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On some properties of the neutrino in the early universe

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Abstract

Properties of the neutrino in the early universe have been investigated incorporating a small inhomogeneity in the mass density of the early universe. Dependence on this factor is found in studying mean free path and mass bound of neutrinos. The mass bound of neutrino is found to be in the range between 14 MeV and 76 MeV corresponding to the radiation and matter dominated eras respectively. The oscillation length for neutrinos have been estimated to be 86 km for beam energy 2.754 MeV and 220 km for beam energy 7 MeV.

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1. Introduction

The neutrino was the first elementary particle proposed that was not a constituent of ordinary matter, and even now the true nature of the particle is not clearly understood. Neutrinos, we find, are involved only in weak interactions and the Standard Model (SM) predicts them to be massless. A large body of observational and experimental efforts has been devoted to verifying whether neutrinos are massive and oscillate from one flavor to another [1–9]. The Super Kamiokande experiments [1] suggested electron neutrino mass $<$ 0.07 eV. The experiment also found evidence for neutrino oscillations and hence mass in atmospheric neutrinos. Current limits on neutrino mass suggest that $m_{\nu_e} < 2$ eV, $m_{\nu_\mu} < 0.19$ MeV and $m_{\nu_\tau} < 18.2$ MeV [10]. Dodelson et al. have derived an upper limit for the sum of neutrino masses as $\sum m_\nu < 0.17$ eV (95% CL) [11]. Though experimental estimates of neutrino mass are far from being perfect, no evidence for neutrinos being massless is yet available [12, 13]. The expanding universe puts a serious constraint on the neutrino mass. The helium abundance problem does not have any satisfactory explanation in the theory of thermonuclear fusion or in nucleosynthesis processes in stars, but may be explained by primeval nucleosynthesis. At cosmic temperatures greater than 10^{13} K, there were few neutrons and photons and the universe had a large number of electrons, positrons and neutrinos. Light elements (deuteron) gradually synthesized to an abundance of helium and to relative abundance of baryon to photon or neutrinos. The upper bound on primordial helium provides an upper bound on the total energy density ρ at that moment of the nucleosynthesis, which is expressed in terms of the allowed number of light neutrino families as $N_\nu < 3 + \delta N_\nu$ [14], where δN_ν includes any extra contribution beyond three left- handed neutrinos.

The neutrino has a significant contribution to the cooling of the universe. Hot systems radiate their excess energy usually as photons or neutrinos. There are various processes by which the neutrinos were produced in the early universe such as neutrino pair bremsstrahlung by nucleons, collective electron plasma excitation, etc. To first order of approximation, the relative order of various processes depends on the equations of state. Neutrinos are supposed to be massless in the SM, but Gerstein [15] proposed the massive neutrino from observational cosmology and relic neutrino abundance. In the SM, light neutrinos are kept in equilibrium with charged leptons and photons in the cosmic plasma due to weak interactions. But above a temperature $T = 1$ MeV, as the expansion rate of the universe increases rapidly, it leads to an early freeze out of the weak interaction and the number of neutrinos becomes constant. The number of photons increases by e^+e^- annihilation. It has been suggested that the present number density of neutrinos per species is $n_{\nu_i} = 3/11 n_\gamma = 102$ cm^{-3} [14]. Hence the Big Bang cosmology which establishes the expansion of the universe from an extremely high temperature and density and Hubble expansion has direct consequences on the neutrino density in the universe.

Theoretically, the SM can include the possibility of massive neutrinos. As the neutrino mass eigenstates and weak eigenstates are not necessarily coincident, there may exist neutrino mixing as described by the Cabbibo angle in the GIM scheme [16] and generalized in Kobayashi- Maskawa [17] six-quark scheme.

Neutrino oscillation is the most reliable observation that predicts neutrino mass. It is a phenomenon in which a neutrino of a particular leptonic flavor, over time, transforms to a different flavor. The idea of neutrino oscillation was first introduced by Pontecorvo [18] in 1957 and subsequently the solar neutrino deficit was observed. Solar neutrino deficit could be explained theoretically by a vacuum oscillation of neutrino flavors. A number of experiments followed, such as by Davis [19], Hirata [20, 21] and others.

Experiments to understand and observe neutrino oscillations have many and multifarious in approach [1–4]. The MINOS experiment in 2006 [3] involved, sending a high intensity beam of muon neutrinos to a particle detector, where it was found that a significant fraction of the original particles had disappeared. The NOvA experiment [4], designed and developed by Fermi lab, will to send neutrino beams through distances $\sim$800 km to obtain improved measurements of mass differences among the neutrino flavors. It also hopes to shed light on the ordering of neutrino mass states; the data is expected to be available by 2010–11.

The current limits on the oscillation parameter are $\Delta m^2_{21} = \Delta m^2_{\rm sol} = 8.0^{+0.6}_{-0.4} \times 10^{-5}$ eV^2 [5] and $\Delta m^2_{31} = \Delta m^2_{32} = \Delta m^2_{\rm atm} = 2.4^{+0.6}_{-0.5} \times 10^{-5}$ eV^2 [6].

We have studied the evolution of the universe with a mass fractal dimension d as an exponent of the density [22–24] and come across many interesting consequences for the early universe lying between the matter and radiation dominated eras. The speed of sound in the mixed phase of radiation and matter dominated era has already been investigated by us [25] considering the cosmological constant to be zero.

In the present work, we have studied the same for a non-zero value of the cosmological constant. We have investigated neutrino opacity in the early universe taking into account the mass fractal dimension d, which has been incorporated in the FRW model of the universe. In inflationary cosmology, the notion of fractal dimension appears at the stage where at each particular point it is possible to consider the universe as a Friedmann universe described by a single scale factor. We use the fractal concept to study the formation and structure of the universe between matter and radiation dominated eras and explore the evolution of the early universe.

We have investigated neutrino opacity in the early universe with this approach. The equation of state for the universe is found to yield some interesting results on neutrino density, mean free path of neutrinos and a mass bound. Vacuum oscillations of neutrino flavors have also been studied considering the evolution of the mass eigenstates as free wave packets. The neutrino states are wave packets with a certain spreading in momentum, and hence such an approach is justified. An expression for probability of oscillation (considering only two flavors) is deduced and compared to other similar results.

2. Velocity of sound

The standard cosmological model of the universe is based on the FRW metric. Einstein's equation runs as [26],

$$\left[\frac{\mathrm{d}R(t)/\mathrm{d}t}{R}\right]^2 = \frac{8\pi G\rho}{3} - \frac{K}{R^2(t)} + \Lambda, \tag{1}$$

where $R(t)$ is the Robertson-Walker scale factor which describes the expansion of the universe and the three space curvature is related to the constant K. G is the gravitational constant, ρ is the energy density and Λ is the cosmological constant. With $\Lambda = 0$, the energy conservation demands that

$$\frac{\mathrm{d}(\rho R^3)}{\mathrm{d}R} = -3PR^2, \tag{2}$$

where P is the isotropic pressure. With a small inhomogeneity d as a mass fractal dimension in the density distribution of the early universe, where $0 < d < 1$ [27], and energy density behaves as $\rho(\mathrm{R}) \sim R^{-d-3}$, we have

$$\frac{\mathrm{d}(R^{-d})}{\mathrm{d}R} = -3PR^2. \tag{3}$$

We presume an adiabatic expansion of state as $P = A\rho^\gamma$ $(\gamma > 1)$, where A is any constant corresponding to the rapid expansion era of the early universe. The energy conservation equation can be generalized through a fractional differentiation of the n^{th} order as [27]

$$\frac{\mathrm{d}^n R^{-d}}{\mathrm{d}R^n} = -3AR^{-(d+3)\gamma+2} \tag{4}$$

where n is, in general, a fractional number. As density fluctuation scales like a fractal as R^{-d-3}, we have replaced the flux $\frac{\mathrm{d}(R^{-d})}{\mathrm{d}R}$ in (3) by $\frac{\mathrm{d}^\mu R^{-d}}{\mathrm{d}R^\mu}$. With $\mu = 1$, $\gamma = 1$, we recover the usual conservation equation of (3). For a flat universe ($k = 0$) we get from (1) with $\Lambda = 0$ and $\rho \sim R^{-d-3}$ [27],

$$R(t) \sim t^\beta \tag{5}$$

where $\beta = 2/d + 3$. The bounds $0 < d < 1$ corresponds to the coexistence phase of matter- and radiation-dominated universe and $d = 0$ represents the matter dominated era; whereas $d = 1$ corresponds to the radiation dominated era. We have also derived the result $t = \beta H^{-1}$ where H is the Hubble parameter [22–24]. With the non-zero value of the cosmological constant the energy conservation equation runs as [28]

$$\frac{\mathrm{d}(\rho R^3)}{\mathrm{d}R} + P\frac{\mathrm{d}R^3}{\mathrm{d}R} + \frac{\alpha R^3}{3} \cdot \frac{\mathrm{d}\Lambda}{\mathrm{d}R} = 0. \tag{6}$$

Assuming the cosmological constant Λ as an explicitly scale-dependent quantity having the form

$$\Lambda \sim \beta' R^K, \tag{7}$$

where β' is an arbitrary constant and $\beta' > 0$, K is an exponent, we may recast the equation (6) as:

$$\frac{\mathrm{d}(\rho R^3)}{\mathrm{d}R} + \frac{\alpha K \beta'}{3(K+3)} \cdot \frac{\mathrm{d}R^{K+3}}{\mathrm{d}R} = -3PR^2, \tag{8}$$

where $\alpha = \frac{3}{8}\pi G$. With adiabatic pressure $P = A\rho^\gamma\,(\gamma \to 1)$, $\rho \sim R^K$, where $K = -d - 3$ (the energy conservation equation (6) is satisfied provided Λ scales in the same manner as the density distribution scales in the early universe) we have,

$$-3A = K\left(\frac{\alpha\beta'}{3} + 1\right) + 3. \tag{9}$$

For radiation and matter dominated era where $A > 0$, we must have $K(\frac{\alpha\beta'}{3} + 1) + 3 < 0$. Hence,

$$\mid K \mid > 3/\left(\frac{\alpha\beta'}{3} + 1\right). \tag{10}$$

Hence the velocity of sound can be expressed as

$$v_s = \left(\frac{\delta P}{\delta \rho}\right)^{\frac{1}{2}}. \tag{11}$$

Thus we have for $\Lambda = 0$ and $\gamma \to 1$ [25] the proportion

$$v_s \propto d^{\frac{1}{2}} \tag{12}$$

and for $\Lambda \sim \beta' R^K$,

$$v_s \simeq \frac{1}{\sqrt{3}}\left[\frac{(3+\alpha\beta')d}{3} + \alpha\beta'\right]^{\frac{1}{2}}. \tag{13}$$

For $\Lambda = 0$ and $\gamma \to 1$, this reduces to $v_s \sim \sqrt{d}$. The velocity of sound, we see, depends on the mass fractality or the inhomogeneity of the medium.

3. Mean free path

In the early universe during the primeval nucleosynthesis process the neutrinos produced lose their energy due to scattering from the electrons and nucleons at the energy scale $T \approx 10^{15}\,\mathrm{K}$. A neutrino propagating in a medium of electron gas has the mean free path [29]

$$\lambda_\nu \approx \frac{\rho^{-4/3}}{\varepsilon_\nu^3} \approx \frac{R^{1.33(d+3)}}{\varepsilon_\nu^3}. \tag{14}$$

where λ_ν is the neutrino mean free path, ρ is nuclear density and ε_ν is neutrino energy. For a degenerate Fermi gas of neutrons we arrive at a neutrino mean free path of

$$\lambda_\nu = \frac{\kappa_T}{\epsilon_\nu^2 (kT) n_n^2}. \tag{15}$$

Here, κ_T is the bulk modulus of the medium, ϵ_ν is the neutrino energy and assumed to be $\epsilon_\nu = kT$ and n_n is number of neutrons/fm^3.

With $v_s = \sqrt{\frac{\gamma P}{\rho}}$ (where γ is adiabatic constant of the medium), we can rewrite the above expression as

$$\lambda_\nu \approx \frac{v_s^2 \rho}{\gamma (kT)^3 n_n^2} \tag{16}$$

with $R = t^{\frac{2}{d+3}}$ and $t = \beta H^{-1}$ [22–24]. We now arrive at

$$\lambda_\nu \sim (d+3)^2 H^2. \tag{17}$$

Similarly, taking into the contribution from Λ, we get the relation

$$\lambda_\nu \sim (d+3)^2 H^2. \tag{18}$$

Thus we find from equations (18) and (19) that the mean free path of neutrino depends on the mass fractal dimension of the medium. The expansion of the universe affects the mean free path of the neutrino and consequently affects the scattering cross section.

4. Mass bound

One of the most important cosmological constraints on stable neutrino is its mass bound. In SM the interaction is kept in equilibrium with charged lepton and photons until a temperature $T \approx 1\,\mathrm{MeV}$. In the early universe during the nucleosynthesis the numbers of photons and neutrinos are the same at about $T \simeq 10^{15}\mathrm{K}$ [14]. After that the rapid expansion of the universe freezes the weak process, fixing the neutrino density. However the e^+e^- annihilation increases the photon density. In the context of investigating the fractal structure of the universe [27], we have come to the expression

$$\frac{2\pi}{3}(d+3)^2 G\rho t^2 = 1, \tag{19}$$

where d is the mass fractal dimension, $\rho = (g/2)\rho_\gamma$, ρ_γ is the photon energy density which may be considered to be equal to the neutrino density ρ_ν in the early phase of the universe between matter and radiation dominated era, and g is the corresponding degrees of freedom. Now, from the above equation we have estimated the neutrino energy density with $\rho \approx (g/2)\rho_\nu$, $t = 10^{-4}\,\mathrm{sec}$, $G = 6.673 \times 10^{-8}\,\mathrm{cm}^3\mathrm{gm}^{-1}\mathrm{sec}^{-2}$, as

$$\rho_\nu = \frac{0.1413}{g(d+3)^2} \times 10^{16}\ \mathrm{gm/cm}^3. \tag{20}$$

Now the number density of the neutrino n_ν is almost equal to the number of photons at temperature on the order of $T = 10^{15}\,\mathrm{K}$ and $n_\gamma = n_\nu \approx T^3$ [30]. If the neutrinos are massive, the energy density of a neutrino is $\rho_\nu = m_\nu n_\nu$, where m_ν is the mass of the neutrino. Thus from expression (20), we have

$$m_\nu = \frac{1.43}{g(d+3)^2} \times 10^{-30}\ \text{gms} \tag{21}$$

To get an estimate of the mass of the neutrino we use $g \approx 427/4$ at the electroweak scale [31] and come across $m_\nu = 2.29 \times 10^{-32}(d+3)^2$ gms. Thus the mass of the neutrino depends on the mass fractal dimension of the universe and m_ν is found to lie between 14 eV and 76 eV, between radiation ($d = 1$) and matter ($d = 0$) dominated eras.

Assuming left handed neutrinos are ever in equilibrium in the primordial plasma, Gerstein et al. [15] have predicted $m_{\nu_i} \leq 37$ eV in a matter dominated universe with zero cosmological constant. They have emphasized the fact that the nature of the matter, radiation and cosmological constant very much influence the value of Ω_0. With $0.25 \leq \Omega_0 \leq 0.4$, they have obtained $0.27\,\mathrm{eV} < \Sigma m_{\nu_i} < 0.37\,\mathrm{eV}$ (where i varies from 1 to 3). m_ν has also been estimated from $\frac{n_B}{n_\gamma}$ [32].

The fraction of the critical mass taken by neutrinos has been estimated to be $\approx$ 20 eV assuming $\Omega = 1$ (which implies the neutrino provide enough mass to close the universe).

Here, we observe that inhomogeneous nature or mass fractal dimension limits the mass of the neutrino in between 14 eV to and 76 eV in the coexistence phase of matter and radiation. So it may be suggested that if we consider the universe starts with a small inhomogeneity, the energy density or the mass is greatly influenced by it.

5. Neutrino oscillation

Neutrino oscillations, to date, remain the most promising avenue of exploration to find neutrino mass. We study the vacuum oscillation of neutrinos considering their evolution as free wave packets. We observe that significant suppression in one type of flavor demands very small momentum difference in the corresponding flavors. Theoretically, neutrino mixing gives rise to the phenomenon of neutrino oscillation. In this phenomenon a neutrino born in an eigenstate, after a certain time interval, finds itself in a mixture of states. Confining ourselves to the first two generations, we assume that the flavor eigenstates ν_e, ν_μ are produced at time $t = 0$. The corresponding mass eigenstates are ν_1, ν_2. We express the weak eigenstates as a linear combination of the mass eigenstates [32] with θ as an angle that parametrizes the mixing, which can be calculated if we know the interaction that gave rise to the masses. A non-zero value of θ implies that some neutrino masses are non-zero and that the mass eigenstates are non- degenerate. If $m_{\nu_i} = 0$, then there is no way to distinguish the weak eigenstates from the mass eigenstates, so the states could always be expanded in a new set and the angle θ rotated to zero. For the time evolution of the mass eigenstates we consider the propagation of the free particle as suggested by Kleber [33]. The expression runs as

$$\nu_i(r,t) = \frac{m_i}{2iht\pi}^{3/2} e^{im_i r^2/ht} \int \mathrm{d}^3 r' e^{im_i (r')^2/2ht} e^{-im_i r r'/ht} \nu_i(r,0). \tag{22}$$

Without any loss of generality we assume that $\nu_i(0)$ is concentrated around $\overrightarrow{r} = 0$. Hence we may rewrite equation (22) as

$$\nu_i(r,t) = \nu_i(0,0)\frac{m_i}{2iht\pi}^{3/2} e^{im_i r^2/ht} \int_0^a 4\pi(r')^2 dr' e^{im_i(r')^2/2ht} e^{-im_i rr'/ht}, \tag{23}$$

where a is the cutoff parameter which is presumed to represent the oscillation length. The expression represents the evolution of the mass eigenstate at some time $t > 0$. At time $t = 0$ we assume the mass eigenstate and weak eigenstate to be identical, and $\nu_\mu(0)$ is orthogonal to $\nu_e(0)$. The mass eigenstates vary with time as per the above equation, and they are free particles after they are produced. Expressing the weak eigenstates as a combination of the mass eigenstates we see the time evolution through the wave packet description. On writing ν_μ(t) we find that at time $t = 0$ what was a pure state $\nu_\mu(0)$ has now become a mixed state containing contribution from both $\nu_\mu(0)$ and $\nu_e(0)$. The probability that the ν_μ beam after a time t will contain ν_e, on calculation is found to be given by

$$p(\nu_\mu \to \nu_e) = | < \nu_e(0)|\nu_\mu(t) > |^2 = \frac{B\sin^2 2\theta}{t}[1 - \cos(E_2 - E_1)t], \tag{24}$$

where $B = \frac{\pi m a^2}{2i^3\hbar}$ and E_i are the energies corresponding to the mass eigenstates ν_i. This describes the simplest type of neutrino flavor oscillation, in which the amplitude of oscillation is found to depend on the mixing angle θ and time t. The oscillation frequency is found to be proportional to Δm^2 and momentum p, by rewriting $(E_2 - E_1)t \simeq \frac{m_2^2 - m_1^2}{2p}t$. We define the oscillation length a by putting $\cos(\frac{m_2^2 - m_1^2}{2p})t \simeq \cos\frac{2\pi x}{a}t$ where $x \simeq ct$ is the distance traveled by the beam in time t. Now, $a = \frac{4\pi p}{m_2^2 - m_1^2}$, where a is an effective length that determines the distance over which one might expect to see the effect.

6. Conclusion

In the present work we have investigated some interesting features of neutrino properties in the early universe in between matter and radiation dominated era. The velocity of sound when calculated fora non-zero value of the cosmological constant, is found to depend on the mass fractal dimension d. We find that the velocity of sound in radiation dominated phase is much greater than in the matter dominated phase, as expected.

An expression for the mean free path of the neutrino is deduced and hence we find that neutrino opacity is influenced by the inhomogeneity of the medium, which we have incorporated through a mass fractal dimension, in the early universe. It may be pointed out here that although the nucleon-neutrino scattering process is not that much dominant in the early universe, it would be very interesting to investigate the influence of the fractal dimension in the scattering process via nucleon-neutrino process during the primeval nucleosynthesis. The mass of the neutrino has been found to be influenced by the fractal dimension in the present work, whereas Gerstein et al. [15] has emphasized the fact that the mass of the neutrino is greatly influenced by the value of Ω_0. It would be relevant to mention here that the large neutrino mass could make the universe finite and closed. The critical density has the value $\approx 58 \times 10^{-30}$ g/cm^3. Considering present day photon and neutrino density almost equal to (400/cm^3) the contribution from the neutrino mass will exceed the critical density if m_ν is more than 10^{-32} g. The last word on neutrino mass, is however far from being spoken, especially since we are still in the dark regarding the nature of dark matter.

The mass bound of the neutrino as we estimate it is between 14–76 eV. Almost all recent research agrees on putting ν_e mass at less than 50 eV, with even lower masses also being suggested [1, 10]. The sum of the three neutrino masses is required to be very small in order that the universe does not become a close one.

Neutrino oscillations have been studied in detail with the results throwing up a time- dependent amplitude of oscillation. Also the frequency of oscillation provides us with a relation connecting oscillation length, the neutrino momentum and mass difference squared. Taking the mass difference squared as 7.59×10^{-5} eV^2 and beam energy 2.754 MeV from SNO results [34] we calculate the oscillation length to be 86 km. For the same mass difference if the beam energy is 7 MeV [1, 2], the oscillation length is found to be 220 km. It may be mentioned that in the Super Kamiokande experiment the detector is placed at a distance of approximately 250 km to observe neutrino oscillations. As per current estimates of neutrino masses, the mass difference squared is expected to be very small, which can only be detected for large value of oscillation lengths.

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