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Tilted Bianchi Type I dust fluid magnetized cosmological model in general relativity

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Abstract

Tilted Bianchi Type I perfect fluid cosmological model in presence of magnetic field is investigated. To get a determinate solution, we assume $p = 0$ and $A = BC$, where A, B and C are metric potentials. A special model is also investigated in the absence of magnetic field. The various physical and geometrical aspects of both the models are also discussed. The effect of the magnetic field on the model is also discussed.

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1. Introduction

Homogeneous and anisotropic cosmological models have been studied widely in the framework of general relativity. These models are more restricted than the inhomogeneous models. In spite of this, they explain a number of observed phenomena quite satisfactorily. In recent years, there has been a considerable interest in investigating spatially homogeneous and anisotropic cosmological models in which matter does not move orthogonal to the hyper surface of homogeneity. Such types of models are called tilted cosmological models.

The general dynamics of these cosmological models have been studied in detail by King and Ellis [1], Ellis and King [2], Collins and Ellis [3]. Ellis and Baldwin [4] have investigated that we are likely to be living in a tilted universe and they have indicated that how we may detect it. King and Ellis [1] have found that there is no Bianchi Type I tilted models if it has been obtained under the assumption that matter takes the perfect fluid form in which

$$T_i^j = (\varepsilon + p)\nu_i \nu^j + p g_i^j + q_i \nu^j + \nu_i q^j + E_i^j$$

$$\nu_i \nu^j = -1 \qquad \varepsilon > 0, p > 0.$$

ν^i is the velocity flow vector and ε, p are the density and pressure of the fluid, and T_i^j denotes the energy-momentum tensor. Dunn and Tupper [5] have shown that a Bianchi tilting universe is possible when an electromagnetic field is present.

Many authors have considered the behavior of individual Bianchi models that contain either a pure magnetic field or magnetic field plus fluid. The magnetic field has significant role in cosmological scale and is present in galactic and intergalactic spaces. Bianchi Type I magnetized orthogonal cosmological models have been studied in detail due to their simplicity. The tilted cosmological models, in presence of magnetic fields are more complicated than those of orthogonal universe.

Bianchi type I cosmological models have been studied by several authors in various context: such as Mazumdar [6], Aguirregabiria [7], Yavuz [8], Beesham [9], Singh and Gupta [10], among others. Primordial magnetic field of cosmological origin have been speculated by Asseo and Sol [11]. Mukherjee [12] has investigated tilted Bianchi Type I universe with heat flux in general relativity, and has shown that the universe assumes a pancake shape. The velocity vector is not geodesic and heat flux is comparable to energy density. Cosmological models with heat flow have been studied by several authors[13–17]. Recently, Bali and Sharma [18] investigated Tilted Bianchi Type I models with heat conduction filled with disordered radiation of perfect fluid in general relativity. Bali and Meena [19] investigated magnetized stiff fluid tilted universe for perfect fluid distribution in general relativity. Bali and Meena [20] also derived flat tilted Bianchi Type V cosmological model in general relativity.

In general relativity, a dust solution is an exact solution of the Einstein field equation in which the gravitational field is produced entirely by the mass, momentum and stress density of a perfect fluid which has positive mass energy density but vanishing pressure. Banerjee and Banerjee [21] studied stationary distribution of dust and electromagnetic fields in general relativity. Bali and Sharma [22] have investigated tilted Bianchi Type I dust fluid and shown that model has cigar type singularity when $T = 0$.

Banerjee et al. [23] have investigated an axially symmetric Bianchi Type I string dust cosmological model in presence and absence of magnetic field. Recently, Bali and Upadhaya [24] have investigated LRS Bianchi Type I strings dust-magnetized cosmological models using the condition $\sigma \propto \theta$, where σ is shear and θ is scalar of expansion which leads to $A = \alpha \mathrm{B}^n$, where n is constant. Concerning the tilted perfect fluid models, Bradley [25] has stated that a tilted dust self-similar model does not exist.

In this paper, we have investigated Bianchi Type I tilted dust fluid cosmological model in the presence and absence of magnetic field. To get a determinate solution, we have also assumed a supplementary condition A=BC between metric potential. The various physical and geometrical aspects of the models are discussed. The effect of magnetic field has also been studied. The magnetic field is due to an electric current produced along x-axis.

2. The Metric and Field Equations

We consider the metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2, \tag{1}$$

where A, B and C are functions of cosmological time t alone.

The energy-momentum tensor for perfect fluid distribution with heat conduction is taken into the form given by Ellis [26] as

$$T_i^j = (\varepsilon + p)\nu_i\nu^j + pg_i^j + q_i\nu^j + \nu_i q^j + E_i^j, \tag{2}$$

together with

$$g_{ij}\nu^i\nu^j = -1, \tag{3}$$

$$q_i q^j > 0, \tag{4}$$

$$q_i\nu^i = 0. \tag{5}$$

Here, E_i^j is the energy-momentum tensor of electromagnetic field given by Lichnerowicz [27] as

$$E_i^j = \bar{\mu}\left[|h|^2\left(\nu_i\nu^j + \frac{1}{2}g_i^j\right) - h_i h^j\right], \tag{6}$$

where $\bar{\mu}$ is magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}}\varepsilon_{ijk\ell}F^{k\ell}\nu^j. \tag{7}$$

$F_{k\ell}$is the electromagnetic field tensor and $\varepsilon_{ijk\ell}$ is the Levi-Civita tensor density, p is pressure, ε is density, and q^i is heat conduction vector orthogonal to the fluid flow vector ν^i. The fluid flow vector ν^i has the components $((\sinh\lambda)/A, 0, 0, \cosh\lambda)$, satisfying condition (3); and λ is the tilt angle. Thus

$$\nu^1 = \frac{\sinh\lambda}{A}, \nu^2 = 0, \nu^3 = 0, \nu^4 = \cosh\lambda \tag{8}$$

We take the incident magnetic field in the direction of x-axis so that

$$h_1 \neq 0, h_2 = 0, h_3 = 0, h_4 \neq 0.$$

Due to the assumption of infinite electrical conductivity of the fluid we also find that

$F_{14} = 0 = F_{24} = F_{34}$. Using these assumption in the equation (7), it is easy to get $F_{12} = F_{13} = 0$. Thus the only non-vanishing component of F_{ij} is F_{23}.

The first set of Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0, \tag{9}$$

leads to

$$\left[\frac{\partial F_{ij}}{\partial x^k} - \Gamma^{\ell}_{jk} F_{i\ell} - \Gamma^{\ell}_{ik} F_{\ell j}\right] + \left[\frac{\partial F_{jk}}{\partial x^i} - \Gamma^{\ell}_{ki} F_{j\ell} - \Gamma^{\ell}_{ji} F_{\ell k}\right] + \left[\frac{\partial F_{ki}}{\partial x^j} - \Gamma^{\ell}_{ij} F_{k\ell} - \Gamma^{\ell}_{kj} F_{\ell i}\right] = 0$$

or

$$\left[\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j}\right] - \Gamma^{\ell}_{jk}(F_{i\ell} + F_{\ell i}) - \Gamma^{\ell}_{ki}(F_{j\ell} + F_{\ell j}) - \Gamma^{\ell}_{ij}(F_{k\ell} + F_{\ell k}) = 0,$$

which again leads to

$$\left[\frac{\partial F_{ij}}{\partial x^k} + \frac{\partial F_{jk}}{\partial x^i} + \frac{\partial F_{ki}}{\partial x^j}\right] = 0, \tag{10}$$

which shows that $\frac{\partial F_{23}}{\partial x^4} = 0$. (11)

Above, the F's in (9) denote covariant derivatives and Γ is a Christoffel symbol.

Therefore, F_{23} = constant = H (say).

From equation (7), we have

$$h_1 = \frac{AH}{\bar{\mu}BC} cosh\lambda \quad h_4 = \frac{-H}{\bar{\mu}BC} sinh\lambda, \tag{11}$$

and

$$|h|^2 = h_i h^i = h_1 h^1 + h_4 h^4 = g^{11}(h_1)^2 + g^{44}(h_4)^2 = \frac{H^2 \cosh^2 \lambda}{\bar{\mu}^2 B^2 C^2} - \frac{H^2 \sinh^2 \lambda}{\bar{\mu}^2 B^2 C^2} = \frac{H^2}{\bar{\mu}^2 B^2 C^2}. \tag{12}$$

In (11), $\bar{\mu}$ is magnetic permeability. Hence equation (6) leads to

$$E^1_1 = \frac{-H^2}{2\bar{\mu}B^2C^2} = -E^2_2 = -E^3_3 = E^4_4. \tag{13}$$

The Einstein's field equation is

$$R^j_i - \frac{1}{2} R g^j_i = -8\pi T^j_i, \quad (c = G = 1). \tag{14}$$

The field equation for the line element (1) hence leads to the following set of equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi \left[(\varepsilon + p) sinh^2\lambda + p + 2q_1 \frac{sinh\lambda}{A} - \frac{H^2}{2\bar{\mu}B^2C^2}\right], \tag{15}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = -8\pi \left[p + \frac{H^2}{2\bar{\mu}B^2C^2}\right], \tag{16}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -8\pi \left[p + \frac{H^2}{2\bar{\mu}B^2C^2}\right], \tag{17}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} = -8\pi \left[-(\varepsilon + p) cosh^2\lambda + p - 2q_1 \frac{sinh\lambda}{A} - \frac{H^2}{2\bar{\mu}B^2C^2}\right], \tag{18}$$

$$(\varepsilon + p) A sinh\lambda\, cosh\lambda + q_1 cosh\lambda + q_1 \frac{\sinh^2 \lambda}{cosh\lambda} = 0, \tag{19}$$

where the suffix "4" denotes ordinary differentiation with respect to cosmic time t alone.

3. Solution of the Field Equations

Equations from (15) to (19) are five equations in seven unknowns: A, B, C, ε, p, λ and q_1. For complete determination of these quantities, we assume two more conditions:

(i) The model is filled with dust or perfect fluid which leads to

$$p = 0. \tag{20}$$

(ii) A metric potential such that

$$A = BC. \tag{21}$$

Equations (15) and (19) lead to the relation

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 8\pi\left(\varepsilon - p\right) + \frac{8\pi H^2}{\bar{\mu}B^2C^2}. \tag{22}$$

From equations (20) and (22), we have

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 8\pi\varepsilon + \frac{8\pi H^2}{\bar{\mu}B^2C^2}, \tag{23}$$

which leads to the relation

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 8\pi\varepsilon + \frac{K}{B^2C^2}, \tag{24}$$

where

$$K = \frac{8\pi H^2}{\bar{\mu}}. \tag{25}$$

From equations (16) and (17), we have the relation

$$\frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{C_{44}}{C} - \frac{A_4C_4}{AC} = 0, \tag{26}$$

which leads to

$$\frac{\nu_4}{\nu} = \frac{a}{\mu^2}, \tag{27}$$

where $BC = \mu$, $\nu = \frac{B}{C}$ and a is constant of integration.

With the help of (25), equations (16) and (17) can also yield

$$\frac{2A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \frac{A_4B_4}{AB} = -16\pi p - \frac{K}{B^2C^2}. \tag{28}$$

For the dust fluid, equation (28) reduces to

$$\frac{2A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \frac{A_4B_4}{AB} = \frac{-K}{B^2C^2}. \tag{29}$$

Using (21) in equation (29), equation (29) becomes

$$\frac{6\mu_{44}}{\mu} + \frac{\mu_4^2}{\mu^2} + \frac{\nu_4^2}{\nu^2} = \frac{-2K}{\mu^2}. \tag{30}$$

With the help of (27), (30) reduces in the form

$$\frac{6\mu_{44}}{\mu} + \frac{\mu_4^2}{\mu^2} + \frac{a^2}{\mu^4} = \frac{-2K}{\mu^2}. \tag{31}$$

Equation (31) leads to

$$2ff^1 + \frac{f^2}{3\mu} = \frac{-a^2}{3\mu^3} - \frac{2K}{3\mu}, \tag{32}$$

where $\mu_4 = \mathrm{f}(\mu)$.

Equation (32) leads to

$$f^2 = \frac{1}{5\mu^2}\left[a^2 - 10K\mu^2 + 5b\mu^{5/3}\right], \tag{33}$$

where b is constant of integration.

From the equation (27), we have

$$log\nu = a\sqrt{5}\int \frac{d\mu}{\mu\sqrt{a^2 - 10K\mu^2 + 5b\mu^{5/3}}}. \tag{34}$$

Hence the metric (1) becomes

$$ds^2 = -\frac{d\mu^2}{f^2} + \mu^2 dx^2 + \mu\nu dy^2 + \frac{\mu}{\nu}dz^2, \tag{35}$$

where ν is determined by (34).

Introducing the transformations

$T = \mu$, $X = x$, $Y = y$, $Z = z$,

the metric (35) reduces to the form

$$ds^2 = \left[\frac{-5T^2}{a^2 - 10KT^2 + 5bT^{5/3}}\right] dT^2 + T^2 dX^2 + T\nu dY^2 + \frac{T}{\nu}dZ^2, \tag{36}$$

where

$$log\nu = a\sqrt{5}\int \frac{dT}{T\sqrt{a^2 - 10KT^2 + 5bT^{5/3}}}. \tag{37}$$

4. Some Physical and Geometrical Features

The density for the model (36) is given by

$$8\pi\varepsilon = \frac{\left(5b - 18KT^{1/3}\right)}{6T^{7/3}}. \tag{38}$$

The reality conditions given by Ellis,

(i) $\varepsilon + p > 0$,

(ii) $\varepsilon + 3p > 0$,

leads to the condition

$$T^{1/3} > \frac{5b}{18K}. \tag{39}$$

The tilt angle λ for the model (36) is given by

$$cosh\lambda = \frac{1}{2}\sqrt{\frac{36KT^{1/3} - 15b}{9KT^{1/3} - 5b}}, \tag{40}$$

$$sinh\lambda = \frac{1}{2}\sqrt{\frac{5b}{9KT^{1/3} - 5b}}. \tag{41}$$

The scalar of expansion θ calculated for the flow vector v^i is given by

$$\theta = \frac{3}{4T^2}\left(108KT^{2/3} - 110KbT^{1/3} + 25b^2\right) \times \sqrt{\frac{(a^2 - 10KT^2 + 5bT^{5/3})}{5(36KT^{1/3} - 15b)(9KT^{1/3} - 5b)^3}}. \tag{42}$$

The components of fluid flow vector v^i and heat conduction vector q^i for the model (36) are given by

$$v^1 = \frac{1}{2T}\sqrt{\frac{5b}{9KT^{1/3} - 5b}}, \tag{43}$$

$$v^4 = \frac{1}{2}\sqrt{\frac{36KT^{1/3} - 15b}{9KT^{1/3} - 5b}}, \tag{44}$$

$$q^1 = \frac{(36KT^{1/3} - 15b)}{192\pi T^{10/3}}\sqrt{\frac{5b}{9KT^{1/3} - 5b}}, \tag{45}$$

$$q^4 = -\frac{5b}{192\pi T^{7/3}}\sqrt{\frac{36KT^{1/3} - 15b}{9KT^{1/3} - 5b}}. \tag{46}$$

The non-vanishing components of shear tensor (σ_{ij}) and rotation tensor (ω_{ij}) are given by the following relations:

$$\sigma_{11} = \frac{1}{48}\sqrt{\frac{(a^2 - 10KT^2 + 5bT^{5/3})(36KT^{1/3} - 15b)}{(9KT^{1/3} - 5b)^5}} \times \left[2(36KT^{1/3} - 15b)(9KT^{1/3} - 5b) - 30KT^{1/3}\right] \tag{47}$$

$$\begin{aligned}\sigma_{22} \quad &= \quad \frac{\nu}{12T}\sqrt{\frac{(a^2 - 10KT^2 + 5bT^{5/3})(36KT^{1/3} - 15b)}{(9KT^{1/3} - 5b)}} \times \\ &\left[\frac{15bKT^{1/3}}{(36KT^{1/3} - 15b)(9KT^{1/3} - 5b)} + \frac{3\sqrt{5}a}{\sqrt{a^2 - 10KT^2 + 5bT^{5/3}}} - 1\right]\end{aligned} \tag{48}$$

$$\sigma_{33} = \frac{1}{12\nu T}\sqrt{\frac{(a^2-10KT^2+5bT^{5/3})(36KT^{1/3}-5b)}{5(9KT^{1/3}-5b)}} \times \left[\frac{15bKT^{1/3}}{(36KT^{1/3}-15b)(9KT^{1/3}-5b)} - \frac{3\sqrt{5}a}{\sqrt{a^2-10KT^2+5bT^{5/3}}} - 1\right] \tag{49}$$

$$\sigma_{44} = \frac{\sqrt{5}b}{24T^2}\sqrt{\frac{(a^2-10KT^2+5bT^{5/3})(36KT^{1/3}-15b)}{(9KT^{1/3}-5b)^3}} \times \left[1 - \frac{15bKT^{1/3}}{(9KT^{1/3}-5b)(36KT^{1/3}-15b)}\right] \tag{50}$$

$$\sigma_{14} = \frac{5}{24T}\left(1320bKT^{1/3} - 1134K^2T^{2/3} - 375b^2\right) \times \sqrt{\frac{(a^2-10KT^2+5bT^{5/3})5b}{(9KT^{1/3}-5b)^5}} \tag{51}$$

$$\omega_{14} = \sqrt{\frac{b(a^2-10KT^2+5bT^{5/3})}{(9KT^{1/3}-5b)^5}}\left[\frac{432K^2T^{2/3} - 45KbT^{1/3} + 25b^2}{32}\right]. \tag{52}$$

The rates of expansion H_i in the direction of the x, y and z axes are given by

$$H_1 = \frac{1}{\sqrt{5}T^2}\sqrt{a^2-10KT^2+5bT^{5/3}}, \tag{53}$$

$$H_2 = \frac{1}{2\sqrt{5}T^2}\left[\sqrt{a^2-10KT^2+5bT^{5/3}} + \sqrt{5}a\right], \tag{54}$$

$$H_3 = \frac{1}{2\sqrt{5}T^2}\left[\sqrt{a^2-10KT^2+5bT^{5/3}} - \sqrt{5}a\right]. \tag{55}$$

5. Special Model

When the magnetic field is absent i.e. $K = 0$, equation (34) reduces to

$$log\nu = \sqrt{5}a\int \frac{d\mu}{\mu\sqrt{a^2+5b\mu^{5/3}}}. \tag{56}$$

Putting $a^2 + 5b\mu^{5/3} = \xi^2$ in (56), we have

$$\nu = \ell\left[\frac{\sqrt{5b\mu^{5/3}+a^2}-a}{\sqrt{5b\mu^{5/3}+a^2}+a}\right]^{\frac{3}{\sqrt{5}}}, \tag{57}$$

where ℓ is a constant of integration.

Using the value of ν from (57), and after making suitable transformations, metric (35) reduces to

$$ds^2 = -\left[\frac{5T^2}{5bT^{5/3}+a^2}\right]dT^2 + T^2dX^2 + T\left[\frac{\sqrt{5bT^{5/3}+a^2}-a}{\sqrt{5bT^{5/3}+a^2}+a}\right]^{3/\sqrt{5}} dY^2 + \int T\left[\frac{\sqrt{5bT^{5/3}+a^2}+a}{\sqrt{5bT^{5/3}+a^2}-a}\right]^{3/\sqrt{5}} dZ^2 \tag{58}$$

By setting $\mu = T$, where $A = \text{(BC)} = \mu,\ B/C = \nu$.

There is a Cigar type singularity in the model at $T = 0$. Near the origin, the metric (58) reduces (for small T) to the form

$$ds^2 = -\left(\frac{5T^2}{a^2}\right) dT^2 + T^2 dX^2 + \left(\frac{5b}{4a^2} T^{1+\sqrt{5}}\right) dY^2 + \left(\frac{5b}{4a^2} T^{1-\sqrt{5}}\right) dZ^2. \tag{59}$$

After making some suitable transformations it reduces to the form

$$ds^2 = -d\tau^2 + \tau dX^2 + \tau^{\frac{1+\sqrt{5}}{2}} dY^2 + \tau^{\frac{1-\sqrt{5}}{2}} dZ^2. \tag{60}$$

This is analogous to the Kasner's form as obtained by Bali and Sharma [28].

For the model (58), density ε and tilt angle λ are given by

$$\varepsilon = \frac{5b}{48\pi T^{7/3}}, \tag{61}$$

$$cosh\,\lambda = \sqrt{3}/2. \tag{62}$$

Reality conditions given by Ellis leads to $b < 0$.

Scalar of expansion θ, the non-vanishing components of (σ_{ij}) and (ω_{ij}) are given by the following set of equations:

$$\theta = \frac{1}{T^2}\sqrt{\frac{3(a^2 + 5bT^{5/3})}{5}}, \tag{63}$$

$$\sigma_{11} = \frac{3}{8}\sqrt{\frac{3}{5}\,(a^2 + 5bT^{5/3})}, \tag{64}$$

$$\sigma_{22} = \frac{1}{4\sqrt{20}\,T}\left[\frac{\sqrt{5bT^{5/3} + a^2} - a}{\sqrt{5bT^{5/3} + a^2} + a}\right]^{3/\sqrt{5}} \times \left[3\sqrt{5}\,a - \sqrt{a^2 + 5bT^{5/3}}\right], \tag{65}$$

$$\sigma_{33} = \frac{-1}{4\sqrt{20}\,T}\left[\frac{\sqrt{5bT^{5/3} + a^2} + a}{\sqrt{5bT^{5/3} + a^2} - a}\right]^{3/\sqrt{5}} \times \left[3\sqrt{5}\,a - \sqrt{a^2 + 5bT^{5/3}}\right], \tag{66}$$

$$\sigma_{44} = \frac{-1}{8\sqrt{15}\,T^2}\sqrt{a^2 + 5bT^{5/3}}, \tag{67}$$

$$\omega_{14} = \frac{1}{32}\sqrt{\frac{-(a^2 + 5bT^{5/3})}{5}}. \tag{68}$$

6. Concluding Remarks

The model described in (36 starts to expand with a big-bang at $T = 0$ and decreases as T increases, then stops at $T = \infty$. The model has a real singularity at $T = 0$. The model in general represents a shearing, rotating and tilted universe in the presence of magnetic field. At the initial stage from where the model starts to expand the energy density $\varepsilon = \infty$ where as $\varepsilon = 0$ at $T = \infty$, thus the metric is asymptotically empty. Initially, tilt angle is given by $cosh\,\lambda = \sqrt{3}/2$ and at $T = \infty$, the model reduces to a non-tilted universe in nature and fluid distribution tends to be comoving. The velocity components of the fluid flow $\nu^1 = \infty$ and $\nu^4 = \sqrt{3}/2$ at $T = 0$. The components of heat conduction vector q^1 and q^4 become infinitely large $(+\infty)$ and (t∞) initially and approaches zero asymptotically, so at this stage they become ineffective. The Hubble components also become infinitely large $(+\infty)$ at this stage whereas at $T = \infty$, the velocity components $\nu^1 = 0$ and $\nu^4 = 1$. Hubble components also vanish asymptotically. The reality conditions put a restriction on b as, $b > 0$. It is interesting that for $T = \left(\frac{5b}{12K}\right)^3$, model (36) stops to expand; when $T = \left(\frac{5b}{9K}\right)^3$, it expands infinitely. This model does not approach isotropy for large values of T. We emphasize (36) is a tilted model; but if we consider the constant of integration $b = 0$, then the tilt angle $\lambda = 0$. Therefore, in this case, there is no tilt in the model and the model reduces to a non-tilted case which is the basic requirement for an orthogonal metric. Furthermore, shear component $\sigma_{44} = 0$ and rotation component ω_{14} also vanish. This is due to the orthogonality of shear and rotation to four velocity, the rotation tensor $\omega_{14} = 0$, $q^1 = 0$, $q^4 = 0$, $\nu^1 = 0$ and $\nu^4 = 0$. In this case model reduces to a non-tilted model as per the requirement of King and Ellis. But in this situation we cannot study the effect of the magnetic field on the model. So, we did not consider $b = 0$.

In the absence of the magnetic field, (36) reduces to (58). The initial singularity of the model is of Cigar type. The model in this case represents, in general, a rotating, shearing and tilted universe. The model starts to expand with a big-bang at $T = 0$ and continues decreasing as T increases, and finally stops at $T = 0$. At the initial stage density becomes infinitely large (t∞) and for the later stage metric becomes asymptotically empty. The tilt nature of the model is also preserved as constant throughout the evolution. This model also does not approach isotropy for large values of T.

Thus, to see the effect of the magnetic effect, we can conclude that as the magnetic field increases, the value of cosh λ decreases therefore tilt angle also decreases. Therefore, higher magnetic field is required to maintain its non-tilt nature where as on the later stages model automatically approaches to non-tilted one. The fluid flow velocity components ν^1 and ν^4 also decrease as the magnetic field increases. The heat conduction component q^1 increases with the magnetic field but q^4 decreases in opposite direction. Hubble components decrease as magnetic field increases. For both the cases the model does not approach isotropy for large values of T.

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