# Magnetic Moment Coupling to Circularly Polarized Photons 

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#### Abstract

Exact stationary solutions of the wave equation are obtained to describe the interaction between magnetic moment of elementary particle and circularly polarized photons. The obtained solutions substantially modify the conventional model of field-matter interaction. It follows from them that magnetic moment couples to photons, and this coupling leads to bound particle-photon states with different energies for different orientations of magnetic moment. As a consequence, the interaction splits particle states differing by directions of total angular momentum. Stationary spin splitting, induced by photons, and concomitant effects can be observed for particles exposed to a lasergenerated circularly polarized electromagnetic wave.


PACS numbers: 03.65.Ge, 03.65.Pm, 12.20.Ds

The interaction between magnetic moment of elementary particle and electromagnetic field is one of fundamental interactions in the nature. The quantum theory to describe one was elaborated at the dawn of quantum mechanics and has taken place in textbooks (e.g., [1, 2]). However, this theory deals with electromagnetic field in the framework of classical electrodynamics. A theory, capable of describing the interaction for quantized electromagnetic field, was unknown up to the present. This gap in the fundamental area of quantum physics arises from formidable mathematical obstacles to find solutions of the wave equation in the case of intensive quantized field. A small parameter, that would help to find the solution as a series expansion, is absent in the problem. As a consequence, the standard quantum-electrodynamic methodology, based on the perturbation theory, is not applicable here. Therefore there is no other way to work out the problem but solve this equation accurately. Unfortunately, a regular method to solve the wave equation is unknown, and success in seeking exact solutions for special cases depends on the Fortune. That is the reason why the problem was not elaborated before now. The author of the Letter was lucky to find exact stationary solutions of the wave equation describing the interaction between magnetic moment of particle and intensive quantized circularly polarized electromagnetic field. Since the obtained solutions substantially modify the conventional model of particle-field interaction, they are of broad interest to the physics community.

First of all, let us consider the interaction between a monochromatic circularly polarized electromagnetic wave and an electrically neutral particle with a magnetic moment $\mu$ and the spin- $1 / 2$ (for example, neutron). The interaction Hamiltonian, written in the conventional form, is given by $\hat{\mathcal{H}}_{\text {int }}=-\hat{\boldsymbol{\mu}} \mathbf{H}$, where $\hat{\boldsymbol{\mu}}=\mu \hat{\boldsymbol{\sigma}}$ is operator of the magnetic moment, $\hat{\boldsymbol{\sigma}}$ is the Pauli matrix operator, and $\mathbf{H}$ is classical magnetic field of the wave 1]. Considering the problem within the standard quantum-field approach [2], the field, $\mathbf{H}$, should be replaced with the field operator, $\hat{\mathbf{H}}$. Assuming the wave to be clockwise-
polarized, this operator can be written as

$$
\begin{equation*}
\hat{\mathbf{H}}=i \widetilde{H}_{0}\left[\mathbf{e}_{z} \times\left(\mathbf{e}_{+} \hat{a} e^{i k_{0} z}-\mathbf{e}_{-} \hat{a}^{\dagger} e^{-i k_{0} z}\right)\right] \tag{1}
\end{equation*}
$$

where $\widetilde{H}_{0}=\sqrt{2 \pi \hbar \omega_{0} / V}, \omega_{0}$ is frequency of the wave, $V$ is volume, $\mathbf{e}_{ \pm}=\left(\mathbf{e}_{x} \pm i \mathbf{e}_{y}\right) / \sqrt{2}$ are polarization vectors, $\mathbf{e}_{x, y, z}$ are unit vectors directed along the $x, y, z$-axes, $k_{0}=$ $\omega_{0} / c$ is wave vector assumed to be directed along the $z$ axis, $\hat{a}$ and $\hat{a}^{\dagger}$ are operators of destruction and creation of photons in the wave, respectively [2]. The replacement leads to the Hamiltonian of the particle-photon system

$$
\begin{equation*}
\hat{\mathcal{H}}=\frac{\hat{\mathbf{p}}^{2}}{2 m}+\hbar \omega_{0} \hat{a}^{\dagger} \hat{a}-\sqrt{2} \mu \widetilde{H}_{0}\left(\hat{\sigma}_{+} \hat{a} e^{i k_{0} z}+\hat{\sigma}_{-} \hat{a}^{\dagger} e^{-i k_{0} z}\right), \tag{2}
\end{equation*}
$$

where the first term on the right-hand side describes kinetic energy of the particle, the second term corresponds to field energy, the third term is the interaction Hamiltonian, $\hat{\mathcal{H}}_{\text {int }}$, rewritten in quantum-field form, $\hat{\mathbf{p}}$ is momentum operator of the particle, $m$ is mass of the particle, and $\hat{\sigma}_{ \pm}=\left(\hat{\sigma}_{x} \pm i \hat{\sigma}_{y}\right) / 2$ are step-up and step-down operators for the $z$-projection of the particle spin, $\mathbf{S}$.

To describe the particle-photon system, let us use the notation $\left|S_{z}, N\right\rangle$ which indicates that the particle is in quantum state with the spin projection $S_{z}=+1 / 2$ or $S_{z}=-1 / 2$ and the wave is in quantum state with the photon occupation number $N$. Then the exact stationary solutions of the wave equation with the Hamiltonian (2), $\psi_{+1 / 2, N_{0}}$ and $\psi_{-1 / 2, N_{0}}$, can be written as

$$
\begin{gather*}
\left|\psi_{ \pm 1 / 2, N_{0}}\right\rangle=\frac{e^{i \mathbf{k r}}}{\sqrt{V}}\left[\sqrt{\frac{\Omega_{ \pm}+\omega_{ \pm}}{2 \Omega_{ \pm}}}\left| \pm 1 / 2, N_{0}\right\rangle\right. \\
\left. \pm e^{\mp i k_{0} z} \sqrt{\frac{\Omega_{ \pm}-\omega_{ \pm}}{2 \Omega_{ \pm}}}\left|\mp 1 / 2, N_{0} \pm 1\right\rangle\right] e^{-i \varepsilon_{ \pm 1 / 2, N_{0}} t / \hbar} \tag{3}
\end{gather*}
$$

where $N_{0}$ is photon occupation number of the unperturbed wave, $\mathbf{k}$ is wave vector of the particle, $\mathbf{r}$ is radiusvector of the particle, energies of the particle-photon system, $\varepsilon_{+1 / 2, N_{0}}$ and $\varepsilon_{-1 / 2, N_{0}}$, are given by

$$
\begin{equation*}
\varepsilon_{ \pm 1 / 2, N_{0}}=\frac{\hbar^{2} k^{2}}{2 m}+N_{0} \hbar \omega_{0} \pm \frac{\hbar \omega_{ \pm}}{2} \mp \frac{\hbar \Omega_{ \pm}}{2} \tag{4}
\end{equation*}
$$

frequencies $\Omega_{+}$and $\Omega_{-}$are

$$
\Omega_{ \pm}=\sqrt{8\left(N_{0}+1 / 2 \pm 1 / 2\right)\left(\mu \widetilde{H}_{0} / \hbar\right)^{2}+\omega_{ \pm}^{2}}
$$

and $\omega_{ \pm}=\omega_{0}\left(1-\hbar k_{z} / m c \pm \hbar k_{0} / 2 m c\right)$. The subscript indexes in Eqs. (3)-(4) indicate genesis of the bound particle-photon states, i.e. the state $\left|\psi_{ \pm 1 / 2, N_{0}}\right\rangle$ turns into the state $\left| \pm 1 / 2, N_{0}\right\rangle$ when the particle-photon interaction vanishes $(\mu=0)$. The solutions (3)-(4) can be easily verified by direct substitution into the wave equation $i \hbar \partial \psi_{ \pm 1 / 2, N_{0}} / \partial t=\hat{\mathcal{H}} \psi_{ \pm 1 / 2, N_{0}}$ with the Hamiltonian (2), keeping in mind the trivial relations [1, 2]

$$
\begin{aligned}
\hat{\sigma}_{ \pm}|\mp 1 / 2, N\rangle & =| \pm 1 / 2, N\rangle, \quad \hat{\sigma}_{ \pm}| \pm 1 / 2, N\rangle=0 \\
\hat{a}^{\dagger}| \pm 1 / 2, N\rangle & =\sqrt{N+1}| \pm 1 / 2, N+1\rangle \\
\hat{a}| \pm 1 / 2, N\rangle & =\sqrt{N}| \pm 1 / 2, N-1\rangle
\end{aligned}
$$

The first term on the right-hand side of Eq. (4) is energy of noninteracting particle, the second term is energy of noninteracting photons, and other terms arise from the particle-photon interaction. This implies that the ground energy, $\varepsilon_{+1 / 2, N_{0}}$, is less than an energy of the noninteracting particle-photon system. Therefore the considered interaction results in stable particle coupling to photons. It follows from the equality

$$
\begin{equation*}
\left\langle\psi_{ \pm 1 / 2, N_{0}}\right| \hat{\boldsymbol{\sigma}}\left|\psi_{ \pm 1 / 2, N_{0}}\right\rangle= \pm \omega_{ \pm} \mathbf{e}_{z} / \Omega_{ \pm} \tag{5}
\end{equation*}
$$

that the states $\psi_{+1 / 2, N_{0}}$ and $\psi_{-1 / 2, N_{0}}$ correspond to mutually opposite orientations of averaged particle spin. Thus the difference in energy of these states, $\Delta \varepsilon=$ $\varepsilon_{-1 / 2, N_{0}}-\varepsilon_{+1 / 2, N_{0}}$, should be interpreted as spin splitting induced by photons. Let us emphasize that the Hamiltonian (2) holds true only for the wave intensive enough. If the photon occupation number of the unperturbed wave, $N_{0}$, is small, intensities of the particle interaction with photons from the wave and with vacuum states of other photons are comparable. Therefore for small occupation numbers of clockwise-polarized photons in the wave, the Hamiltonian (2) should be supplemented with terms describing the interaction between magnetic moment and vacuum states of counterclockwise-polarized photons with the same wave vector $k_{0}$. With these terms accounted, the photon-induced spin spitting $\Delta \varepsilon$ vanishes for $N_{0}=0$, as expected. In what follows we shall be to assume the wave to be intensive $\left(N_{0} \gg 1\right)$ and the particle to be nonrelativistic $\left(\omega_{ \pm} \approx \omega_{0}\right)$. Then the frequencies $\Omega_{ \pm}$in Eqs. (3)-(4) can be replaced with the frequency

$$
\begin{equation*}
\Omega=\sqrt{\left(2 \mu H_{0} / \hbar\right)^{2}+\omega_{0}^{2}} \tag{6}
\end{equation*}
$$

where $H_{0}=\sqrt{2 N_{0}} \widetilde{H}_{0}$ is classical amplitude of magnetic field of the wave. Thus the circularly polarized electromagnetic wave leads to the stationary spin splitting

$$
\begin{equation*}
\Delta \varepsilon=\sqrt{\left(2 \mu H_{0}\right)^{2}+\left(\hbar \omega_{0}\right)^{2}}-\hbar \omega_{0} \tag{7}
\end{equation*}
$$

which cannot be described by the conventional model of particle-field interaction [1, 2], based on the classical electrodynamics. It follows from the aforesaid that this splitting should be considered as novel quantum-field effect arising from the dressing of particle by circularly polarized photons. For neutrons exposed to the wave generated by a modern petawatt laser, the splitting (7) may approach the electron-Volt level.

The problem solved above for the spin- $1 / 2$ can be generalized for a particle with arbitrary total angular momentum, J. In this case, the magnetic moment operator can be written as $\hat{\boldsymbol{\mu}}=(\mu / J) \hat{\mathbf{J}}$, where $\hat{\mathbf{J}}$ is operator of total angular momentum [1]. Substitution of this magnetic moment operator into the interaction Hamiltonian, $\hat{\mathcal{H}}_{\text {int }}$, results in replacing the operators $\hat{\sigma}_{ \pm}$with the operators $\hat{J}_{ \pm}=\left(\hat{J}_{x} \pm i \hat{J}_{y}\right) / 2 J$ in the complete Hamiltonian of the particle-photon system (2). The regular procedure to solve accurately the wave equation with the modified Hamiltonian (2) is as follows. The solutions, $\psi_{j, N_{0}}$, should be sought in the form

$$
\begin{align*}
\left|\psi_{j, N_{0}}\right\rangle & =\frac{e^{i \mathbf{k r}}}{\sqrt{V}} \sum_{n=-J}^{J} C_{j, N_{0}}^{(n)} e^{i(n-j) k_{0} z}\left|n, N_{0}+j-n\right\rangle \\
& \times e^{-i \varepsilon_{j, N_{0}} t / \hbar} \tag{8}
\end{align*}
$$

where the notation $\left|J_{z}, N\right\rangle$ indicates that the particle is in quantum state with the total angular momentum projection $J_{z}$ and the wave is in quantum state with the photon occupation number $N$. Keeping in mind the relation [1]

$$
\begin{aligned}
\left\langle J_{z}, N\right| \hat{J}_{+}\left|J_{z}-1, N\right\rangle & =\left\langle J_{z}-1, N\right| \hat{J}_{-}\left|J_{z}, N\right\rangle \\
& =\sqrt{\left(J+J_{z}\right)\left(J-J_{z}+1\right)} / 2 J
\end{aligned}
$$

substitution of the function (8) into the wave equation with the modified Hamiltonian (22) results in the system of $2 J+1$ homogeneous algebraic equations

$$
\begin{array}{r}
{\left[\frac{\hbar^{2} k^{2}}{2 m}+N_{0} \hbar \omega_{0}+(j-n) \hbar \omega_{j-n}-\varepsilon_{j, N_{0}}\right] C_{j, N_{0}}^{(n)}} \\
-\frac{\mu \widetilde{H}_{0}}{\sqrt{2} J}\left[\sqrt{\left(N_{0}+j-n\right)(J+n+1)(J-n)} C_{j, N_{0}}^{(n+1)}\right. \\
\left.+\sqrt{\left(N_{0}+j-n+1\right)(J+n)(J-n+1)} C_{j, N_{0}}^{(n-1)}\right]=0 \\
n=-J,-J+1, \ldots, J-1, J,
\end{array}
$$

where $\omega_{l}=\omega_{0}\left(1-\hbar k_{z} / m c+l \hbar k_{0} / 2 m c\right)$. The well known procedure of solving such an algebraic system leads to $2 J+1$ sets of solutions, $\left\{\varepsilon_{j, N_{0}}, C_{j, N_{0}}^{(n)}\right\}$, which define $2 J+1$ wave functions (8). The parameter $j$, undefined before, should be specified independently for each of the sets in order to turn the bound particle-photon state $\left|\psi_{j, N_{0}}\right\rangle$ into the state $\left|j, N_{0}\right\rangle$ when the particle-photon interaction vanishes (i.e. for $\mu=0$ ). It appears that this parameter is equal to different values $-J,-J+1, \ldots, J-1, J$ for different wave functions (8) and should be interpreted as
the $z$-projection of total angular momentum of the noninteracting particle. As expected, for particles with the total angular momentum $J=1 / 2$ the described procedure leads to the solutions (3)-(4). Omitting relativistically small terms $\left(\omega_{l} \approx \omega_{0}\right)$ and keeping in mind that the wave is intensive $\left(N_{0} \gg 2 J\right)$, the energy of the particlephoton system for arbitrary $J$ can be written as

$$
\begin{align*}
\varepsilon_{j, N_{0}} & =\frac{\hbar^{2} k^{2}}{2 m}+\left(N_{0}+j\right) \hbar \omega_{0}-j \sqrt{\left(\mu H_{0} / J\right)^{2}+\left(\hbar \omega_{0}\right)^{2}} \\
j & =-J,-J+1, \ldots, J-1, J \tag{9}
\end{align*}
$$

Thus the interaction leads to different energies for different $z$-projections, $j$, of total angular momentum, J. It should be noted that Eq. (9) is applicable also to atoms which can formally be considered as electrically neutral particles, if a wave field is much weaker than an intraatomic field. In this case the wave does not change substantially an intra-atomic structure, and $\mu$ can be interpreted as known effective magnetic moment of unperturbed atom. Then the total angular momentum, J, includes a nuclear spin, electron spins and orbital angular momentums of electrons. As a result, the atom-photon interaction splits atom energy levels totally. If the wave is generated by an usual laser with the wavelength $\sim \mu \mathrm{m}$ and the intensity of radiation $\sim 10^{8} \mathrm{~W} / \mathrm{cm}^{2}$, the Eq. (9) can be used to find the photon-induced spin splitting of the ground $1 S$-state in a hydrogen atom. Neglecting the small nuclear magnetic moment, the splitting is described by Eq. (7), where $\mu$ is equal approximately to the electron Bohr magneton. The calculation leads to the splitting value $\Delta \varepsilon \sim 10^{-4} \mathrm{eV}$ that is tens times as large as the usual hyperfine splitting of the $1 S$-state [1]. Naturally, besides the energy splitting in isolated atoms, circularly polarized photons can also induce the energy gap opening in molecules and different condensed-matter structures, that will be analyzed elsewhere.

To complete the analysis, let us consider the photon coupling to an electrically charged particle with the spin$1 / 2$ (for example, electron). It is well known that interaction between a circularly polarized electromagnetic wave and a free charged particle leads to rotation of the particle [3]. This rotation causes the additional (spin-orbit) interaction between magnetic moment of the particle and electric field of the wave. As a consequence, we need to modify the Hamiltonian (2) for this case. Firstly, the Hamiltonian should be supplemented with a term describing the spin-orbit interaction. In the framework of classical electrodynamics the term has the form [2]

$$
\begin{equation*}
\hat{\mathcal{H}}_{s o}=-\left(\mu_{a}+\mu_{B} / 2\right) \hat{\boldsymbol{\sigma}}[\mathbf{E} \times \hat{\mathbf{v}} / c] \tag{10}
\end{equation*}
$$

where $\mu_{a}$ is anomalous magnetic moment of the particle, $\mu_{B}=e \hbar / 2 m c$ is Bohr magneton of the particle, $e$ is electric charge of the particle, $\hat{\mathbf{v}}=\hat{\mathbf{p}} / m$ is operator of particle velocity, and $\mathbf{E}$ is electric field of the wave. Secondly, the momentum operator $\hat{\mathbf{p}}$ should be replaced
with the operator $\hat{\mathbf{p}}-e \mathbf{A} / c$, where $\mathbf{A}$ is vector potential of the wave. Thirdly, the classical fields, $\mathbf{A}$ and $\mathbf{E}$, should be replaced with the field operators, $\hat{\mathbf{A}}$ and $\hat{\mathbf{E}}$, respectively. Using the well known expressions for these field operators [2], the modified Hamiltonian (2) can be written as $\hat{\mathcal{H}}=\hat{\mathcal{H}}^{\prime}+\hat{\mathcal{H}}^{\prime \prime}$, where

$$
\begin{aligned}
\hat{\mathcal{H}}^{\prime} & =\hat{\mathbf{p}}^{2} / 2 m-\sqrt{2} \mu \widetilde{H}_{0}\left(\hat{\sigma}_{+} \hat{a} e^{i k_{0} z}+\hat{\sigma}_{-} \hat{a}^{\dagger} e^{-i k_{0} z}\right)+\hat{a}^{\dagger} \hat{a} \\
& \left.\times\left[\hbar \widetilde{H}_{0}^{2} / m c \omega_{0}^{2}\right)\left(e c+\left(2 \mu_{a}+\mu_{B}\right) \omega_{0} \hat{\sigma}_{z}\right)\right], \\
\hat{\mathcal{H}}^{\prime \prime} & =-\left(\widetilde{H}_{0} / m\right)\left(e / \omega_{0}+\mu_{a} / c+\mu_{B} / 2 c\right) \\
& \times\left(\hat{p}_{+} \hat{a} e^{i k_{0} z}+\hat{p}_{-} \hat{a}^{\dagger} e^{-i k_{0} z}\right),
\end{aligned}
$$

and $\hat{p}_{ \pm}=\left(\hat{p}_{x} \pm i \hat{p}_{y}\right) / \sqrt{2}$. Exact solutions of the wave equation with the Hamiltonian $\hat{\mathcal{H}}^{\prime}$ can be found in the form (8) with $J=1 / 2$ by invoking the procedure described above. As to the Hamiltonian $\hat{\mathcal{H}}^{\prime \prime}$, it can be accounted by using the standard perturbation theory [1] which is applicable for states (8) with small wave vector components $k_{x}$ and $k_{y}$. In this way we shall obtain expressions for energy levels $\varepsilon_{+1 / 2, N_{0}}$ and $\varepsilon_{-1 / 2, N_{0}}$. As a result, the photon-induced spin splitting, $\Delta \varepsilon=$ $\varepsilon_{-1 / 2, N_{0}}-\varepsilon_{+1 / 2, N_{0}}$, is given by

$$
\begin{equation*}
\Delta \varepsilon=2 \sqrt{\left[\mu H_{0}\right]^{2}-\left[\frac{\hbar e H_{0}^{2}}{m c}\right]\left[\mu_{a}+\frac{\mu_{B}}{2}\right]+\left[\frac{\hbar \omega_{0}}{2}\right]^{2}}-\hbar \omega_{0} \tag{11}
\end{equation*}
$$

for $p_{0} / m c \ll 1$, where $p_{0}=e H_{0} / \omega_{0}$ is momentum of the rotating particle [3]. Since magnetic moment of the particle is $\mu=\mu_{a}+\mu_{B}$, the splitting (11) for charged particles in vacuum has the form

$$
\begin{equation*}
\Delta \varepsilon=\sqrt{\left(2 \mu_{a} H_{0}\right)^{2}+\left(\hbar \omega_{0}\right)^{2}}-\hbar \omega_{0} \tag{12}
\end{equation*}
$$

and depends only on anomalous part of magnetic moment, $\mu_{a}$. Considering free electrons in condensed matter, their mass $m$ in Eq. (11) should be interpreted as effective electron mass, $m_{0}^{*}$, while the Bohr magneton $\mu_{B}=e \hbar / 2 m_{0} c$ depends on electron mass in vacuum, $m_{0}$. In this case the sign of $\Delta \varepsilon$ depends on $m_{0}^{*}$. Neglecting the small quantity $\mu_{a}$ for electrons, we obtain from Eq. (11) that for $m_{0}<m_{0}^{*}$ the ground electron state in condensed matter is $\varepsilon_{+1 / 2, N_{0}}$, and for $m_{0}>m_{0}^{*}$ one is $\varepsilon_{-1 / 2, N_{0}}$.

If the particle rotation induced by the wave is suppressed, the interaction (10) does not influence on the spin splitting. Then the interaction Hamiltonian can be written in the same form, $\hat{\mathcal{H}}_{\text {int }}=-\hat{\boldsymbol{\mu}} \hat{\mathbf{H}}$, as for uncharged particles. As a result, in this case the expressions (7) and (9), obtained before for electrically neutral particles, can be used for charged particles as well. Such a suppression takes place for confined charged particles, including electrons in atoms and nanostructures, as well as for free electrons in condensed matter for $\omega_{0} \tau \ll 1$, where $\tau$ is electron mean free time. For electrons the splitting (12) is much less than the splitting (7) because of the ratio
$\mu_{a} / \mu \sim 10^{-3}$. Therefore electron systems, where the rotation is suppressed, are most suitable for observation of the discussed effect.

Formally, the results described above are obtained for nonrelativistic particles. To describe the relativistic case, we have to start from the Hamiltonian, $\hat{\mathcal{H}}=$ $\hbar \omega_{0} \hat{a}^{\dagger} \hat{a}+\hat{\mathcal{H}}_{D}(\hat{\mathbf{p}}-e \hat{\mathbf{A}} / c)$, based on the Dirac Hamiltonian $\hat{\mathcal{H}}_{D}(\hat{\mathbf{p}})$ [2], that will be done elsewhere. Running ahead, it should be noted that the relativistic analysis predicts novel effects (for instance, the increasing of particle mass, arising from the photon dress of particle), but leads to the same above-described expressions in the limiting case of small particle velocities, as expected.

Finalizing the Letter, let us duscuss possible observable consequences of the particle-photon coupling described by Eqs. (3)-(7). The first effect of the photoninduced spin splitting (7) is magnetization of particles exposed to the wave. It follows from Eq. (5) that the particle, being in the ground state $\varepsilon_{+1 / 2, N_{0}}$, is spinpolarized along the $z$-axis. This implies that an equilibrium gas of particles, exposed to the circularly polarized wave, will be spin-polarized along angular momentum vector of photons. In the particular case of nondegenerate gas, the magnetization vector can be written as $\mathbf{M}=\left(\mu n \omega_{0} / \Omega\right) \tanh (\Delta \varepsilon / 2 T) \mathbf{e}_{z}$, where $T$ is temperature, and $n$ is density of the gas.

The second effect is optical transitions with frequencies different from the wave frequency, $\omega_{0}$. That effect arises from interaction between the bound particlephoton states (3) and free photons of other kinds. Nonzero magnetodipole moments for the transitions are given by

$$
\begin{align*}
\left.\left|\left\langle\psi_{ \pm 1 / 2, N_{0} \mp 1}\right| \mathbf{e}_{\mp} \hat{\boldsymbol{\mu}}\right| \psi_{ \pm 1 / 2, N_{0}}\right\rangle \mid & =|\mu| \frac{\sqrt{\Omega^{2}-\omega_{0}^{2}}}{\sqrt{2} \Omega} \\
\left.\left|\left\langle\psi_{\mp 1 / 2, N_{0}}\right| \mathbf{e}_{\mp} \hat{\boldsymbol{\mu}}\right| \psi_{ \pm 1 / 2, N_{0}}\right\rangle \mid & =|\mu| \frac{\Omega+\omega_{0}}{\sqrt{2} \Omega} \\
\left.\left|\left\langle\psi_{\mp 1 / 2, N_{0} \pm 2}\right| \mathbf{e}_{ \pm} \hat{\boldsymbol{\mu}}\right| \psi_{ \pm 1 / 2, N_{0}}\right\rangle \mid & =|\mu| \frac{\Omega-\omega_{0}}{\sqrt{2} \Omega} \\
\left.\left|\left\langle\psi_{\mp 1 / 2, N_{0} \pm 1}\right| \mathbf{e}_{z} \hat{\boldsymbol{\mu}}\right| \psi_{ \pm 1 / 2, N_{0}}\right\rangle \mid & =|\mu| \frac{\sqrt{\Omega^{2}-\omega_{0}^{2}}}{2 \Omega} \tag{13}
\end{align*}
$$

As a result, transitions between the states (3) with nonzero matrix elements (13) can be accompanied by magnetodipole emission and absorption of electromagnetic radiation. It follows from the expressions (13) and (4) that there are the three new transition frequencies, $\Omega, \Omega-\omega_{0}, \Omega+\omega_{0}$. Allowed optical transitions with these frequencies from the ground state $\varepsilon_{+1 / 2, N_{0}}$ are pictured in Fig. 1. by arrows. It should be stressed that the frequency (6) depends on the magnetic field amplitude, $H_{0}$, that leads to dependence of the transition frequencies on a wave intensity. Besides transitions with the above-mentioned new frequencies, there are optical transitions with the wave frequency, $\omega_{0}$. They are described by the first of the matrix elements (13). These transitions


FIG. 1: Schematics of optical transitions between bound particle-photon states.
change the photon occupation number of the wave by one and should be interpreted as scattering of photons by the particle. It should be reminded that optical transitions are accompanied by momentum transfer to particles from photons, that can change mechanical energy of particles (the quantum recoil effect). As a consequence, the abovementioned frequencies describe the optical transitions accurately when the quantum recoil can be neglected. This takes place, particularly, for confined particles. Let us stress that the bound particle-photon states, $\psi_{ \pm 1 / 2, N}$, can be classified by the $z$-projection of angular momentum of the particle-photon system, $l_{z}=N \pm 1 / 2$. As expected, the nonzero matrix elements (13) correspond to transitions between states (3) with projections $l_{z}$ different by $-1,0,1$.

It should be reminded that the electromagnetic wave has been assumed to have the clockwise polarization. If the wave is counterclockwise-polarized, the operators $\hat{\sigma}_{+}$ and $\hat{\sigma}_{-}\left(\hat{J}_{+}\right.$and $\left.\hat{J}_{-}\right)$in the Hamiltonian (2) should be permuted. In this case the expressions following from the Hamiltonian retain their form but particle spin (or, generally, total angular momentum of particle) changes its direction to the opposite. As to other kinds of photon polarization, it can be shown that magnetic moment couples to elliptically polarized photons weaker than to circularly polarized ones, and in the limiting case of linear polarization the coupling vanishes.

It follows from the aforesaid that the predicted particle coupling to photons is fundamental quantum effect unexplored before. Certainly, the presented first analysis does not exhaust the problem entirely but forms a basis for further experimental and theoretical studies.

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