Is magnetic flux quantized inside a solenoid?

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Abstract

In some textbooks on quantum mechanics, the description of flux quantization in a superconductor ring based on the Aharonov-Bohm effect may lead some readers to a (wrong) conclusion that flux quantization occurs as well for a long solenoid with the same quantization condition in which the charge of cooper pair $2e$ is replaced by the charge of one electron e. It is shown how this confusion arises and how can one avoid it.

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1 Introduction

Aharonov-Bohm effect is a quantum mechanical phenomena in which a charged particle in a region free of magnetic field is affected by the vector potential which produces that magnetic field [1]. One immediate result of this effect is to explain the magnetic flux quantization in a superconductor ring. This is done based on the uniqueness of the wave function after a 2π rotation around the direction of magnetic field. One may usually think that such a description is also applied for a magnetic field trapped inside a long solenoid so that the corresponding flux is quantized as well as superconductor case. However, it is easily shown that in case of such quantization in a long solenoid applied in the Aharonov-Bohm effect, we would not observe the shift of interference pattern in this effect. In this short article, we try to understand the reasons for flux quantization in the superconductor ring and flux non-quantization in a long solenoid.

2 Aharonov-Bohm effect

The Schrodinger equation for a charged particle e in the presence of an electromagnetic field $(\mathbf{A}, \phi)$ is given by

$$\frac{1}{2m}(-i\hbar\nabla - \frac{e}{c}\mathbf{A})^2\psi + e\phi\psi = E\psi. \tag{1}$$

This equation is invariant under gauge transformation

$$\mathbf{A} \rightarrow \mathbf{A} + \nabla\lambda, \tag{2}$$

where λ is an arbitrary function of space and time. This invariance causes the wave function to acquire an extra phase, in passing along a path through a region free of magnetic field, according to

$$\phi = \frac{e}{\hbar c}\int_p \mathbf{A}.\mathbf{dx}. \tag{3}$$

The phase difference between the two cases where the charged particle is moved along two different pathes with the same endpoints encompassing the magnetic flux Φ is given by

$$\Delta\phi = \frac{e}{\hbar c}\Phi. \tag{4}$$

One may observe this phase difference by locating a long solenoid having a variable magnetic flux between the two slits in the two slits experiment. In fact, the relative phase difference of the wave functions at a given point on the screen when the particle (electron) passes the first or the second slit depends on the magnetic flux encompassed by the long solenoid and any change in the flux results in a shift in the interference pattern which is observable as the Aharonov-Bohm effect.

3 Flux quantization in a superconductor

Consider a ring of superconductor containing a trapped magnetic flux with a constant number of flux lines. The Schrodinger equation for a unit charged particle, namely the cooper pair $2e$

is the same as (1) with e replaced by $2e$. The wave function of this pair after a 2π rotation around the direction of magnetic field inside the ring is given by

$$\phi = \frac{2e}{\hbar c}\int_p \mathbf{A}.\mathbf{dx} = \frac{2e}{\hbar c}\Phi. \tag{5}$$

The uniqueness of wave function then requires this phase to be an integer multiplications of 2π so that the magnetic flux is quantized as follows

$$\Phi = \frac{2\pi\hbar c}{2e}n, \quad n = 0, \pm 1, \pm 2, ... \tag{6}$$

In other words, the numbers of trapped flux lines can just be integer multiplications of $\frac{2\pi\hbar c}{2e}$. Flux quantization in a superconductor was experimentally observed in 1961 by Deaver and Fairbank [2].

4 Flux non-quantization in a long solenoid

In some standard textbooks on quantum mechanics [3] the subject of flux quantization and Aharonov-Bohm effect are so vaguely presented that one may mistakenly realizes the flux quantization is a specific property of any magnetic flux such as the one inside a long solenoid, whereas this is not the case and flux quantization occurs just inside a superconductor ring. On the other hand, in some other books the flux non-quantization in a long solenoid is explicitly mentioned without a detailed explanation [4].

Consider a two slit experiment with a long solenoid, having a magnetic flux Φ, located between the two slits. At a given point on the screen, the superposition of two wave functions of the charged particle received by two slits $1, 2$ in the presence of the long solenoid is given by

$$\psi = (\psi_1 e^{ie\Phi/\hbar c} + \psi_2) e^{ie/\hbar c \int_2 \mathbf{A}.\mathbf{dx}}. \tag{7}$$

The magnetic flux Φ is then responsible for a relative phase difference between the two wave components received by two slits $1, 2$. This relative phase may shift the interference patter and this is experimentally observed. It is immediately seen that if the magnetic flux would be quantized according to

$$\Phi = \frac{2\pi\hbar c}{e}n, \quad n = 0, \pm 1, \pm 2, ... \tag{8}$$

the above phase difference would be vanished and no shift in the interference patter would be observed, a result which is in sharp contrast with the observation. Therefore, we conclude that flux quantization does not occur inside a long solenoid. However, a question is immediately raised: Why is that the uniqueness of wave function of the charged particle e after a 2π rotation around the long solenoid does not lead to the flux quantization (8)?

5 Simply and non-simply connectedness

The wave function of the cooper pair inside a superconductor ring should vanish on the interior walls of the ring and it simply means that there is no cooper pair outside the ring, especially in the interior hole encompassing the magnetic flux. Since there is no wave function outside the ring then the closed path integral around the magnetic flux is limited and confined to the interior region of the ring where there are cooper pairs. This means the space is non-simply connected so that one can not continuously contract the closed integral to zero. Therefore, the phase acquired by the wave function in this case is a topological one and the uniqueness condition of wave function leads the magnetic flux to be quantized according to (6).

In the case of a long solenoid, however, there is no limitation or confinement on the closed integral taken over a closed path around the solenoid because there is no limitation on the presence of electron in the region of magnetic flux inside the solenoid. This is because, contrary to the superconductor ring there is no boundary condition imposed on the wave function of an electron. In principle, after a 2π rotation of an electron around the long solenoid one can continuously contract the resultant closed integral (total phase) to zero with no difficulty, because the space in this case is simply connected, namely the electron in principle can be present every where in space. Therefore, the total phase is no longer a topological phase and so one can not expect a quantization condition (8), rather we observe a continuous magnetic flux whose continuous variation may lead to the Aharonov-Bohm effect in the two slits experiment including a long solenoid containing magnetic flux. It is worth noticing that although the magnetic flux in the superconductor ring is quantized but the Aharonov-Bohm effect is observed for a superconductor ring, as well. This is because, the flux lines are integer multiplications of $\frac{\pi\hbar c}{e}$ and if we substitute it into the relative phase in (7) the phase shift occurs for the odd numbers of flux lines.

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