differential structures and have an interplay between Hodge duality and two-forms [22].) However, quantionic research is just beginning and there are many open problems. Structural unification does not offer yet answers for quantum gravity, or even for the complete Standard Model.

Non-commuting geometry [4] provides the Lagrangeian for the entire Standard Model, but the "clothes for the SM beggar" as Connes put it do not have a clear physics origin, or a provable uniqueness associated with it. (It is also possible, and probably a better explanation, that the lack of clear physics origin is only a reflection of the author's lack of understanding of non-commuting geometry.) In quantionic physics, the natural symmetry is $U_q(1) = U(1) \times SU(2)$, and determining the origin of the strong force $SU(3)$ symmetry is an open problem currently under vigorous research. Increasing the available degrees of freedom by considering $U_q(2)$ can lead to $SU(3)$, but the question becomes why stop here and not consider for example $U_q(17)$, or any arbitrarily high number. What is the distinguishing property of $SU(3)$ from quantionic perspective? Preliminary results appear to answer fully this question, but it is premature to present them here.

Another open problem is the elimination of split complex numbers. From the point of view of degeneracy, split complex numbers have a different degeneracy, that

of identity operation and of complex conjugation. They form a non-division algebra, but the concept of charge is not well defined. In this case the $SO(2,4)$ group is replaced most likely by the $SO(3,3)$ group.

In terms of quantum gravity, there are links between the $SO(2,4)$ group and loop quantum gravity [23],[24] and between twistors and string theory [25]. The major problems of general relativity such as renormalizability, singularities, and global structure do not yet get much clarification from quantionic physics. Structural unification provides some backing for the impossibility of existence of closed time-like curves (CTCs), because in this case quantionic algebra is not possible. The very point of Grgin complexification and of the linear space $L^{(2,4)}$ was to find a space where reflections cannot be undone by continuous transformations, and on a CTC space one can undo the reflections. This corresponds to a particle being created at a point, going back in time and acquiring a phase shift, and then being reabsorbed at the creation point thus loosing the phase information [26] and breaking unitarity [27].

Second quantization and spontaneous symmetry breaking are not yet researched in quantionic theory. As non-Abelian gauge theories, electroweak theory is renormalizable and the strong force is renormalizable only at high energy or small distances. Is renormalizability always satisfied in quantionic physics? Probably yes and the best way to prove it is to interpret quantionic physics as a Yang-Mills theory using $U_q(1)$. This should not be very hard since electroweak theory is a Yang-Mills quantum field theory already. Then one can use either the massless or the massive on-shell Yang-Mills renormalizability property [28] or other more laborious methods.

However, although not exceptionally hard, second quantization and obtaining (at least) the electroweak interaction are far from being a trivial task either. First, let us view quantions a as a number system and compare their internal symmetry $U_q(1)$ with the $U(1)$ symmetry of the complex numbers. QED is a far more sophisticated theory than what one might expect from the complex number symmetry alone. If quantions are simply a number system, then one would not expect a lot of physics to follow from it. Existing and preliminary unpublished results shows however that critical features of the Standard Model are naturally appearing in quantionic algebra and this looks to be more than just a mathematical coincidence. In electroweak and strong force, nature exhibits nonlinear self-interaction, and quantions are a linear algebra. The author's expectation is that composability principle (which may also include split-complex composability) and the Yang-Mills local symmetry principle would generate the full axiomatization of the Standard Model. The main thrust of Grgin's research is however in a slightly different direction. The current aim of quantionic research is to discover first all the "inherent" properties of quantionic algebra without using gauge symmetry or any other concepts outside quantions.

From the principle of local symmetry, converting the global symmetries of quantions into local gauge symmetries resulting in a quantionic Yang-Mills theory will introduce nonlinearity. The following important questions will then arise: how does the CPT quantionic property, the space-time dimensionality, and all other inherent quantionic properties survive the opposite local to global transition?

If we analyze isomorphisms between unitary and orthogonal groups, at low dimensionality, we have only four such cases:

$$
\begin{aligned}
\mathrm{U}(1) &\approx \mathrm{SO}(2) \\
\mathrm{SU}(2) &\approx \mathrm{SO}(3) \\
\mathrm{SU}(4) &\approx \mathrm{SO}(6) \\
\mathrm{SU}(2,2) &\approx \mathrm{SO}(2,4)
\end{aligned}
\tag{38}
$$

If quantizing general relativity does not take the route of quantions, then one is restricted to using one or all of the top three isomorphisms above instead. The first isomorphism is too simplistic and the $SU(4) \approx SO(6)$ does not yet appear to play any significant role in physics. The $SU(2) \approx SO(3)$ isomorphism is used by loop quantum gravity, and from the renormalizability property, if true, it is at least conceivable that we may have a dual unification problem: a canvas space-time quantization, and a matter quantization using quantions. The strong force may then arise out of the necessity of making the two unification approaches compatible. Using any of the first three isomorphism above (and in particular the $SU(2) \approx SO(3)$ isomorphism) has a major disadvantage in terms of the time problem for canonical quantum gravity [29], but $SU(2)$ is a core symmetry of quantionic physics and the link between loop quantum gravity and quantions could appear naturally [23],[24].

Standard Model has the $U(1) \times SU(2) \times SU(3)$ symmetry, and Geoffrey Dixon proposed using the algebra $\mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$[30]. From quantionic algebra, we can see that using only norm division algebras is not enough to construct the correct axiomatization of the Standard Model.

In terms of normed division algebras, one has the following isomorphisms:

$$
\begin{aligned}
\mathrm{sl}(2,\, \mathbb{R}) &\approx \mathrm{so}(2,1) \\
\mathrm{sl}(2,\, \mathbb{C}) &\approx \mathrm{so}(3,1) \\
\mathrm{sl}(2,\, \mathbb{H}) &\approx \mathrm{so}(5,1) \\
\mathrm{sl}(2,\, \mathbb{O}) &\approx \mathrm{so}(9,1)
\end{aligned}
\tag{39}
$$

Quantions are related to the second isomorphism, while the last isomorphism is related to the 10-dimensional superstring theory [1], and to supersymmetric gauge theories [31]. Given the correct space-time dimensionality predicted by $\mathrm{sl}(2,\mathbb{C})$ under the structural relativity-quantum mechanics unification, the space-time dimensionality predicted by superstring theory, and the rigidity of the algebraic structures, then the likelihood of string theory to be the correct fundamental physics theory is now much lower.

As a speculation, in the dual unification approach, maybe the space-time quantization should be done using split-complex composability, the electroweak quantization should be done using quantionic physics, and the strong force is a mixed object of both split-complex and regular composability [32], within the Pati-Salam $SU(4) \times SU(4)$ grand unification theory [33] (The AdS-CFT correspondence[34] hints towards the deep connection between gravity and strong force.). However, three serious arguments count against the dual composability approach: the von Newman uniqueness property of quantum mechanics based on complex numbers [8], the non-commuting geometry framework for the Standard Model using only complex numbers [4], and the inherent Standard Model properties contained inside quantionic algebra which is also based on elliptical composability. Also it is not clear at this point if quantizing gravity is even possible in a split-complex quantum mechanics. However, the non-uniqueness of the split-complex quantum mechanics may not be that serious of a problem on curved space-time. Even in standard quantum mechanics uniqueness is not absolute since the Unruh effect shows that the number of particles is not globally definable. Standard composability leads to elliptical quantum mechanics, while split-complex composability leads to hyperbolic quantum mechanics. Due to its unbounded nature, hyperbolic quantum mechanics may explain the inflation period before the Big Bang and the current value of the cosmological constant Λ. If this were true, then cosmology would run along the lines of Penrose's before the Big Bang ideas [35]. Expanding on those ideas, the universe is always dominated by hyperbolic composability, experiences locally a parabolic fluctuation leading to inflation and Big Bang, followed by the normal cosmic evolution. Some of the fundamental constants may incorporate the remnants of the frozen original interaction between hyperbolic and elliptic composability[3], and our universe may be only one of uncountable other universes, majority of them being uninteresting due to the lack of stable atoms[4]. Later, after all black holes evaporate and all matter decays, the original traces of the elliptical composability are erased and the cycle can start anew.

Speculation aside, the major quantionic problem is then this: can the unification of gravity and relativity be worked out completely inside quantionic physics, or are we faced with a double unification problem? Either way, quantionic renormalizability, asymptotic freedom, quark charges, split-complex composability, AdS-CFT correspondence, non-commutative geometry, Higgs mechanism, and the $U_q(2)$ symmetries are the major puzzle pieces in constructing a coherent theory of nature.

The author conjectures that quantionic physics can be proved always renormalizable. It is unclear if we may be facing in fact a double unification problem with the strong force arising out of the necessity of making the two unification approaches compatible. It is important to find out if gravity can be also quantized using split-complex composability, because the answer can decide if split-complex quantum mechanics will ever play a physical role (like the positron solutions from Dirac's equation), or it is just a mathematical dead end.

A.   Axiomatization of physics

After the Galilean revolution, physics became an experimental science. Now, with quantionic advances in unifying quantum mechanics and relativity, here is the boldest speculation of all: what if nature enjoys uniqueness in the sense that four dimensional space time, general relativity, and quantum mechanics are mandatory consequences of a hypothetical theory of everything? What if all physics can be derived mathematically without the need for experiments in a post Galilean era[5]? Since Gödel's famous incompleteness theorem[36], we know that mathematics is infinite. But how about physics? Is physics axiomatizable? This is not a new question. It was first proposed in 1900 by David Hilbert as problem six of his famous twenty three problems that should define the next century of mathematics [37]. If problem six is solvable, uniqueness results are critical. So far, the author is aware of the following uniqueness results of various strengts: quantions, time, orthogonal groups, and Hilbert space.

When considering this problem, one should consider axioms that are not mere technical postulates, like for example the definition of Hilbert space, but principles that will separate the Platonic world of abstract mathematics from the real physical world. One such postulate is the composability principle discussed above.

[3] Are all of the Standard Model physical constants derivable from fundamental principles? Some of the constants have already been "derived" from some mathematical arguments, but without a complete theory it is easy to dismiss them as "numerology". Grgin makes two arguments in favor of deriving the constants. First, algebraic methods are more powerful than traditional gauge theory methods which have only the power of dimensional analysis. Second, if the constants have some value at some point in space-time why would they not have another value at another point unless there is a mathematical necessity for their value in the first place?

[4] Because this argument is similar with the anthropic principle, it should be used only as a speculative philosophical argument at this time. The author believes that anthropic principal can be used, but only after the fact and not to make predictions.

[5] Those speculations are not new and are periodically rediscovered independently in slightly different forms by the people working in the foundational areas which by its very nature forces to consider them. Big caution has to be exercised however because if the speculation is false, its utopian appeal can have very real negative consequences in terms of pursuing other options as the history of the string theory shows [38]. This is not an argument against string theory per se, since it applies to the quantionic approach as well. It is a warning against abandoning the Galilean era too soon.

Dimensional analysis of Lie groups is a very powerful tool to prove uniqueness, and two important results were obtained in this way. First, in general relativity, if we demand that one needs to support local mathematical structures of infinite complexity (in other words a general ontology), then one necessarily obtains the orthogonal groups [39]. For ontology to be possible, orthogonal groups are required. Second, if quantum mechanics is defined as a framework of reasoning when hypothesis forms a continuum and the maximum evidence accessible through experiment is not allowed to exceed a finite upper bound, then by dimensional analysis one obtains unitary groups and the Hilbert space [40].

Orthogonal groups correspond to real numbers, and if nature were to be described by real numbers only, then EPR's definition of reality would hold: "If, without in any way disturbing a system, we can predict with certainty (...) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." [41]. Creating a universe using only orthogonal and symplectic groups might not be logically possible. If we add general relativity (a subset of the ways orthogonal and symplectic groups can be coherently combined), then there are three supporting arguments for this conclusion. Recall that in general relativity any object falling inside of a black hole will reach the end of a geodesic line in a finite time, meaning that it will be erased out of existence/ontology. While solving the information paradox is extremely hard in quantum gravity, information is guaranteed to be lost for sure in the absence of quantum mechanics. Another argument would be the existence of singularities in general relativity, but the Big Bang may have very well started from a singularity, and therefore this is a weak argument. The third argument is the existence of CTCs in general relativity which can easily lead to paradoxes [42]. (There are counter arguments along the line of the principle of minimal action which try to eliminate the initial conditions leading to paradoxes [43], but those arguments demand that free will is only an illusion, and more importantly, have unintended unphysical consequences [44]). Taking the three arguments together, unitary groups (and global hyperbolicity, because unitary groups do not solve the CTC problem) may be a necessity if nature is to be self consistent. This argument for the necessity of unitary groups is incomplete because we need to prove first the necessity of general relativity. General relativity follows from the equivalence principle, and uses mass as a fundamental concept. As a concept, mass lacks a clear mathematical and physical origin and may force us into a circular argument (the spontaneous symmetry breaking origin of mass requires unitary groups).

Let us continue the discussion by proposing two other principles: the deformability principle and the universal truth property principle.

One principle that should never be used is the anthropic principle because this principle can hide all unsolved problems in a scientific dishonest way. If we strip away the need for the existence of our universe as is, we are left with requiring just its existence (ontology must be present, but we do not specify how), and then this is a scientifically valid principle (falsifiable). To avoid confusion and to keep it consistent with the original paper where it was first mentioned [39], this principle should be called the "deformability" principle. In the original paper context, deformability meant that the local physical structure was allowed to vary freely which corresponded to the requirement that arbitrary matter distributions should be allowed. Expressed in terms free of general relativity concepts, this principle demands the support of local mathematical structures of infinite complexity which in turn imply the existence of orthogonal groups of arbitrary signature $SO(p,q)$. The existence of time, or the transition from $SO(p,q)$ to $SO(1, n-1)$ requires yet another principle: the universal truth property.

In general in mathematics, the truth value of a statement depends on the context. For example, the statement that two parallel lines never meet is true in Euclidean geometry, and false on Riemannian geometry. The mathematical meaning of truth is coded by the Tarski theorem [45] which roughly states that inside an axiomatic system, one cannot define the truth value of its own predicates. Thus, in mathematics, truth means that something is derived from axioms, while in the physical world truth is usually defined as something corresponding to reality and has a ubiquitous non-trivial (but easily overlooked) universal property. In physics, events occurring on the four dimensional event manifold are true for all observers and across all contexts. This is a remarkable property that can be shown to lead to the necessity of time as the only way to avoid self-referencing paradoxes via the Liar's paradox [46].

The original inspiration for this result was Gödel's incompleteness of arithmetic theorem, but this theorem was not used directly due to the continuous nature of the event manifold. The incompleteness theorem shows that mathematics in infinite in the sense that, at least in some cases, one can always find a new statement (or in the mathematical terminology a predicate) p, which cannot be proved or disproved within the existing axiomatic system. If the predicate is then added as a new axiom, the process can be repeated again in the extended axiomatic system. Since the new axiom can be added as either (p) or (not p), the process generates two new incompatible axiomatic systems. This process shows that the outside space and time Platonic world of mathematics is not only infinite, but also filled with contradictory axiomatic systems that cannot be organized into a coherent system.

In the physical world however (which share at least the same complexity as the mathematical world since we can discover the mathematical axioms), the universal truth property (or equivalently global consistency) leads to a constraint which manifests itself as global hyperbolicity, or time.

Gödel's proof can be translated almost one to one into

the time proof theorem with only one twist at the end because event manifolds are "decidable" on the account of the universal truth property. Event manifolds use real numbers which have a complete axiomatization and the difficult part was solving the conceptual problem of finding a mapping between the two domains: formalized arithmetic and continuous event manifolds. The conceptual mapping is as follows: predicate $\leftrightarrow$ event, proof $\leftrightarrow$ affine transformation, gödelization of proofs (assigning numbers to proofs) $\leftrightarrow$ parameterization of initial conditions, diagonal argument $\leftrightarrow$ self-interaction. Once the conceptual problem is solved (which in hindsight looks obvious and trivial, but was particularly hard to discover in the first place), straightforward technical details follow easily. In the end of the proof, unlike Gödel, we are forced to conclude that we do have inconsistency instead of incompleteness for non-globally hyperbolic event manifolds. Only globally hyperbolic event manifolds save the physical world from contradictions.

The proof can also be expressed in terms of time travel and CTC spaces. Here the Liar's paradox is typically presented as the grandfather paradox when one goes back in time to kill his own grandfather and thus is preventing his own birth. Proponents of time travel use quantum (and sometimes classical) mechanics as a justification for Novikov's principle [47] which forbids all self-referencing paradoxes. The justification is that there are no self-referencing paradoxes in classical or quantum mechanics (see also the no clone theorem of quantum mechanics [48]) due to the symplectic structure contained in both of them. (In phase space for example, closed self-consistent evolution loops do exist.) However, analyzing its consequences, this principle leads to infinities [44]. Gödel's incompleteness proof relies on multiplication to perform the " gödelization of proofs", while the necessity of time proof relies on the ability to have initial conditions leading to paradoxes (or the ability to extend the local parameterization of the initial conditions into globally inconsistent self-interaction). To the extent that this is not possible due to the symplectic structure of either classical or quantum mechanics, the infinities mentioned above play a critical role in completing the proof[6]. The impossibility of CTC spaces does not provide a mechanism for which the creation of wormholes or CTC spaces is impossible. This remains an important open problem to be solved under a complete unified theory.

The last (conceptual) problems in a hypothetical theory of everything are the problem of free will and the total information of the universe. In a totally deterministic universe, free will is just an illusion while in a completely chaotic universe, there is no controllability and hence free will does not exist as well. Chaitin's algorithmic information theory shows that "if one has ten pounds of axioms and a twenty-pound theorem, then that theorem cannot be derived from those axioms" as Chaitin puts it [49]. So why do we have free will and how come we can discover the infinite world of mathematics if physics is truly axiomatizable using only a handful of axioms?

The composability principle may provide the answer to both of those questions. Here are the extremely high level heuristic arguments.

First, free will is equivalent with the ability to set the orientation of physics detectors [50] (for example the spin orientation of an electron does not have a definite value before measurement) which corresponds to the ability to split a composed system (or generators and observables for a single particle) into sub-systems in an arbitrary fashion.

Second, in terms of information, quantum mechanics is equivalent with having a continuous set of possibilities and only a finite set of answers. The total information of a system able to be decomposed in an infinite number of ways is infinite and this is why our infinitely complex universe can exist and mathematicians can continue to discover new mathematical axioms. As a speculation, in conjunction to this, hyperbolic quantum mechanics and Penrose's before the Big Bang ideas may explain the low entropy at the time of the Big Bang and the arrow of time.

B. Uniqueness of structural unification

Structural unification of non-relativistic quantum mechanics and relativity requires changes on either quantum mechanics or relativity. Quantions take the algebraic route of changing quantum mechanics by removing the unnecessary division property. On the geometric route, adding division to relativity is impossible because it would either contradict the experimental evidence (a Galilean era argument), or will violate the universal truth property (a post-Galilean era argument). Therefore the geometric route to structural unification is not allowed. Only within the boundaries of the more "complicated" Hilbert space (relative to the Lorenz metric signature), the non-relativistic quantum mechanics can be changed (see Eq. 13 and its similarity with complex numbers). Another route on the geometric track is the conformal compactification of the Minkowski space (using the $SO(2,4)$ group). In this case, a null ray is mapped to a point, and a point on the Minkowski space becomes a Riemann sphere transforming geometry into complex numbers. As seen from quantionic research, the $SO(2,4)$ group expands the quantionic centralizer, and while the twistor space possesses shared relativity and quantum mechanics characteristics, this unification goes outside the regular Hilbert space and special relativity.

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[6] The proof of the necessity of time did not originate from an attempt to prove the impossibility of time travel, but it naturally led there. On this (easier) track, all that remained to be proven was the rejection of Novikov's principle. Since this principle is sometimes understood as a tautology, the way to reject it was not to prove it wrong directly, but to analyze its consequences [44].

In the original derivation of quantions [12] a mistake was uncovered [15] and as a result the strength of the uniqueness result was weakened. The only open problem at this point is to seek a stronger proof of uniqueness of structural unification which would include large dimensionality and this is currently under active research.

C. Discussion summary

Quantionic research is the latest attempt in constructing a unified physics theory. The $SU(3)$ symmetry research effort in the quantionic program is the most active area right now and holds the promise of unexpected new insights. In particular, it may lead to the correct grand unification theory (GUT), if nature does indeed have one. Second quantization, renormalizability, nonlinear self-interaction, spontaneous symmetry breaking, the links with non-commuting geometry, canonical quantum gravity, and string theory are not yet researched in this new approach. If composability is to be taken seriously[7] and physics is axiomatizable, then either hyperbolic quantum mechanics is a real physical phenomenon which may explain the positive cosmological constant, or some other fundamental principle is yet to be discovered.

In the end, following the mathematics will lead us into the right direction, but at this point, the author offers the following conjectures: quantionic physics will be proved always renormalizable[8]. The second (not so novel) con-

jecture is that physics is axiomatizable and Hilbert's sixth problem is completely solvable. Three new physics principles which clearly separate the real physical world from the Platonic world of mathematics were identified so far: composability from quantionic research, deformability principle from Lie group dimensional analysis, and the universal truth property from the impossibility of closed timelike curves. In addition to the Yang-Mills local gauge symmetry principle, they may completely explain the Standard Model and may possibly lead to GUT and quantum gravity[9].

Lacking the critical mass evidence to make it a conjecture but presented as a speculation backed by indirect supporting results, split-complex hyperbolic quantum mechanics may get to play an actual physical role in the gravity unification problem and in the strong force. Two core results are needed to settle this question: research the hyperbolic equivalent of quantions, and the feasibility of using split-complex numbers into a modified diffeomorphism invariant quantum gravity theory.

VI. ACKNOWLEDGMENTS

[7] Elliptic quantum mechanics and parabolic classical mechanics are real phenomena. Is hyperbolic quantum mechanics a real phenomenon as well?

[8] Because gravity is not renormalizable, this may force us to consider a dual unification problem.

[9] It is unclear if Hamilton's principle is really needed since Dirac's equation can be obtained directly from quantions. In a field theory context we may be forced to start from a Lagrangian.

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