Quantum metrology to probe atomic parity violation

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An entangled state prepared in the decoherence free sub-space together with a Ramsey type measurement can probe parity violation in heavy alkali ions like Ba^+ or Ra^+. Here I propose an experiment with Ba^+ ions as an example to measure the small parity violating effect in this system. I will also show that the accuracy of such an experiment can be three orders of magnitude better for a day of measurement as compared to the single ion experiment.

Measurement of parity violation (PV) in the 6S-7S transition of neutral Cs has been performed with an uncertainty reaching 0.35% [1, 2, 3]. This was achieved in several steps over a period of three decades. The main difficulty in atomic PV experiments is to measure the small size of an observable which arises from the interference of a PV electric dipole transition amplitude which is of the order of $10^{-11}ea_0$ with a much larger electromagnetic transition amplitude. For the Cs experiment an equally demanding theoretical effort by atomic theorists [4] led to the evaluation of the weak nuclear charge Q_W which is a unique low energy test of the standard electroweak theory. Further improvement will lead to reducing the limits on the mass of an eventually additional light or heavy boson [5]. Apart from the necessity of improving the PV measurement in Cs, it would be worthwhile to consider the possible measurement of PV in other atomic systems. Cesium is so far the only candidate which has reached an accuracy of less than one percent. In this letter, I propose a novel way of improving the signal-to-noise ratio in the measurement of atomic PV, thereby reducing the experimental uncertainty.

A newly proposed method adopted for the Cs measurement involved left-right asymmetry of the forbidden transition rate involving 6S-7S transition [6, 7]. This method is presently been persuade for Fr, the heaviest alkali [8, 9]. Unfortunately, the need of a large number of atoms to observe the asymmetry limits this experiment. Recently, it has been proposed to observe a linear Stark shift in an interferometric measurement with small number of atoms of Fr [10]. The first proposal of parity violating light-shift in a heavy ion like Ba^+ was put forward in 1993 by Fortson [11]. Presently it is being pursued at different experimental laboratories. Initial radio frequency (RF)spectroscopy on Ba^+ has also been performed to observe the light shifts of different Zeeman sub-levels. These measurements are limited by magnetic field noise as well as laser light frequency noise [12]. To finally observe the PV light shift it is necessary to achieve an uncertainty well below one Hertz in the ground state Larmor frequency since even in the presence of a strong electric field, the shift is only of the order of 0.2 Hz. In what follows, I outline the basic principle of such a measurement.

Parity violation in an atomic system leads to a small mixing of odd and even parity states. For Ba^+ this leads to a small electric dipole transition amplitude between $6S_{1/2}$-$5D_{3/2}$ states which can be otherwise connected by predominantly an electric quadrupole transition. Compared to the electromagnetically allowed electric quadrupole transition rate, the weak PV electric dipole transition rate is several orders of magnitude smaller. Therefore a direct measurement of the latter is not possible. Instead of a direct measurement, Fortson proposed to measure an observable arising from the interference between the two transitions on a laser cooled single trapped ion. To measure such an observable one needs to consider the light shift on the Zeeman sub-levels of the ground state ($S_{\pm1/2}$) after shining in two laser lights in a standing wave configuration. The ion should be placed at the maximum of the laser field for the PV electric dipole transition while at the maximum of the field gradient for the quadrupole transition as prescribed in [11]. The total light-shift due to these lasers will consist of the shift due to the electric quadrupole transition as well as weak PV dipole transition. Fortunately, the light-shift caused by the quadrupole transition laser is independent of the magnetic sub-state, unlike the PV light-shift. Therefore one can measure the ground state Larmor frequency in the presence and in the absence of the standing wave lasers. The difference will give the PV induced light shift. Achieving one percent uncertainty in such a measurement is still a challenge.

Here it will be shown that indeed such a precise measurement can be performed using maximally correlated quantum states. The primary objective of a PV light-shift measurement is a frequency measurement which is a metrology problem. The best possible metrology is currently performed by observing the phase evolution of superposed relevant atomic states. Maximally entangled states for quantum metrology is a rather recent field and it has been implemented in a relatively few cases [13, 14, 15]. They have been used to improve the signal-to-noise ratio [16], to efficiently detect quantum state [17], to measure scattering length [18] and to do spectroscopy in decoherence free sub-space [19]. An entangled state in a decoherence free sub-space (DFS) [20, 21] makes any measurement immune to environment changes. Here I will discuss a possibility to use a maximally correlated state in a DFS to measure a parity violating light shift in an heavy atom like Ba^+ or Ra^+. Employing the generalized Ramsey interferometric technique to maximally correlated

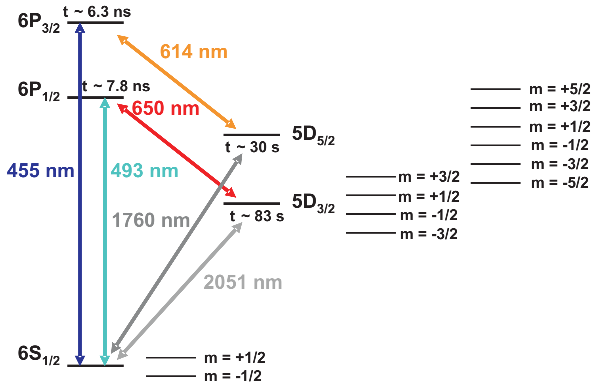

FIG. 1: Relevant atomic levels of Ba^+. The Zeeman sublevels are also shown for clarity.

atomic state it will be possible to determine the PV light-shift. Under free precision a maximally entangled atomic state $\psi(0) = \frac{1}{\sqrt{2}}(|u_1\rangle |u_2\rangle + |v_1\rangle |v_2\rangle)$ evolves into $\psi(\tau) = \frac{1}{\sqrt{2}}(|u_1\rangle |u_2\rangle + \exp^{i\Delta\lambda\tau} |v_1\rangle |v_2\rangle)$ after a time τ. The phase evolution rate $\Delta\lambda = [(E_{u_1} + E_{u_2}) - (E_{v_1} + E_{v_2})]/\hbar$ corresponds to the energy difference between the atomic states u_k and v_k. The real part of the phase factor $\exp^{i\Delta\lambda t}$ can be measured by projecting the ions on the states $|\pm\rangle = \frac{1}{\sqrt{2}}(|u\rangle_k \pm |v\rangle_k)$ and measuring the parity operator. For states in the DFS the free precision time τ can be made very long and the phase can be measured accurately [21, 22]. It will be shown that the use of maximally correlated state will enhance 3-orders of magnitude higher signal-to-noise ratio as compared to the original proposal. By a careful choice of the state it is possible to measure the PV-light-shift in DFS thereby avoiding the possible systematic effects in coupling to the environment.

In the following I shall consider two ions confined in a linear Paul trap. In general the ion can either be Ba^+ or Ra^+ depending on the experimental goal. For simplicity let us consider only Ba^+ whose relevant electronic levels are shown in fig. 1. Instead of a single ion, a pair of ions will be Doppler cooled [23] by using 493 nm and 650 nm lasers for cooling re-pumping respectively. A 2.05 μm laser will be employed to perform sideband ground state cooling of the center-of-mass mode of the ion string [23]. In this way the initialization of the two ion string can be achieved and therefore we can prepare the following state of the two ion system

$$(|\,0\rangle_1 \;|\,0\rangle_2) \;|\,0\rangle, \tag{1}$$

where the last multiplicative state denotes the common center of mass (COM) motional state of the two ion system. Starting from this initial state by application of a pulse sequence on blue sidebands of the s-d transition it is possible to create a maximally entangled Bell state like [19]

$$\frac{1}{\sqrt{2}}(|0\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2), \tag{2}$$

where $|0\rangle$ and $|1\rangle$ denotes the $S_{1/2}$ and $D_{3/2}$ states respectively. By application of only carrier pulses it is possible to transfer this state to the $S_{1/2}$-level Zeeman manifold [22]

$$\frac{1}{\sqrt{2}}(|-1/2\rangle_1 |+1/2\rangle_2 + |+1/2\rangle_1 |-1/2\rangle_2) |0\rangle\,. \tag{3}$$

Therefore for our experiment we will have a maximally entangled state of two ions in the Zeeman manifold of the electronic ground state. The Zeeman shifts of the two parts of the entangled state cancels out in absence of magnetic field gradient along the trap axis. The presence of the two ion state makes it decoherence free as compared to a superposition state of a single ion. If left unperturbed this state will evolve with time as

$$|-1/2\rangle_1 |+1/2\rangle_2 + \exp(i\Delta\lambda\tau) |+1/2\rangle_1 |-1/2\rangle_2\,, \tag{4}$$

where $\Delta\lambda$ denotes the phase evolution rate which corresponds to the energy difference between the two parts of the entangled state.

After having the state initialized, two laser beams in a standing wave configuration are applied as shown in fig. 2. The corresponding electric fields [11] are

$$E' = \hat{x} E'_0 \cos kz \tag{5}$$

and

$$E'' = i\hat{z} E''_0 \sin kx. \tag{6}$$

The E'' field is applied along the trap axis which is the $x-$axis. The ions can be placed at the desired antinode position of the standing wave by tuning the dc trap voltage along the axis. In this case multiples of 2.05 μm will be necessary. This separation is sufficient to address the ions individually. The applied fields will lead to interaction matrix elements

$$\Omega^{\rm PV}_{m'm}(1) = -\frac{1}{2\hbar}\epsilon^{\rm PV}_{m'm}|_x E'_0$$
$$\Omega^{\rm quad}_{m'm}(1,2) = -\frac{1}{2\hbar}\epsilon^{\rm quad}_{m'm}|_{zx} i E''_0,$$

where ϵ stands for transition amplitudes and 1,2 stands for the two ions. The $S_{1/2}$ state magnetic quantum numbers are denoted by m while m' stands for those in the $D_{3/2}$ state. Considering the quantization axis to be along the $z-$axis, $\Delta m = \pm 1$ quadrupole and PV dipole transitions are allowed. Eq. 7 needs to be evaluated at the ion positions only $e.i.$ $(x, z) = (0, 0)$ and $(x, z) = (2\pi/k)$

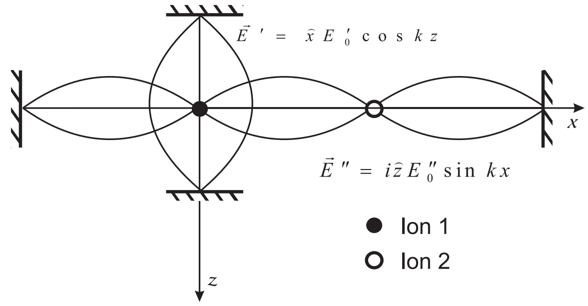

FIG. 2: A cartoon of two ions placed in a linear ion trap and interrogated by two standing wave lasers. The laser to excite the electric quadrupole transition is propagating along the trap axis. The PV electric dipole transition is excited by the laser propagating along the z axis. Both ions are placed at the anti-node (maximum potential gradient) of the electric quadrupole transition laser. The first ion only is placed at the node (maximum field) of the PV dipole transition laser. The amplitude E_0' should be orders of magnitude smaller as compared to E_0''.

respectively for ion 1 and 2. The observable quantity is given by [11]

$$|\Omega_{m'm}|^2 \approx |\Omega^{\rm quad}_{m'm}|^2 + 2Re(\Omega^{\rm PV*}_{m'm}\Omega^{\rm quad}_{m'm}) \qquad (7)$$

The interference is of importance for us since it is proportional to the $E1$ PNC amplitude $\epsilon^{\rm PV}_{m'm}$. Light-shifts $\Delta\omega^{\rm PV}_m$ and $\Delta\omega^{\rm quad}_m$ occur due to the PV dipole and quadrupole transitions respectively between the S and D states. This is obtained by summing over all m' states. Fortson [11] approximately calculated these shifts for Ba^+ to be

$$\Delta\omega^{\rm PV}_m = \pm 1 \times 10^{-11}\frac{ea_0}{2\hbar}E_0' \qquad (8)$$

$$\Delta\omega^{\rm quad}_m = -2 \times 10^{-4}\frac{ea_0}{2\hbar}E_0''. \qquad (9)$$

The sign of PV light-shift depends on the initial magnetic sub-state of the ground state S. The electric field and not the gradient of it interacting with the atom leads to a PV dipole transition. Therefore as per the two standing wave configurations, the E' field is the one which is responsible for the PV light-shift. As seen in fig. 2, it is only applied to "ion 1". To maximize the PV light-shift the electric field amplitude E_o' at the ion 1 position should be as large as possible while the field gradient of E_0'' as small as possible. Using comparable electric fields amplitudes the light-shifts are seven orders of magnitude different. Fortunately the light-shift for the electric quadrupole transition laser is independent of the $m-$state. So a change in the Larmor frequency shift in presence and in absence of the standing wave fields will directly measure the PV light-shift, provided the Larmor frequency can be measured with uncertainty well below one Hertz.

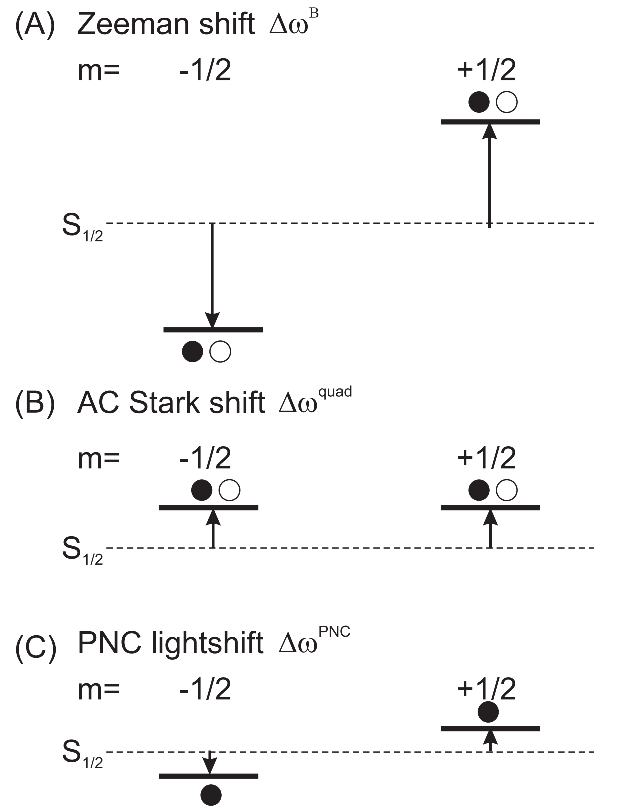

FIG. 3: A cartoon of two ions placed in a linear ion trap and interrogated by two standing wave lasers.

After initializing the two ion state to a maximally correlated state Eq. 3, the two standing wave lasers are applied as shown in fig. 2 for a time interval τ. The entangled state evolves according to the energy difference of the two parts as mentioned in Eq. 4. Figure 3 depicts the level-shifts that occur during the evolution time τ due to the presence of two standing wave lasers at 2.05 μm frequency.

In presence of only small magnetic field the Zeeman levels are shifted up(down) according to $m = +0.5(-0.5)$. Maximized field gradient E_0'' will have the m levels further shifted but in the same direction due to the ac Stark shift [11, 24]. The direction of the PV light-shift which is only applicable to ion 1, is $m-$state dependent. Therefore the energy difference between the two parts of the entangled state is given by

$$\begin{aligned}&[E(|-1/2\rangle_1) + E(|+1/2\rangle_2)] - [E(|+1/2\rangle_1) + E(|-1/2\rangle_2)]\\ =\ &[2(\Delta\omega^{B_2} - \Delta\omega^{B_1}) + 2\Delta\omega^{\rm PV}]\hbar,\end{aligned}$$

where Δ^{B_k} denotes the Zeeman shift at the kth ion position taking into account a possible magnetic field gradient along the trap axis. The measurement of parity in the $|\pm\rangle$ basis for different evolution time τ gives the right side of eq. 10. In absence of the two standing wave lasers the same measurement yields $2(\Delta^{B_2} - \Delta^{B_1})$. Therefore by subtraction one obtains the required PV light-shift $2\Delta^{\rm PV}$.

The uncertainty in the PV light-shift measurement will be determined by the decoherence time of the maximally entangled state which in our case is practically infinite. For measurements performed for a time period T on N maximally entangled atomic systems the uncertainty will be inversely proportional to NT [13]. Therefore the measurement accuracy can be approximated as

$$\frac{\epsilon^{\rm PV}}{\delta\epsilon^{\rm PV}} \approx \frac{\epsilon^{\rm PV} E_0'}{\hbar} f NT, \tag{10}$$

where f denotes the measurement efficiency. It is determined by how well the entangled state is formed and by the decoherence time of the state. In our case it can be close to one since it has been shown that such state can be prepared with a fidelity of nearly 95 %. N in this case is 2 since two ion maximally correlated state is used for the measurement. This proportionality with NT comes from the correlation as formulated Bollinger et.al. [13] instead of $\sqrt{NT}$ in case of uncorrelated systems. Therefore this figure-of-merit will be $\sqrt{2T}$ times higher as compared to the original proposal of Fortson [11]. Therefore for one day of measurement data the statistical uncertainty will be more than 2 orders of magnitude smaller.

In order to finally extract the PV induced $E1$ amplitude it is necessary to know the electric field at the ion position $E_0'(0)$, $E_0''(0)$ and the AC Stark shift induced by the $x-$propagating laser. The electric fields could be measured by off-resonant excitations. The AC Stark shift can as well be measured by using the technique of generalized Ramsey interference experiment [24, 25]. Using two ions instead of one ion in a linear ion trap may lead to unwanted stray electric fields which is a major concern for parity mixing. Since the ions are side band cooled to the ground state of their COM mode, the field at the ion equilibrium position must be zero. The ions in a linear string of Coulomb crystal have a wavepacket span which is negligible as compared to the wavelength of the standing wave. Therefore they can be considered to be at rest. At the ion's equilibrium position there is no electric field and therefore the PNC like shifts due to stray electric fields are negligibly small as well. The first order effects due to stray fields as well as the trapping potential are not only displaced from the PV transition by multiples of trap frequency but are also negligibly small due to sideband cooling. Therefore they do not interfere with it.

I have shown that a two ion entangled state is a better option for the measurement of parity violating light shift as compared to the single ion experiment. The experimental techniques that are involved are regularly in use by the quantum computation community. Therefore it is feasible with today's technology. The problem of placing the ion at a required position is a demanding task but can be achieved [26].

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