Different classes of quantum gates entanglers

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Abstract

We construct quantum gates entanglers for different classes of multipartite states. In particular we construct entangler operators for W and GHZ classes of multipartite states based on the construction of the concurrence classes. We also in detail discuss these two classes of the quantum gates entanglers for three-partite states.

1 Introduction

Entangled states are very important building blocks of fault-tolerant quantum computer. In general, a quantum computer is build using quantum circuit containing wires and elementary quantum gates. For an entangled based implementation of the quantum computer one also needs to construct quantum gates entanglers. It is also important that these quantum gates entanglers are able to produce any desire class of multipartite entangled quantum states. Recently, L. Kauffman and S. Lomonaco have constructed topological quantum gate entangler for two-qubit state [1]. These topological operators are called braiding operators that can entangle quantum states. These operators are also unitary solution of quantum Yang-Baxter equation. We have also recently construct quantum gate entangler for multi-qubit states based on a topological and geometrical method [2]. In particular, we have constructed unitary operators that can entangle multi-qubit quantum states if they satisfy the completely separability condition defined by the ideal of the Segre embedding. In this paper, we will construct quantum gates entanglers for different classes of multipartite states based on the definition of W and GHZ concurrence classes which is also similar to the construction of these unitary operators [3, 4]. In section 2 we will give an introduction to the construction of Artin braid group and Yang-Baxter equation. In section 3 we review the basic construction of concurrence classes based on orthogonal complement of positive operator valued measure (POVM) on quantum phase. Next, in section 4 which is also the main part of this paper, we will construct quantum gates entanglers for different classes of entangled multipartite states. These operators can entangle a specific class of entangled multipartite state if they satisfy separability conditions for W and GHZ class of states. Finally, in section 5 we visualize these classes of quantum gates entanglers for three-partite states. For example, we will explicitly define the W class and GHZ class quantum gates entanglers for such a quantum state.

2 Quantum gate entangler based on Artin braiding operator

In this section we will study relation between topological and quantum entanglement by investigating the unitary representation of Artin braid group. Here are some general references on quantum group and low-dimensional topology [5, 6]. The Artin braid group B_n on n strands is generated by $\{b_n : 1 \leq i \leq n-1\}$. The generators of the group B_n satisfy the following relations

i) $b_i b_j = b_j b_i$ for $|i-j| \geq 2$ and

ii) $b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1}$ for $1 \leq i < n$.

Each generator or product of generators admits an interesting n-strand graphical presentations. We can also find a matrix representation of the group B_n in the resulting space $\mathcal{V}^{\otimes m} = \mathcal{V} \otimes \mathcal{V} \otimes \cdots \otimes \mathcal{V}$, where $\mathcal{V}$ is a complex vector space. Now, for two strands braid there is associated an operator $R : \mathcal{V} \otimes \mathcal{V} \longrightarrow \mathcal{V} \otimes \mathcal{V}$. Moreover, let $\mathcal{I}$ be the identity operator on $\mathcal{V}$. Then, the quantum Yang-Baxter equation is defined by

$$(R \otimes \mathcal{I})(\mathcal{I} \otimes R)(R \otimes \mathcal{I}) = (\mathcal{I} \otimes R)(R \otimes \mathcal{I})(\mathcal{I} \otimes R). \tag{2.0.1}$$

The Yang-Baxter equation represents the fundamental topological relation in the Artin braid group. The inverse to R will be associated with the reverse elementary braid on two strands. Next, we define a representation τ of the Artin braid group to the automorphism of $\mathcal{V}^{\otimes m}$ by

$$\tau(b_i) = \mathcal{I} \otimes \cdots \otimes \mathcal{I} \otimes R \otimes \mathcal{I} \otimes \cdots \otimes \mathcal{I}, \tag{2.0.2}$$

where R act on $\mathcal{V}_i \otimes \mathcal{V}_{i+1}$. This equation describes a representation of the braid group if R satisfies the Yang-Baxter equation and is also invertible. Moreover, this representation of braid group is unitary if R is also unitary operator. Thus R being unitary indicated that this operator can performs topological entanglement and it also can be considers as quantum gate. It has been show in [1] that R can also perform quantum entanglement by acting on qubits states. An associative unital algebra A with homomorphism $\Delta : A \longrightarrow A \otimes A$ is called co-multiplication and also co-associative, that is for all $a \in A$ we have

$$(\mathcal{I} \otimes \Delta)(\Delta(a)) = (\Delta \otimes \mathcal{I})(\Delta(a)). \tag{2.0.3}$$

We can also construct another co-product $\Delta^{'} = \sigma \circ \Delta$, where, σ is a permutation defined by $\sigma(a_1 \otimes a_2) = \sigma(a_2 \otimes a_1)$. Next, let $\mathcal{R} \in A$ such that $(\Delta^{'}(a)) = \mathcal{R}(\Delta(a))\mathcal{R}^{-1}$. The Hopf algebra A is called quasi-triangular if

$$(\Delta \otimes \mathcal{I})(\mathcal{R}) = \mathcal{R}_{13}\mathcal{R}_{23}, \text{ and } (\mathcal{I} \otimes \Delta)(\mathcal{R}) = \mathcal{R}_{13}\mathcal{R}_{12}, \tag{2.0.4}$$

where $\mathcal{R}_{12} = \sum_i a_i \otimes b_i \otimes \mathcal{I}$, $\mathcal{R}_{12} = \sum_i \mathcal{I} \otimes a_i \otimes b_i$, and $\mathcal{R}_{13} = \sum_i a_i \otimes \mathcal{I} \otimes b_i$. Moreover, R is called a universal $\mathcal{R}$-matrix and satisfies the following relation

$$\mathcal{R}_{12}\mathcal{R}_{13}R_{23} = \mathcal{R}_{23}\mathcal{R}_{13}\mathcal{R}_{12}. \tag{2.0.5}$$

This equation implies that the matrix $R = \Pi\mathcal{R}$, where Π is a permutation operator satisfies quantum Yang-Baxter equation 2.0.1.

3 Concurrence classes for general multipartite states

In this section we will review the construction of concurrence classes based on orthogonal complement of a POVM on quantum phase. Let us denote a general, multipartite quantum system with m subsystems by $\mathcal{Q} = \mathcal{Q}_m(N_1, N_2, \ldots, N_m)$ $= \mathcal{Q}_1\mathcal{Q}_2 \cdots \mathcal{Q}_m$, consisting of a state

$$|\Psi\rangle = \sum_{j_1=1}^{N_1} \sum_{j_2=1}^{N_2} \cdots \sum_{j_m=1}^{N_m} \alpha_{j_1 j_2 \cdots j_m} |j_1 j_2 \cdots j_m\rangle \tag{3.0.6}$$

and, let $\rho_{\mathcal{Q}} = \sum_{n=1}^{\mathrm{N}} p_n |\Psi_n\rangle\langle\Psi_n|$, for all $0 \leq p_n \leq 1$ and $\sum_{n=1}^{\mathrm{N}} p_n = 1$, denote a density operator acting on the Hilbert space $\mathcal{H}_{\mathcal{Q}} = \mathcal{H}_{\mathcal{Q}_1} \otimes \mathcal{H}_{\mathcal{Q}_2} \otimes \cdots \otimes \mathcal{H}_{\mathcal{Q}_m}$, where the dimension of the jth Hilbert space is given by $N_j = \dim(\mathcal{H}_{\mathcal{Q}_j})$. Moreover, let us introduce a complex conjugation operator $\mathcal{C}_m$ that acts on a general state $|\Psi\rangle$ of a multipartite state as $\mathcal{C}_m|\Psi\rangle = \sum_{j_1=1}^{N_1} \sum_{j_2=1}^{N_2} \cdots \sum_{j_m=1}^{N_m} \alpha^*_{j_1 j_2 \cdots j_m} |j_1 j_2 \cdots j_m\rangle$. The density operator $\rho_{\mathcal{Q}}$ is said to be fully separable, which we will denote by $\rho^{sep}_{\mathcal{Q}}$, with respect to the Hilbert space decomposition, if it can be written as $\rho^{sep}_{\mathcal{Q}} = \sum_{n=1}^{\mathrm{N}} p_n \bigotimes_{j=1}^{m} \rho^n_{\mathcal{Q}_j}$, $\sum_{n=1}^{\mathrm{N}} p_n = 1$, for some positive integer N, where p_n are positive real numbers and $\rho^n_{\mathcal{Q}_j}$ denote a density operator on Hilbert space $\mathcal{H}_{\mathcal{Q}_j}$. If $\rho^p_{\mathcal{Q}}$ represents a pure state, then the quantum system is fully separable if $\rho^p_{\mathcal{Q}}$ can be written as $\rho^{sep}_{\mathcal{Q}} = \bigotimes_{j=1}^{m} \rho_{\mathcal{Q}_j}$, where $\rho_{\mathcal{Q}_j}$ is a density operator on $\mathcal{H}_{\mathcal{Q}_j}$. If a state is not separable, then it is called an entangled state.

Now, a general and symmetric POVM in a single N_j-dimensional Hilbert space $\mathcal{H}_{\mathcal{Q}_j}$ is given by

$$\Delta(\varphi_{1_j,2_j}, \ldots, \varphi_{1_j,N_j}, \varphi_{2_j,3_j}, \ldots, \varphi_{N_j-1,N_j}) = \sum_{l_j}^{N_j} \sum_{k_j=1}^{N_j} e^{i\varphi_{k_j,l_j}} |k_j\rangle\langle l_j|, \tag{3.0.7}$$

where $|k_j\rangle$ and $|l_j\rangle$ are the basis vectors in $\mathcal{H}_{\mathcal{Q}_j}$ and the quantum phases satisfy the following relation $\varphi_{k_j,l_j} = -\varphi_{l_j,k_j}(1 - \delta_{k_j l_j})$, see Ref. [7]. The POVM is a function of the $N_j(N_j-1)/2$ phases $(\varphi_{1_j,2_j}, \ldots, \varphi_{1_j,N_j}, \varphi_{2_j,3_j}, \ldots, \varphi_{N_j-1,N_j})$. It is now possible to form a POVM of a multipartite system by simply forming the tensor product

$$\Delta_{\mathcal{Q}}(\varphi_{k_1,l_1}, \ldots, \varphi_{k_m,l_m}) = \Delta_{\mathcal{Q}_1}(\varphi_{k_1,l_1}) \otimes \cdots \otimes \Delta_{\mathcal{Q}_m}(\varphi_{k_m,l_m}),$$

where, e.g., φ_{k_1,l_1} is the set of POVMs phase associated with subsystems $\mathcal{Q}_1$, for all $k_1, l_1 = 1, 2, \ldots, N_1$, where we need only to consider when $l_1 > k_1$. Let us now construct concurrence classes for general multipartite states $\mathcal{Q}_m(N_1, \ldots, N_m)$. In order to simplify our presentation, we will use $\Omega_m = \varphi_{k_1,l_1}, \ldots, \varphi_{k_m,l_m}$ as an abstract multi-index notation. The unique structure of our POVM enables us to distinguish different classes of multipartite states, which are inequivalent under LOCC operations. In the m-partite case, the off-diagonal elements of the matrix corresponding to

$$\widetilde{\Delta}_{\mathcal{Q}}(\Omega_m) = \widetilde{\Delta}_{\mathcal{Q}_1}(\varphi_{k_1,l_1}) \otimes \cdots \otimes \widetilde{\Delta}_{\mathcal{Q}_m}(\varphi_{k_m,l_m}), \tag{3.0.8}$$

have phases that are sum or differences of phases originating from two and m subsystems. That is, in the later case the phases of $\widetilde{\Delta}_{\mathcal{Q}}(\varphi_{k_1,l_1}, \ldots, \varphi_{k_m,l_m})$ take the form $(\varphi_{k_1,l_1} \pm \varphi_{k_2,l_2} \pm \ldots \pm \varphi_{k_m,l_m})$ and identification of these joint phases makes our classification possible. Thus, we can define linear operators for the $EPR_{\mathcal{Q}_{r_1}\mathcal{Q}_{r_2}}$ class based on our POVM which are sum and difference of phases of two subsystems, i.e., $(\varphi_{k_{r_1},l_{r_1}} \pm \varphi_{k_{r_2},l_{r_2}})$. That is, for the $EPR_{\mathcal{Q}_{r_1}\mathcal{Q}_{r_2}}$ class we have

$$\begin{aligned}\widetilde{\Delta}^{EPR}_{\mathcal{Q}_{r_1,r_2}}(\Omega_m) &= \mathcal{I}_{N_1} \otimes \cdots \otimes \widetilde{\Delta}_{\mathcal{Q}_{r_1}}(\varphi^{\frac{\pi}{2}}_{k_{r_1},l_{r_1}}) \otimes \cdots \otimes \widetilde{\Delta}_{\mathcal{Q}_{r_2}}(\varphi^{\frac{\pi}{2}}_{k_{r_2},l_{r_2}}) \\ &\otimes \cdots \otimes \mathcal{I}_{N_m}. \end{aligned} \tag{3.0.9}$$

For the GHZ^m class, we define the linear operators based on our POVM which are sum and difference of phases of m-subsystems, i.e., $(\varphi_{k_{r_1},l_{r_1}} \pm \varphi_{k_{r_2},l_{r_2}} \pm \ldots \pm \varphi_{k_m,l_m})$. That is, for the GHZ^m class we have

$$\begin{aligned}\widetilde{\Delta}^{GHZ^m}_{\mathcal{Q}_{r_1,r_2}}(\Omega_m) &= \widetilde{\Delta}_{\mathcal{Q}_{r_1}}(\varphi^{\frac{\pi}{2}}_{k_{r_1},l_{r_1}}) \otimes \widetilde{\Delta}_{\mathcal{Q}_{r_2}}(\varphi^{\frac{\pi}{2}}_{k_{r_2},l_{r_2}}) \otimes \\ &\widetilde{\Delta}_{\mathcal{Q}_{r_3}}(\varphi^{\pi}_{k_{r_3},l_{r_3}}) \otimes \cdots \otimes \widetilde{\Delta}_{\mathcal{Q}_m}(\varphi^{\pi}_{k_{r_m},l_{r_m}}), \end{aligned} \tag{3.0.10}$$

where by choosing $\varphi^{\pi}_{k_j,l_j} = \pi$ for all $k_j < l_j,\ j = 1, 2, \ldots, m$, we get an operator which has the structure of Pauli operator σ_x embedded in a higher-dimensional Hilbert space and coincides with σ_x for a single-qubit. There are $C(m, 2)$ linear operators for the GHZ^m class and the set of these operators gives the GHZ^m class concurrence.

4 Multipartite quantum gate entangler

In this section we will construct quantum gates entanglers for W and GHZ classes of multipartite quantum states. Our construction is based on definition of W and GHZ concurrence classes for general multipartite states [3, 4]. Let us consider quantum system $\mathcal{Q}^p_m(N, N, \ldots, N)$, where $N_1 = N_2 = \cdots = N_m = N = 2$ and p indicates that we consider a quantum system in pure state. Then, for a multipartite state a topological unitary transformation $\mathcal{R}_{N^m\times N^m}$ that create multipartite entangled state is defined by $\mathcal{R}_{N^m\times N^m} = \mathcal{R}^d_{N^m\times N^m} + \mathcal{R}^{ad}_{N^m\times N^m}$, where

$$\mathcal{R}^a_{N^m\times N^m} = \mathrm{diag}(\alpha_{11\cdots 1}, 0, \ldots, 0, \alpha_{NN\cdots N}) \tag{4.0.11}$$

is a diagonal matrix and

$$\mathcal{R}^{ad}_{N^m\times N^m} = \mathrm{antidiag}(0, \alpha_{NN\cdots N-1}, \alpha_{NN\cdots N-1N}, \ldots, \alpha_{1\cdots 21}, \alpha_{1\cdots 12}, 0) \tag{4.0.12}$$

is an anti-diagonal matrix. Now, we have the following proposition for multipartite states.

Proposition 4.0.1 *If elements of $\mathcal{R}_{N^m\times N^m}$ satisfy*

$$\langle\Psi|\widetilde{\Delta}^{EPR}_{\mathcal{Q}_{r_1,r_2}}(\Omega_m)\mathcal{C}_m\Psi\rangle \neq 0$$

then the state $\mathcal{R}_{N^m\times N^m}(|\psi\rangle \otimes |\psi\rangle \otimes \cdots \otimes |\psi\rangle)$, with $|\psi\rangle = |1\rangle + |2\rangle + \ldots + |N\rangle$ is a EPR or W class entangled.

The proof follows from the construction of $\mathcal{R}_{N^m\times N^m}$ which is based on separable elements of multipartite states given by EPR class of operator presented in [3, 4].

Proposition 4.0.2 *If elements of $\mathcal{R}_{N^m\times N^m}$ satisfy*

$$\langle\Psi|\widetilde{\Delta}^{GHZ^m}_{\mathcal{Q}_{r_1,r_2}}(\Omega_m)\mathcal{C}_m\Psi\rangle \neq 0 \tag{4.0.13}$$

then the state $\mathcal{R}_{N^m\times N^m}(|\psi\rangle\otimes|\psi\rangle\otimes\cdots\otimes|\psi\rangle)$, with $|\psi\rangle = |1\rangle+|2\rangle+\ldots+|N\rangle$ is a GHZ class entangled.

The proof follows from the construction of $\mathcal{R}_{N^m\times N^m}$ which is based on separable elements of multipartite states defined by GHZ class of operator [3, 4]. Note that this operator is a quantum gate entangler since

$$\begin{aligned}\tau_{N^m\times N^m} &= \mathcal{R}_{N^m\times N^m}\mathcal{P}_{N^m\times N^m} \\ &= \operatorname{diag}(\alpha_{1\cdots 11},\alpha_{1\cdots 12},\ldots,\alpha_{N\cdots NN})\end{aligned} \tag{4.0.14}$$

is a $N^m\times N^m$ phase gate and $\mathcal{P}_{N^m\times N^m}$ is $N^m\times N^m$ a "swap gate". However, we need to imposed some constraints on the parameters $\alpha_{k_1k_2\cdots k_m}$ to ensure that our $\mathcal{R}$ is unitary. This is our main result and in following section we will illustrate it by some examples.

5 Quantum gate entangler for three-partite states

In this section, we will construct a quantum gate entangler for three-partite quantum systems. Let us consider quantum system $\mathcal{Q}^p_m(N,N,N)$, where $N_1 = N_2 = N_3 = N = 2$. Then, for a general three-partite state a topological unitary transformation $\mathcal{R}_{N^3\times N^3}$ that create multipartite entangled state is defined by $\mathcal{R}_{N^3\times N^3} = \mathcal{R}^d_{N^3\times N^3} + \mathcal{R}^{ad}_{N^3\times N^3}$, where

$$\mathcal{R}^a_{N^3\times N^3} = \operatorname{diag}(\alpha_{111},0,\ldots,0,\alpha_{222}) \tag{5.0.15}$$

is a diagonal matrix and

$$\mathcal{R}^{ad}_{N^3\times N^3} = \operatorname{antidiag}(0,\alpha_{221},\alpha_{212},\ldots,\alpha_{121},\alpha_{112},0) \tag{5.0.16}$$

is an anti-diagonal matrix. Moreover, for three-partite states we have the following EPR class operators

$$\widetilde{\Delta}^{EPR}_{\mathcal{Q}_{1,2}}(\Omega_3) \;=\; \widetilde{\Delta}_{\mathcal{Q}_1}(\varphi^{\frac{\pi}{2}}_{k_1,l_1})\otimes\widetilde{\Delta}_{\mathcal{Q}_2}(\varphi^{\frac{\pi}{2}}_{k_2,l_2})\otimes\mathcal{I}_{N_3}.$$

$\widetilde{\Delta}^{EPR}_{\mathcal{Q}_{1,3}}(\Omega_3)$ and $\widetilde{\Delta}^{EPR}_{\mathcal{Q}_{2,3}}(\Omega_3)$ are defined in a similar way. Now, based on proposition 4.0.1, If elements of $\mathcal{R}_{N^3\times N^3}$ satisfies

$$\langle\Psi|\widetilde{\Delta}^{EPR}_{\mathcal{Q}_{1,2}}(\Omega_3)\mathcal{C}_3\Psi\rangle = \sum^{N_1}_{l_1>k_1=1}\sum^{N_2}_{l_2>k_2=1}\{\sum^{N_3}_{k_3=l_3=1}(\alpha_{k_1l_2k_3}\alpha_{l_1k_2l_3}-\alpha_{k_1k_2k_3}\alpha_{l_1l_2l_3})\}\neq 0, \tag{5.0.17}$$

$$\langle\Psi|\widetilde{\Delta}^{EPR}_{\mathcal{Q}_{1,3}}(\Omega_3)\mathcal{C}_3\Psi\rangle = \sum^{N_1}_{l_1>k_1=1}\sum^{N_3}_{l_3>k_3=1}\{\sum^{N_2}_{k_2=l_2=1}(\alpha_{k_1k_2l_3}\alpha_{l_1l_2k_3}-\alpha_{k_1k_2k_3}\alpha_{l_1l_2l_3})\}\neq 0, \tag{5.0.18}$$

$$\langle\Psi|\widetilde{\Delta}^{EPR}_{\mathcal{Q}_{2,3}}(\Omega_3)\mathcal{C}_3\Psi\rangle = \sum^{N_2}_{l_2>k_2=1}\sum^{N_3}_{l_3>k_3=1}\{\sum^{N_1}_{k_1=l_1=1}(\alpha_{k_1k_2l_3}\alpha_{l_1l_2k_3}-\alpha_{k_1k_2k_3}\alpha_{l_1l_2l_3})\}\neq 0, \tag{5.0.19}$$

then the state $\mathcal{R}_{N^3\times N^3}(|\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle)$, with $|\psi\rangle = |1\rangle + |2\rangle$ is a W class entangled.

The second class of three-partite state that we would like to consider is the GHZ^3 class. For this class, we have

$$\widetilde{\Delta}^{GHZ^3}_{\mathcal{Q}_{1,2}}(\Omega_3) \quad = \quad \widetilde{\Delta}_{\mathcal{Q}_1}(\varphi^{\frac{\pi}{2}}_{k_1,l_1}) \otimes \widetilde{\Delta}_{\mathcal{Q}_2}(\varphi^{\frac{\pi}{2}}_{k_2,l_2}) \otimes \widetilde{\Delta}_{\mathcal{Q}_3}(\varphi^{\pi}_{k_3,l_3}).$$

$\widetilde{\Delta}^{GHZ^3}_{\mathcal{Q}_{1,3}(\Omega_3)}$ and $\widetilde{\Delta}^{GHZ^3}_{\mathcal{Q}_{2,3}}(\Omega_3)$ are defined in a similar way. Now, based on proposition 4.0.2, If elements of $\mathcal{R}_{N^3\times N^3}$ satisfies

$$\langle\Psi|\widetilde{\Delta}^{GHZ^3}_{\mathcal{Q}_{1,2}}(\Omega_3)\mathcal{C}_3\Psi\rangle = \sum^{N_1}_{l_1>k_1=1}\sum^{N_2}_{l_2>k_2=1}\sum^{N_3}_{l_3>k_3=1} \tag{5.0.20}$$
$$\{\alpha_{k_1l_2l_3}\alpha_{l_1k_2k_3} + \alpha_{k_1l_2k_3}\alpha_{l_1k_2l_3} - \alpha_{k_1k_2,l_3}\alpha_{l_1l_2k_3} - \alpha_{k_1k_2,k_3}\alpha_{l_1l_2l_3}\} \neq 0,$$

$$\langle\Psi|\widetilde{\Delta}^{GHZ^3}_{\mathcal{Q}_{1,3}}(\Omega_3)\mathcal{C}_3\Psi\rangle = \sum^{N_1}_{l_1>k_1=1}\sum^{N_2}_{l_2>k_2=1}\sum^{N_3}_{l_3>k_3=1} \tag{5.0.21}$$
$$\{\alpha_{k_1l_2l_3}\alpha_{l_1k_2k_3} - \alpha_{k_1l_2k_3}\alpha_{l_1k_2l_3} + \alpha_{k_1k_2l_3}\alpha_{l_1l_2k_3} - \alpha_{k_1k_2k_3}\alpha_{l_1l_2l_3}\} \neq 0,$$

$$\langle\Psi|\widetilde{\Delta}^{GHZ^3}_{\mathcal{Q}_{2,3}}(\Omega_3)\mathcal{C}_3\Psi\rangle = \sum^{N_1}_{l_1>k_1=1}\sum^{N_2}_{l_2>k_2=1}\sum^{N_3}_{l_3>k_3=1} \tag{5.0.22}$$
$$\{-\alpha_{k_1l_2l_3}\alpha_{l_1k_2k_3} + \alpha_{k_1l_2k_3}\alpha_{l_1k_2l_3} + \alpha_{k_1k_2l_3}\alpha_{l_1l_2k_3} - \alpha_{k_1k_2k_3}\alpha_{l_1l_2l_3}\} \neq 0,$$

then the state $\mathcal{R}_{N^3\times N^3}(|\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle)$, with $|\psi\rangle = |1\rangle + |2\rangle$ is a GHZ class entangled. Thus, we have constructed two different classes of quantum gates entanglers for multipartite states based on construction of the concurrence. This result is the first step toward the construction of quantum gates entangler for specific classes of multipartite entangled states. These set of operators are very important for design of entangled based quantum computer and needs further investigation.

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