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Kantowski-Sachs viscous fluid cosmological model with a varying Λ

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Abstract

We study the Kantowski-Sachs cosmological model filled with viscous fluid in the presence of cosmological term Λ. To get a deterministic solution, a condition between metric potentials is used. The viscous coefficient of bulk viscous fluid is assumed to be a power function of mass density, whereas coefficient of shear viscosity is considered proportional to scale of expansion in the model. It is found that the cosmological constant Λ is positive and is a decreasing function of time, which is supported by results from recent supernova observations. Some physical and geometrical properties of the models are also discussed.

Key Words: Kantowski-Sachs viscous fluid model, cosmology, variable cosmological constant.

1. Introduction

Investigation of relativistic cosmological models usually involves the energy momentum tensor of matter generated by a perfect fluid. To consider more realistic models one must take into account viscosity mechanisms; and, indeed, viscosity mechanisms has attracted the attention of many researchers. Misner [1, 2] suggested the strong dissipation due to neutrino viscosity may considerably reduce the anisotropy of the black body radiation. Weinberg [3, 4] suggested that a viscosity mechanism in cosmology can explain the unusual high entropy per baryon in present events. Waga et al. [5], Pachel et al. [6], Guth [7], and Murphy [8] have shown that bulk viscosity associated with the grand unified theory Phase transition (see Langackar in [9]) may lead to an inflationary scenario.

In the modern cosmological theories, the dynamic cosmological term $\Lambda(t)$ remains a focal point of interest as it solves the cosmological constant problem in a natural way. There is significant observational evidence towards identifying Einstein's cosmological constant Λ or a component of material content of the universe that varies slowly with time and space and so acts like Λ. Recent cosmological observations by the

High-z Supernova Team and the Supernova Cosmological Project [10–16] suggest the existence of a positive cosmological constant Λ with magnitude $\Lambda\left(\frac{G\hbar}{c^3}\right) \approx 10^{-123}$.

A uniform cosmological model filled with fluid under pressure and with viscosity has been developed by Murphy [8]. A solution that we have found exhibits an interesting feature where the big bang type singularity appears in the infinite past. Exact solutions for isotropic homogeneous cosmology for open, closed and flat universes have been found by Santos et al. [17] with the bulk viscosity being a power function of energy density.

The effect of bulk viscosity on cosmological evolution has been investigated by a number of authors in the context of general theory of relativity [18–26]. The nature of cosmological solution for homogenous cosmological model was investigated by Belinsky et al. [27] and shown that viscosity can not remove the cosmological singularity but result in a qualitatively new behavior of the solution near singularity. Huang [28] has studied Bianchi type models with bulk viscosity as a power function of energy density and when the universe is filled with stiff matter. The effect of bulk viscosity, with a time varying bulk viscous coefficient, on the evolution of isotropic FRW models in the context of open thermodynamics system was studied by Desikan [29]. Ray et al. [30] have studied anisotropic-charged fluid sphere with varying cosmological constant. Bianchi type I cosmological models with cosmological term Λ was studied by Singh et al. [31], Chakravarty and Biswas [32] and Belinchon [33]. A new class of bulk viscous universe with time dependent declaration parameter and Λ-term was investigated by Pradhan and Otarod [34]. A spherical inhomogeneous dust collapse with position Λ has been studied by Ghose and Deshkar [35]. Pradhan and Otarod [36] have studied the universe with time dependent declaration parameter and Λ term in the presence of a perfect fluid. Bali and Yadav [37] and Bijan and Saha [38] have investigated the Bianchi type IX viscous fluid cosmological models. Weber [39, 40] has done a qualitative study of Kantowski and Sachs [41] cosmological models. Lorentz [42]; Gron [43]; Matravers [44]; Krori et al. [45]; Xanthopoulos et al. [46]; Dabrowski [47]; and Li and Hao [48] have also studied cosmological model for the Kantowski-Sachs space-time. Recently, Pradhan et al. [49] have studied Bianchi type IX viscous fluid cosmological with a varying Λ.

Motivated by the situation discussed above, in this paper we focus upon the problem with cosmological constant in the presence of viscous fluid in evolution of the Kantowski-Sachs cosmological model.

2. Metric and the field equations

We consider the Kantowski-Sachs metric in the form

$$ds^2 = dt^2 - A^2 dr^2 - B^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{1}$$

where A and B are functions of cosmic time t only.

The Einstein's field equations (in gravitational units $c = 1,\ G = 1$) are

$$R_i^j - \frac{1}{2} R g_i^j = -\left(8\pi T_i^j - \Lambda g_i^j\right), \tag{2}$$

where R_i^j is the Ricci tensor; $R = g^{ij} R_{ij}$ is the Ricci scalar; and T_i^j is the energy momentum tensor of a viscous fluid given by

$$T_i^j = (\rho + p')\, u_i u^j - p' \delta_i^j + \eta g^{j\beta}\left[u_{i;\beta} + u_{\beta;i} - u_i u^\alpha u_{\beta;\alpha} - u_\beta u^\alpha u_{i;\alpha}\right], \tag{3}$$

where $p' = p - \left(\xi - \frac{2}{3}\eta\right) u^i_{;i}$. Here, ρ is the energy density, $p$is pressure, and η and ξ are the coefficients of shear and bulk viscosity, respectively. The Semicolon (;) indicates covariant differentiation. Note that the shear and bulk viscosities $\eta$and ξ are positively definite, i.e. $\eta > 0$, $\xi > 0$; and may be either constant or functions of time or energy such as

$$\eta = |a|\ \rho^{\alpha}, \quad \xi = |b|\ \rho^{\beta}\,,$$

where a and b are constants. u_i is the flow vector satisfying the relations

$$g_{ij}u^i u^j = 1. \tag{4}$$

We choose the coordinates to be commoving so that

$$u^1 = u^2 = u^3 = 0, \quad u^4 = 1\,. \tag{5}$$

The Einstein's field equations (2) for the line element (1) has been set up as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} + \Lambda = 8\pi\left[-p' + 2\,\eta\,\frac{\dot{A}}{A}\right], \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \Lambda = 8\pi\left[-p' + 2\,\eta\,\frac{\dot{B}}{B}\right], \tag{7}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} + \Lambda = 8\pi\rho\,, \tag{8}$$

where a dot $(\cdot)$ over a variable denotes ordinary differentiation with respect to time t.

3. Solution of the field equations

Equations (6)–(8) are three independent equations in seven unknowns A, B, ρ, p, η, ξ and Λ. For the complete determinacy of the system, we need extra conditions. First, we assume a relation in metric potential as

$$A = B^m. \tag{9}$$

Second, we assume that the coefficient of shear viscosity is proportional to the scale of expansion, i.e.

$$\eta \propto \theta, \tag{10}$$

where m is a real number and θ is the scalar of expansion given by

$$\theta = u^i_{;\,i}. \tag{11}$$

With these extra conditions, equations (6) and (7) lead to

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} = 16\pi\eta\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right]. \tag{12}$$

From equation (10), we obtain

$$\eta = l\left[\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right], \tag{13}$$

where l is a proportionality constant.

Equations (12) together with (9) and (13) lead to

$$B\ddot{B} + \beta\dot{B}^2 = 1/\left(m-1\right), \tag{14}$$

which can be rewritten as

$$\frac{d}{dB}\left(f^2\right) + \frac{2\beta}{B}\left(f^2\right) = \frac{2}{B\left(m-1\right)}, \tag{15}$$

where

$$\beta = m + 1 + 16\pi\, l\ \left(m+2\right), \tag{16}$$

and

$$\dot{B} = f\ \left(B\right). \tag{17}$$

From (15), we obtain

$$\left(\frac{dB}{dt}\right)^2 = \left[\frac{1}{\beta\left(m-1\right)} + \frac{\alpha}{B^{2\beta}}\right], \tag{18}$$

where α is a constant of integration.

After a suitable transformation of coordinates, the metric (1) reduces to the form

$$ds^2 = \left[\frac{1}{\beta\left(m-1\right)} + \frac{\alpha}{T^{2\beta}}\right]^{-1} dT^2 - T^{2m}dr^2 - T^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \tag{19}$$

where $B = T$. The pressure and density for model (19) are given by

$$8\pi p = \left\{K_1\left[\frac{1}{\beta(m-1)} + \frac{\alpha}{T^{2\beta}}\right] - 1\right\}\frac{1}{T^2} + \frac{\alpha\beta}{T^{2(\beta+1)}} + 8\pi\xi\left(m+2\right)\left[\frac{1}{(m-1)\beta T^2} + \frac{\alpha}{T^{2(\beta+1)}}\right]^{\frac{1}{2}} - \Lambda, \tag{20}$$

$$8\pi\rho = \frac{K_2}{T^2} + \frac{\left(2m+1\right)\alpha}{T^{2(\beta+1)}} + \Lambda, \tag{21}$$

where

$$K_1 = 16\pi\left(m+2\right)\left(ml - \frac{l}{3}\left(m+2\right)\right) - 1\,, K_2 = \frac{\left(2m+1\right)}{\left[m+1+16\pi\, l\ \left(m+2\right)\right]\left(m-1\right)} + 1\,.$$

For the specification of ξ we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{22}$$

where $0 \le \gamma \le 1$ is constant.

Thus, given $\xi(t)$, we can solve for the cosmological parameters. In most of the investigations, bulk viscosity is assumed to be a simple power function of the energy density (see, for examples, Pavon [50], Maartens [20], Zimdahl [21]):

$$\xi(t) = \xi_0 \rho^n, \tag{23}$$

where ξ_0 and n are constants.

If $n = 1$, equation (23) may corresponds to a radiative fluid (see Weinberg in [3, 4]). However, more realistic models (see Santos in [17]) are based on n lying in the regime $0 \leq n \leq \frac{1}{2}$.

On using (23) in (20), we obtain following relation for the pressure:

$$8\pi p = \left[K_1\left(\frac{1}{(m-1)\beta} + \frac{\alpha}{T^{2\beta}}\right) - 1\right]\frac{1}{T^2} + \frac{\alpha\beta}{T^{2(\beta+1)}} + 8\pi\xi_0\rho^n (m+2)\left[\frac{1}{(m-1)\beta T^2} + \frac{\alpha}{T^{2(\beta+1)}}\right]^{\frac{1}{2}} - \Lambda. \tag{24}$$

3.1. Model I: Solution for $\xi = \xi_0$

When $n = 0$ equation (23) reduces to $\xi = \xi_0 =$constant. Hence in this case, equation (24), with the use of (21) and (22), leads to

$$\begin{aligned} 8\pi(1+\gamma)\rho &= \left[K_1\left(\frac{1}{(m-1)\beta} + \frac{\alpha}{T^{2\beta}}\right) + \frac{(2m+1)}{(m-1)\beta}\right]\frac{1}{T^2} + \frac{\alpha(2m+\beta+1)}{T^{2(\beta+1)}} \\ &\quad + 8\pi\xi_0(m+2)\left[\frac{1}{(m-1)\beta T^2} + \frac{\alpha}{T^{2(\beta+1)}}\right]^{\frac{1}{2}}. \end{aligned} \tag{25}$$

Eliminating $\rho(t)$ between equations (21) and (25), we have

$$\begin{aligned} (1+\gamma)\Lambda &= \left\{K_1\left(\frac{1}{(m-1)\beta} + \frac{\alpha}{T^{2\beta}}\right) - \left[1 + \frac{(2m+1)\gamma}{(m-1)\beta} + \gamma\right]\right\}\frac{1}{T^2} + \frac{\alpha[\beta - (2m+1)\gamma]}{T^{2(\beta+1)}} \\ &\quad + 8\pi\xi_0(m+2)\left[\frac{1}{(m-1)\beta T^2} + \frac{\alpha}{T^{2(\beta+1)}}\right]^{\frac{1}{2}}. \end{aligned} \tag{26}$$

3.2. Model II: Solution for $\xi = \xi_0\,\rho$

When $n = 1$ equation (23) reduces to$\xi = \xi_0\rho$. Hence in this case equation (24), with the use of (21) and (22), leads to

$$\begin{aligned} 8\pi\rho &= \frac{1}{\left[1 + \gamma - \xi_0(m+2)\left(\frac{1}{(m-1)\beta T^2} + \frac{\alpha}{T^{2(\beta+1)}}\right)^{\frac{1}{2}}\right]} \\ &\quad \times \left\{\left[K_1\left(\frac{1}{(m-1)\beta} + \frac{\alpha}{B^{2\beta}}\right) + \frac{(2m+1)}{(m-1)\beta}\right]\frac{1}{T^2} + \frac{\alpha(2m+\beta+1)}{T^{2(\beta+1)}}\right\}. \end{aligned} \tag{27}$$

Eliminating $\rho(t)$ between equations (21) and (27), we have

$$\begin{aligned}\Lambda &= \frac{1}{\left[1+\gamma-\xi_0\,(m+2)\left(\frac{1}{(m-1)\beta T^2}+\frac{\alpha}{T^{2(\beta+1)}}\right)^{\frac{1}{2}}\right]} \\ &\times\left\{\left[K_1\left(\frac{1}{(m-1)\,\beta}+\frac{\alpha}{T^{2\beta}}\right)+\frac{(2m+1)}{(m-1)\,\beta}\right]\frac{1}{T^2}+\frac{\alpha\,(2m+\beta+1)}{T^{2(\beta+1)}}\right\}-\left[\frac{K_2}{T^2}+\frac{(2m+1)\,\alpha}{T^{2(\beta+1)}}\right]. \quad (28)\end{aligned}$$

Some physical aspects of the models

A straightforward calculation leads to the following expression for the scalar of expansion θ for the shear σ of the fluid for the metric (19):

$$\theta=(m+2)\left[\frac{1}{(m-1)\,\beta T^2}+\frac{\alpha}{T^{2(\beta+1)}}\right]^{\frac{1}{2}} \quad (29)$$

$$\sigma=\left[\left(m^2+2\right)\frac{\dot{T}^2}{T^2}-\frac{1}{3}\,(m+2)^2\left(\frac{1}{(m-1)\,\beta T^2}+\frac{\alpha}{T^{2(\beta+1)}}\right)\right]^{\frac{1}{2}}. \quad (30)$$

The expansion factor θ decreases as a function of T and asymptotically approaches zero with ρ and $p$also approaching zero as $T\to\infty$.

4. Particular models

If we set $m=2$, the geometry of space-time (19) reduces to the form

$$\begin{aligned}ds^2&=\left[\tfrac{1}{3+64\,\pi\,l}+\tfrac{\alpha}{T^{2(3+64\,\pi\,l)}}\right]^{-1}dT^2\\ &-T^4dr^2-T^2\left(d\theta^2+\sin^2\theta\,d\phi^2\right),\end{aligned} \quad (31)$$

where $\alpha$is a constant of integration. The pressure and density for model (31) are given by

$$\begin{aligned}8\pi p &= \left[\left(\frac{128\pi\,l}{3}-1\right)\left(\frac{1}{3+64\,\pi\,l}+\frac{\alpha}{T^{2(3+64\,\pi\,l)}}\right)-1\right]\frac{1}{T^2}\\ &+\frac{\alpha\,(3+64\,\pi\,l)}{T^{8(1+16\,\pi\,l)}}+32\pi\xi\left[\frac{1}{(3+64\,\pi\,l)\,T^2}+\frac{\alpha}{T^{8(1+16\,\pi\,l)}}\right]^{\frac{1}{2}}-\Lambda\,;\end{aligned} \quad (32)$$

$$8\pi\rho=\left[\frac{5}{3+64\pi\,l}+1\right]\frac{1}{T^2}+\frac{5\alpha}{T^{8(1+16\,\pi\,l)}}+\Lambda\,. \quad (33)$$

4.1. Model I: Solution for $\xi = \xi_0$

When $n = 0$ equation (23) reduces to $\xi = \xi_0$ =constant. Hence in this case equation (32), with the use of (33) and (22), leads to the relation

$$\begin{aligned} 8\pi\,(1+\gamma)\,\rho \;&=\; \left[\left(\frac{128\,\pi\,l}{3}-1\right)\left(\frac{1}{(3+64\,\pi\,l)}+\frac{\alpha}{T^{2(3+64\,\pi\,l)}}\right)+\frac{5}{(3+64\,\pi\,l)}\right]\frac{1}{T^2} \\ &\quad +\frac{\alpha\,(8+64\,\pi\,l)}{T^{8(1+16\pi\,l)}}+32\pi\xi_0\left[\frac{1}{(3+64\,\pi\,l)\,T^2}+\frac{\alpha}{T^{8(1+16\,\pi\,l)}}\right]^{\frac{1}{2}}. \end{aligned} \tag{34}$$

Eliminating $\rho\,(t)$ between equations (33) and (34), we obtain

$$\begin{aligned} (1+\gamma)\,\Lambda \;&=\; \left[\left(\frac{128\,\pi\,l}{3}-1\right)\left(\frac{1}{(3+64\,\pi\,l)}+\frac{\alpha}{T^{2(3+64\,\pi\,l)}}\right)-\frac{5\gamma}{(3+64\,\pi\,l)}-(\gamma+1)\right]\frac{1}{T^2} \\ &\quad +\frac{\alpha\,(8+64\,\pi\,l)-5\,(\gamma+1)}{T^{8(1+16\pi\,l)}}+32\pi\xi_0\left[\frac{1}{(3+64\,\pi\,l)\,T^2}+\frac{\alpha}{T^{8(1+16\,\pi\,l)}}\right]^{\frac{1}{2}}. \end{aligned} \tag{35}$$

4.2. Model II: Solution for $\xi = \xi_0\rho$

When $n = 1$ equation (23) reduces to $\xi = \xi_0\rho$. Hence in this case equation (32), with the use of (33) and (22), leads to the relation

$$\begin{aligned} \rho \;&=\; \frac{1}{8\pi\left[1+\gamma-\frac{4\xi_0}{T}\left[\frac{1}{(3+64\,\pi\,l)T^2}+\frac{\alpha}{T^{2(3+64\,\pi\,l)}}\right]^{\frac{1}{2}}\right]} \\ &\quad \times\left\{\left[\left(\frac{128\,\pi\,l}{3}-1\right)\left(\frac{1}{(3+64\,\pi\,l)}+\frac{\alpha}{T^{2(3+64\,\pi\,l)}}\right)+\frac{5}{(3+64\,\pi\,l)}\right]\frac{1}{T^2}+\frac{\alpha\,(8+64\,\pi\,l)}{T^{8\,(1+16\,\pi\,l)}}\right\} \end{aligned} \tag{36}$$

$$\begin{aligned} \Lambda \;&=\; \frac{1}{8\pi\left[1+\gamma-\frac{4\xi_0}{T}\left[\frac{1}{(3+64\,\pi\,l)T^2}+\frac{\alpha}{T^{2(3+64\,\pi\,l)}}\right]^{\frac{1}{2}}\right]} \\ &\quad \times\left\{\left[\left(\frac{128\,\pi\,l}{3}-1\right)\left(\frac{1}{(3+64\,\pi\,l)}+\frac{\alpha}{T^{2(3+64\,\pi\,l)}}\right)+\frac{5}{(3+64\,\pi\,l)}\right]\frac{1}{T^2}+\frac{\alpha\,(8+64\,\pi\,l)}{T^{8\,(1+16\,\pi\,l)}}\right\} \\ &\quad -\left[\frac{5}{(3+64\,\pi\,l)}+1\right]\frac{1}{T^2}-\frac{5\alpha}{T^{8\,(1+16\,\pi\,l)}}. \end{aligned} \tag{37}$$

Some physical aspects of the models

The expansion factor θ and the shear σ in model class (31) are given by

$$\theta = 4\left[\frac{1}{(3+64\,\pi\,l)\,T^2}+\frac{\alpha}{T^{8(1+16\,\pi\,l)}}\right]^{\frac{1}{2}} \tag{38}$$

$$\sigma = \left[6\frac{\dot{T}^2}{T^2} - \frac{16}{3}\left(\frac{1}{(3+64\,\pi\, l)\,T^2} + \frac{\alpha}{T^{8(1+16\,\pi\, l)}} \right) \right]^{\frac{1}{2}} . \tag{39}$$

Note that the expansion factor θ is a decreasing function of T. Since $\lim\limits_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, isotropy is not approached for large values of T. Also note the presence of a real, physical singularity at $T = 0$.

5. Special models

If we set $m = 2$ and $l = -\frac{1}{32\pi}$, equations (18) lead to

$$\frac{BdB}{\sqrt{B^2+\alpha}} = dt. \tag{40}$$

This on integration gives

$$B^2 = t^2 + \alpha', \tag{41}$$

where $\alpha' = d_1^2 - \alpha$ and d_1 is an integrating constant. Hence we obtain

$$A = B^2 = t^2 + \alpha'. \tag{42}$$

Using the transformation

$$T = t^2 + \alpha', \tag{43}$$

metric (1) takes the form

$$ds^2 = \frac{1}{4\,(T-\alpha')} dT^2 - T^2 dr^2 - T\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) . \tag{44}$$

The pressure and density for the model (44) are given by

$$8\pi\, p = \frac{\alpha}{T^4} - \left[\frac{7}{3}\left(1+\frac{\alpha}{T^2}\right) - 1 \right] \frac{1}{T^2} - \frac{32\pi\xi}{T}\left(1+\frac{\alpha}{T^2}\right)^{\frac{1}{2}} - \Lambda\,, \tag{45}$$

$$8\pi\rho = \left[6 + \frac{5\alpha}{T^4} \right] \frac{1}{T^2} + \Lambda\,. \tag{46}$$

5.1. Model I: Solution for $\xi = \xi_0$

When $n = 0$, equation (23) reduces to $\xi = \xi_0 =$constant. Hence in this case equation (45), with the use of (46) and (22), leads to

$$8\pi\,(1+\gamma)\,\rho = \left[6 + \frac{5\alpha}{T^4} - \frac{7}{3}\left(1+\frac{\alpha}{T^2}\right) + 1 \right] \frac{1}{T^2} + \frac{\alpha}{T^4} + \frac{32\pi\xi_0}{T}\left(1+\frac{\alpha}{T^2}\right)^{\frac{1}{2}} , \tag{47}$$

$$(1+\gamma)\,\Lambda = \left[6 + \frac{5\alpha}{T^4} - \frac{7}{3}\left(1+\frac{\alpha}{T^2}\right) + 1 \right] \frac{1}{T^2} + \frac{\alpha}{T^4} - \frac{\gamma+1}{T^2}\left(6 + \frac{5\alpha}{T^4}\right) + \frac{32\pi\xi_0}{T}\left(1+\frac{\alpha}{T^2}\right)^{\frac{1}{2}} . \tag{48}$$

5.2. Model II: Solution for $\xi = \xi_0 \rho$

When $n = 1$, equation (23) reduces to $\xi = \xi_0 \rho$. Hence in this case equation (45), with the use of (46) and (22), leads to

$$8\pi\,\rho = \frac{\left[7 + \frac{5\alpha}{T^4} - \frac{7}{3}\left(1 + \frac{\alpha}{T^2}\right)\right]\frac{1}{T^2} + \frac{\alpha}{T^4}}{1 + \gamma - \frac{4\xi_0}{T}\left(1 + \frac{\alpha}{T^2}\right)^{\frac{1}{2}}}\,, \tag{49}$$

$$\Lambda = \frac{\left\{\left[7 + \frac{5\alpha}{T^4} - \frac{7}{3}\left(1 + \frac{\alpha}{T^2}\right)\right]\frac{1}{T^2} + \frac{\alpha}{T^4}\right\} - \left(6 + \frac{5\alpha}{T^4}\right)\frac{1}{T^2}}{1 + \gamma - \frac{4\xi_0}{T}\left(1 + \frac{\alpha}{T^2}\right)^{\frac{1}{2}}}. \tag{50}$$

Some physical aspects of the models

The expansion θ and the shear σ in the model (44) are given by

$$\theta = 4\left[\frac{1}{T^2} + \frac{\alpha}{T^4}\right]^{\frac{1}{2}}, \tag{51}$$

$$\sigma = \frac{1}{T^2}\left[6\dot{T}^2 - 16\left(1 + \frac{\alpha}{T^2}\right)\right]^{\frac{1}{2}}. \tag{52}$$

As $T \to \infty$, $\theta \to 0$. Since $\lim\limits_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, isotropy is not approached for the large values of T. Note the presence of a real, physical singularity at $T = 0$.

6. Conclusion

We have presented Kantowski-Sachs models with viscous fluid in presence of cosmological term Λ. In all models, we observe that they do not approach isotropy for large values of time T. We have assumed that the fluid obeys an equation of state of the form $p = \gamma\rho$, and bulk viscosity is assumed to be a simple power function of energy density given by $\xi(t) = \xi_0 \rho^n$. The cosmological constant in all models given in sections 3.1 and 3.2 are decreasing functions of time and all approach a small positive value as time increases. The values of cosmological constant for these models are found to be small and positive and are supported by results recently obtained by the High-z Supernova Search Team and the Supernova Cosmological Project [10–16]. It is observed that the expansion factor θ is a decreasing function of T and approaches 0 as $T \to \infty$. Also from the equation (27), the energy density ρ approaches 0 as $T \to \infty$. Since $\lim\limits_{T\to\infty} \frac{\sigma}{\theta}$ = constant, the model is not isotropic for large value of T.

The particular model for the value of $m = 2$ is also obtained. We observed that ρ and p approach 0 as $T \to \infty$. Thus we have obtained a physically relevant decay law for the cosmological constant without considering an assumption for variation.

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