

# Pure single photon generation by type-I PDC with backward-wave amplification

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## Abstract

We explore a promising method of generating pure heralded single photons. Our approach is based on parametric downconversion in a periodically-poled waveguide. However, unlike conventional downconversion sources, the photon pairs are counter-propagating: one travels with the pump beam in the forward direction while the other is backpropagating towards the laser source. Our calculations reveal that these downconverted two-photon states carry minimal spectral correlations within each photon-pair. This approach offers the possibility to employ a new range of downconversion processes and materials like PPLN (previously considered unsuitable due to its unfavorable phasematching properties) to produce heralded pure single photons over a broad frequency range.

## 1 Introduction

Linear optical quantum computing (LOQC) schemes, such as continuous variable entanglement distillation [1] or single photon quantum gates [2] require sources of pure heralded single photons. Such single photon sources may be realized via photon pair generation by parametric downconversion (PDC). The photon number correlation between the resulting fields, typically labelled signal and idler, can be exploited to herald the existence of one photon by detection of its partner. However the purity of the heralded photon is limited by spatial and spectral correlations within each photon pair arising from energy and momentum conservation between pump, signal and idler photons. One possibility of generating pure heralded single photons without spectral filtering and a reduction in the source brightness is group velocity matching [3]. However this approach to generate separable photon pairs is limited to a few materials and wavelength ranges.

Most PDC experiments to date have been performed in bulk crystals, yet lately a lot of attention has been focused on PDC in waveguides. The main advantage of PDC in rectangular waveguides is the strict collinear propagation of the pump, signal and idler fields, in contrast to angular dispersion in bulk crystal setups. Along with the high modal confinement inside the waveguide this leads to a large increase in collection efficiency [4] and the elimination of spatial correlations. Furthermore, due to the strict collinear propagation of pump, signal and idler beams these sources are much more convenient to handle in the laboratory.

The spectral properties of PDC states are governed by the phasematching properties of the nonlinear material and this determines the frequencies of the downconverted photons. In a bulk nonlinear crystal one can exploit noncollinear PDC to achieve perfect phasematching, but this approach cannot be used in a waveguide structure as the direction of propagation is fixed. Instead, one must adopt quasi-phasematching (QPM) [5, 6]: A spatial periodic variation of the  $\chi_{ijk}^{(2)}$ -nonlinearity in the crystal introduces a new so called quasiphasematching vector ( $k_{QPM} = 2\pi/\Lambda$ ), for a sinusoidal poling with period  $\Lambda$ . In that way it is in principle possible to choose the signal and idler wavelengths freely, under the restriction of energy conservation.

The generation of backward-wave oscillations in three-wave-mixing processes was proposed in 1966 [10]. For the generation of correlated photon pairs in waveguided PDC this approach was revisited in 2002 by Booth et. al. [11]. In this configuration almost all the momentum of the pump photon has to be compensated by the QPM poling structure within the crystal. This requires grating periods in the sub-micron range (0.2 -0.6  $\mu\text{m}$ ) for signal and idler photons generated in the range from 800 to 1600 nm.

Quasiphasematched PDC processes with counterpropagating signal and idler photons and a perpendicularly propagating pump in planar semiconductor waveguides have already been observed [12, 13], and their respective quantum properties have been studied [14]. However, to date, the high absorption in semiconductor materials and tiny interaction volume limit the achievable photon flux. In dielectric materials a breakthrough has been made towards sub-micron poling periods in the past year: Backward-wave oscillation in potassium titanyl phosphate (KTP) has been reported [15], and simultaneously in lithium niobate (LN) new sub-micron poling techniques have been explored [16]. Therefore the required crystals are within reach.

In this paper, we consider a collinear waveguided PDC setup with counterpropagating signal and idler fields. We show that this configuration allows the generation of pure heralded single photons in a large range of nonlinear materials and wavelengths.

## 2 The PDC state

The frequency structure of a downconverted *copropagating* two-photon state is found to be [7]:

$$\begin{aligned}
 |\psi_{s,i}\rangle \approx & |0\rangle + A \int \int d\omega_s d\omega_i \exp \left[ \underbrace{-\frac{(\omega_s + \omega_i - \omega_p)^2}{2\sigma^2}}_{\alpha(\omega_s + \omega_i)} \right] \\
 & \times \underbrace{\text{sinc} \left( \frac{L}{2} \Delta k \right) \exp \left[ -i \frac{L}{2} \Delta k \right]}_{\phi(\omega_s, \omega_i)} \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) |0\rangle \quad (1)
 \end{aligned}$$

The pump distribution  $\alpha(\omega_s + \omega_i)$  is given by the incoming laser. In our case we assume a mode-locked pulsed laser system with a Gaussian frequency distribution, centered around  $\omega_p$  with width  $\sigma$ . The phasematching function  $\phi(\omega_s, \omega_i)$

is governed by the waveguide dimensions and crystal dispersion, ensuring momentum conservation ( $\Delta k = k_p - k_s - k_i - k_{QPM}$ ). Because of the strict collinear propagation inside the waveguide the transverse wavevector mismatch does not enter Eq. (1). The product of these two functions gives the joint spectral amplitude (JSA):  $f(\omega_s, \omega_i) = \alpha(\omega_s + \omega_i) \cdot \phi(\omega_s, \omega_i)$ . For analytic calculations the two-photon state is often simplified with the Gaussian approximation ( $\text{sinc}(x) \approx \exp(-\gamma x^2)$  where  $\gamma \approx 0.193$ ).

Heralding one photon of this PDC-state will in general lead to a mixed heralded single photon state, due to correlations in the JSA ( $f(\omega_s, \omega_i)$ ) [8]. Pure heralded single photons are created if and only if the downconverted two-photon state can be written as a product state  $|\psi_{s,i}\rangle = |\psi_s\rangle \otimes |\psi_i\rangle$ . This requires a separable JSA:  $f(\omega_s, \omega_i) = f(\omega_s)f(\omega_i)$ . In order to quantify the separability of the generated PDC states one has to perform a Schmidt decomposition [9], i.e. a basis transformation into a set of orthonormal Schmidt modes,  $|\psi_s^n(\omega_s)\rangle$  and  $|\psi_i^n(\omega_i)\rangle$ :

$$|\psi_{s,i}(\omega_s, \omega_i)\rangle = \sum_n \sqrt{\lambda_n} |\psi_s^n(\omega_s)\rangle \otimes |\psi_i^n(\omega_i)\rangle \quad (2)$$

The probability to emit a photon pair into one specific pair of Schmidt modes  $|\psi_s^n(\omega_s)\rangle \otimes |\psi_i^n(\omega_i)\rangle$  is given by  $\lambda_n$ , which is monotonically decreasing for successive higher order modes. Thus a perfectly separable state corresponds to a state where the full weight of the probability distribution accumulates on the first pair of Schmidt modes ( $\lambda_0 = 1$ ).

### 3 Backward-wave oscillations

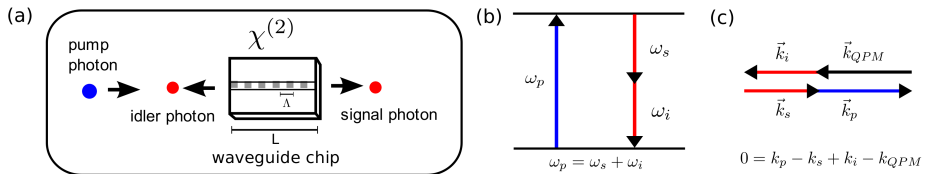


Figure 1: Waveguided parametric downconversion with one backward-wave oscillation: (a) Process scheme, (b) Energy conservation, (c) Momentum conservation.

Fig. 1 illustrates the generation of *counterpropagating* photon pairs and the corresponding energy and momentum conservation conditions. To account for the backward propagating wave in our formalism (Eq.(1)), we have to alter the momentum conservation condition:

$$\Delta k = k_p - k_s + k_i - 2\pi/\Lambda. \quad (3)$$

This has a significant effect on the properties of the generated two-photon-states. The biggest advantage over copropagating PDC are the different requirements to generate separable two-photon states. With the Gaussian approximation and expansion of the wave vector mismatch  $\Delta k$  as a Taylor series up to the first

order, we are able to analytically derive a condition for separability from Eq. (1) and (3) (analogous to [8]):

$$0 = \frac{2}{\sigma^2} + (k'_p - k'_s)(k'_p + k'_i) = \frac{2}{\sigma^2} + \left(\frac{1}{v_p} - \frac{1}{v_s}\right) \left(\frac{1}{v_p} + \frac{1}{v_i}\right). \quad (4)$$

Therefore a waveguide in which the pump pulses propagate faster than the downconverted forward propagating signal pulses ( $v_s < v_p$ ) will generate separable photon pairs. Note that this requirement is much easier to satisfy than that for the usual copropagating case, where the group velocities must satisfy either  $v_s < v_p < v_i$  or  $v_i < v_p < v_s$  [8].

Further insight is obtained by deriving the angle of the phasematching function in the  $\{\omega_s, \omega_i\}$ -plane:

$$\theta = -\arctan \left[ \frac{k'_p - k'_s}{k'_p + k'_i} \right] = -\arctan \left[ \frac{\nu_s - \nu_p}{\nu_i + \nu_p} \frac{\nu_i}{\nu_s} \right] \quad (5)$$

Here  $\theta$  is defined as the angle between the phasematching function and the signal axis. As can be deduced from Eq. (4) and (5), the condition for factorability requires a phasematching angle between  $0^\circ < \theta < 90^\circ$ . Considering the relative magnitude of the numerator and denominator in Eq.(5), identical group velocities for the signal, idler and pump waves will result in a horizontally orientated phasematching function and a factorable JSA. This is in very good agreement with the small group velocity dispersion in common nonlinear materials over the relevant wavelength region  $\nu_s(\omega_s) \approx \nu_i(\omega_i) \approx \nu_p(\omega_s + \omega_i)$ . These requirements are opposed to copropagating decorrelation proposals which rely on different group velocities for the signal, idler and pump waves. In the counterpropagating case very similar group velocities are demanded, a requirement that is much more robust and easier to fulfill.

The highly nonlinear crystal LN, commonly used in PDC experiments to generate photon pairs is unable to produce factorable copropagating photon pairs, but can be used to generate separable pairs in the backward-propagating regime. Fig. 2 illustrates this particular example of a separable counterpropagating two-photon state. It demonstrates several benefits of the backward-wave approach simulated using periodically poled LN (PPLN) as the nonlinear medium. The momentum mismatch changes to a much stricter condition in comparison with copropagating PDC. This leads to an extremely narrow spectral width of the backpropagating photons, almost one order of magnitude smaller than the spectral distribution of the forward propagating photon. In this case, the FWHM of the wavelength distribution is 0.09 nm for the backpropagating photon and 0.73 nm for the forward propagating photon. The narrow frequency bandwidth of the backpropagating photon makes it very well suited for long distance transmission in optical fibers.

It has to be noted that according to [17], the total photon pair production rate will decrease with respect to sources that create strictly forward propagating photons, but the effective generation rate can still exceed bulk crystal setups due to the high nonlinearity and higher collection efficiencies in a waveguide architecture.

In more general terms we would like to emphasise that the separability of a downconverted photon pair is almost independent of the signal and idler frequencies (Fig. 3(a)). With appropriate grating periods it is possible to generate

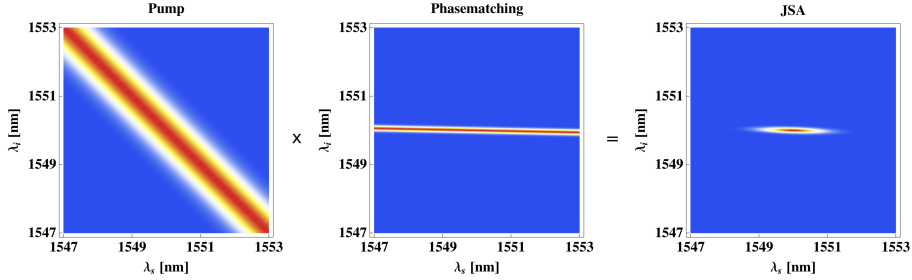


Figure 2: Pump envelope, Phasematching function and JSA plotted in the Gaussian approximation without phase; Parameters: LN, type-I PDC, e-polarized rays, pump central wavelength  $\lambda_p = 775$  nm, FWHM of the pump intensity distribution  $\Delta\lambda_p = 0.58$  nm, waveguide dimensions:  $4 \mu\text{m} \times 4 \mu\text{m} \times 5$  mm, grating period  $\Lambda = 0.35 \mu\text{m}$ .

separable degenerate signal and idler photons from 800 nm (where detectors are most efficient) to 1550 nm (the wavelength with minimal loss in optical fibers). This can be used to create a *tunable* pure heralded single photon source. As depicted in Fig. 3(b), for a fixed grating period different pump wavelengths lead to a change in the downconverted signal wavelength, whereas the separability and the idler frequency remain constant. Hence the wavelength of the signal photon can be tuned by changing the pump wavelength, without impact on the idler frequency and with very little change in the separability.

## 4 Numerical analysis

To quantify the stated benefits, we numerically investigated periodically poled LN (PPLN) and periodically poled KTP (PPKTP) as sources of separable counterpropagating photon pairs. In PPLN we chose the type-I downconversion process with the highest nonlinearity, where extraordinary-polarized pump photons decay into extraordinary-polarized signal and idler photons ( $\chi_{eee}^{(2)} = 63$  pm/V). In KTP we analyzed strictly z-polarized signal, idler and pump waves making use of the largest tensor element  $\chi_{zzz}^{(2)} = 27.4$  pm/V.

The rectangular waveguide embedded in the crystal material was modelled with standard dimensions of  $4 \mu\text{m} \times 4 \mu\text{m} \times 5$  mm and a realistic refractive index step between waveguide and surrounding material of 0.01. To simulate the propagation of our signal, idler and pump waves inside the waveguide we calculated the spatial modes of signal, idler and pump fields according to [18, 19]. The resulting decomposition of the wavevector into its longitudinal and transverse components enabled us to correct the bulk crystal Sellmeier equations and to obtain a modified JSA. In the scope of this paper we assume that the signal, idler and pump fields propagate in the fundamental waveguide mode.

We investigated the possibility of generating decorrelated and degenerate photon pairs in the range of 800 nm to 1600 nm. For each studied signal and idler degeneracy wavelength the pump wavelength was chosen appropriately ( $\lambda_p = \lambda_{s,i}/2$ ), and the grating was matched to give phasematching at the degeneracy point ( $\Lambda = 2\pi/(k_p - k_i - k_s)$ ). For each parameter set ( $\lambda_s = \lambda_i = 2\lambda_p, \Lambda(\lambda_s, \lambda_i)$ )

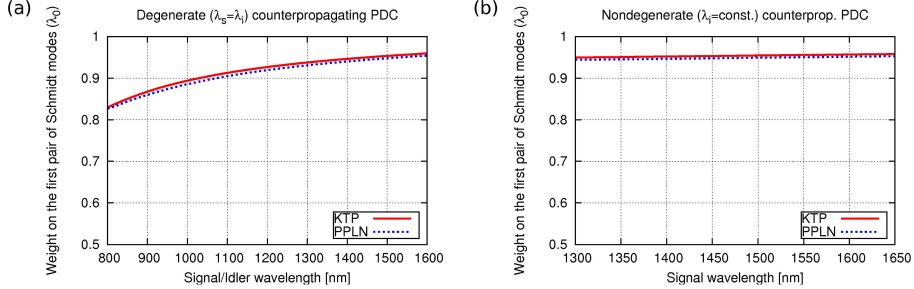


Figure 3: (a) In counterpropagating PDC it is possible to generate separable degenerate signal and idler photons in the range from 800 to 1600 nm. (b) The separability is maintained for nondegenerate PDC ( $\lambda_i = 1550$  nm).

we optimized the pump width  $\Delta\lambda_p$  in the range from  $\Delta\lambda_p = 0.02 - 0.35$  nm to yield a state with maximum separability. The optimum value of  $\Delta\lambda_p$  was determined by performing a Schmidt decomposition according to Eq. (2) for every set of parameters.

The results are depicted in Fig. 3(a): The overall probability of the generated photon pairs to be emitted in the first pair of Schmidt modes ( $\lambda_0$ ) is very high and only differs slightly for KTP and PPLN. The general improvement in separability for higher wavelengths is due to the fact that in this region the group velocities of the signal, idler and pump waves almost perfectly equal each other.

In a similar manner we checked the feasibility of this setup as a frequency-tuneable pure heralded single photon source using nondegenerate PDC (Fig. 3(b)). For a constant grating period  $\Lambda$  yielding phasematching at 1550 nm for signal and idler, we investigated the impacts of tuning the pump central frequency. Due to the horizontal orientation of the phasematching function the frequency distribution of the idler photons remains unchanged. However the frequency of the forward propagating signal photon shifts with the pump frequency. Again the pump width was chosen to yield a maximally separable two-photon states, now in the range from  $\Delta\lambda_p = 0.22 - 0.34$  nm. Once more our results are almost independent of the chosen nonlinear crystal and the constant high level of separability shows that with this setup it will indeed be possible to create a tuneable pure heralded single photon source.

## 5 Conclusion

We have examined the spectral properties of downconverted counterpropagating two-photon states in rectangular waveguides. The major differences in comparison with copropagating downconverted photon-pairs allow us to exploit a wide range of processes and materials for heralded single photon generation. This technique provides separable two-photon states for a wide range of degenerate and non-degenerate signal and idler wavelengths which will be useful for practical purposes such as LOQC. Due to the progress in the production of microstructured waveguides an experimental implementation of our proposal will be feasible in the near future.

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