Correlations in photon-numbers and integrated intensities in parametric processes involving three optical fields

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Two strongly-pumped parametric interactions are simultaneously realized in a single nonlinear crystal in order to generate three strongly correlated optical fields. By combining together the outputs of two of the three detectors measuring intensities of the generated fields, we obtain the joint photocount statistics between the single field and the sum of the other two. Moreover, we develop a microscopic quantum theory to determine the joint photon-number distribution and the joint quasi-distributions of integrated intensities and prove nonclassical nature of the three-mode state. Finally, by performing a conditional measurement on the single field, we obtain a state endowed with a sub-Poissonian statistics, as testified by the analysis of the conditional Fano factor. The role of quantum detection efficiencies in this conditional state-preparation method is discussed in detail.

I. INTRODUCTION

The quantum properties of photon pairs generated in the process of spontaneous parametric down-conversion [1, 2, 3] have been investigated in many experimental and theoretical works during the last 30 years. Pairwise character of the fields generated in this process has been used, e.g., for testing fundamental laws of quantum mechanics [3], as well as for quantum teleportation [4], quantum cryptography [5] and in metrology applications [6]. The theory that describes this kind of interaction has been elaborated from several points of view for fields containing just a fraction of a photon pair [7, 8, 9] and for fields composed of many photon pairs [10]. Also stimulated emission of photon pairs has been addressed [11, 12, 13].

More recently, the development of the field of quantum-information processing [14] has drawn attention to three-field quantum correlations. In fact, interesting correlations can be reached when strongly-pumped parametric down-conversion and parametric amplification are combined together through a common field. Such a system can be built in a single nonlinear crystal oriented in such a way that phase-matching conditions for both the interactions are fulfilled together [15]. It has been demonstrated that the tripartite state generated in this way is endowed with entanglement in the number of photons. In particular as the constant of motion admitted by the hamiltonian that describes the process suggests, we obtain that the number of photons in one of the three generated fields is always equal to the sum of the photons in the other two fields [16]. These properties make the system useful for the generation of nonclassical states by means of a suitable conditional state preparation. In more detail, if a given number n_0 of photons in field a_0 is detected then the remaining two fields a_1 and a_2 are ideally left in a state $|\psi\rangle = \sum_{l=0}^{n_0} c_l |l\rangle_1 |n_0 - l\rangle_2$ entangled in photon numbers, $|l\rangle_i$ means the Fock state with l photons in field a_i and coefficients c_l depend on the nonlinear interaction.

Here we present the experimental realization of this scheme and in particular we study the joint photocount statistics between one field (single field) and the sum of the other two (compound field) in order to find out the experimental conditions in which it is possible to realize a deterministic source of states entangled in the number of photons. We note that this source requires photon-number resolving detectors [17, 18, 19, 20, 21, 22].

Note that fields composed of photon pairs have been experimentally investigated [23, 24, 25, 26, 27] under conditions that allowed having up to several thousands of photon pairs per pump pulse. A detailed theory based on a multi-mode description of the generated fields has been developed for spontaneous [28] as well as for stimulated processes [29] with the aim to interpret the experimental data. In this work, as the photocounts from two of the three detectors are combined together, we also use this theory in order to correctly interpret and process the measurements. In particular, we can determine the joint single-compound-field (JSCF) photon-number distribution and the JSCF quasi-distributions of integrated intensities from the measured JSCF photocount distribution. Moreover, the conditional photocount and photon-number distributions, together with the corresponding Fano factors, are derived in order to point out the versatility of the scheme as a source of nonclassical states. To this aim, the quantum detection efficiency is an important parameter.

Theoretical description of the nonlinear process is presented in Sec. II. Photon-number distributions and quasi-distributions of integrated intensities as well as other properties of the measured fields are derived in Sec. III. Experimental results are discussed in Sec. IV. Sec. V gives conclusions.

II. THREE-MODE PARAMETRIC PROCESS

We consider a three-mode parametric process in which two strong coherent classical fields pump two interlinked nonlinear interactions: a frequency down-conversion and a parametric amplification. The corresponding interaction Hamiltonian can be written as follows [15, 30]:

$$H_{\rm int} = \gamma_0 a_0^\dagger a_2^\dagger + \gamma_1 a_1^\dagger a_2 + {\rm h.c.}, \quad (1)$$

where a_i ($a_i^\dagger$), $i = 0, 1, 2$, are the corresponding photon annihilation (creation) operators and γ_0 and γ_1 are coupling constants for frequency down-conversion and parametric amplification, respectively. Symbol h.c. stands for the hermitian conjugated term. Analytical solutions of the corresponding Heisenberg equations for the annihilation and creation operators are given in [15]. They can be used for the determination of the normal three-mode characteristic function of the spontaneous process:

$$\begin{aligned} C(\beta_0,\beta_1,\beta_2) &= \exp[-B_0|\beta_0|^2 - B_1|\beta_1|^2 - B_2|\beta_2|^2 \\ &+(D_{01}\beta_0^*\beta_1^* + D_{02}\beta_0^*\beta_2^* + \bar{D}_{12}\beta_1\beta_2^* + {\rm c.c.})], \quad (2) \end{aligned}$$

where β_i ($i = 0, 1, 2$) are parameters, c.c. means the complex conjugated term and

$$\begin{aligned} B_0 &= \langle \Delta a_0^\dagger \Delta a_0\rangle = |f_1|^2 + |f_2|^2, \\ B_1 &= \langle \Delta a_1^\dagger \Delta a_1\rangle = |g_0|^2, \\ B_2 &= \langle \Delta a_2^\dagger \Delta a_2\rangle = |h_0|^2, \\ D_{01} &= \langle \Delta a_0 \Delta a_1\rangle = f_0^* g_0, \\ D_{02} &= \langle \Delta a_0 \Delta a_2\rangle = f_0^* h_0, \\ \bar{D}_{12} &= -\langle \Delta a_1^\dagger \Delta a_2\rangle = -h_0 g_0^*. \quad (3) \end{aligned}$$

The functions g_0, h_0, and f_0 are defined in [15]. In order to study the nonclassical nature of the three-mode state, we introduce the determinants

$$\begin{aligned} K_{12} &= B_1 B_2 - |\bar{D}_{12}|^2 = 0, \\ K_{01} &= B_0 B_1 - |D_{01}|^2 = -|g_0|^2 < 0, \\ K_{02} &= B_0 B_2 - |D_{02}|^2 = -|h_0|^2 < 0. \quad (4) \end{aligned}$$

According to Eqs. (4) fields a_1 and a_2 are classically correlated whereas correlations between fields a_0 and a_1 (a_0 and a_2) can lead to nonclassical behavior.

In the experiment, the outputs of the detectors placed on fields a_1 and a_2 have been summed with the aim to measure photocount correlations between the single field a_0 and the compound field formed by fields a_1 and a_2. These correlations are important to test the performance of a source of states entangled in the number of photons and obtained after a conditional measurement of n_0 photons in field a_0. They can be derived from the following normal characteristic function:

$$\begin{aligned} C(\beta_0,\beta_1,\beta_1) &= \exp[-B_0|\beta_0|^2 - B_{12}|\beta_1|^2 \\ &+ (D_{0,12}\beta_0^*\beta_1^* + {\rm h.c.})], \quad (5) \end{aligned}$$

where

$$\begin{aligned} B_{12} &= B_1 + B_2 - \bar{D}_{12} - \bar{D}_{21} = |h_0 + g_0|^2, \\ D_{0,12} &= D_{01} + D_{02} = f_0^* g_0 + f_0^* h_0, \\ K_{0,12} &= B_0 B_{12} - |D_{0,12}|^2 = -|g_0|^2 - |h_0|^2 < 0. \quad (6) \end{aligned}$$

To obtain Eq. (5) we have assumed that γ_1 is real. We note that information about the coupling constants γ_0 and γ_1 can be obtained from the reconstruction of the photocount distributions provided that the interaction time t is known.

We assume that each field is composed of M independent temporal modes [27]. Then the variances of integrated intensities W can be expressed as follows:

$$\begin{aligned} \langle (\Delta W_0)^2\rangle &= M B_0^2, \\ \langle (\Delta W_j)^2\rangle &= M B_j^2, j = 1, 2, \\ \langle \Delta W_0 \Delta W_j\rangle &= M|D_{0j}|^2, j = 1, 2, \\ \langle \Delta W_1 \Delta W_2\rangle &= M|\bar{D}_{12}|^2. \quad (7) \end{aligned}$$

If photocounts belonging to fields a_1 and a_2 are combined together, we can write the variances of the integrated intensities W in the following form:

$$\begin{aligned} \langle (\Delta W_{12})^2\rangle &= \langle (\Delta W_1)^2\rangle + \langle (\Delta W_2)^2\rangle + 2\langle \Delta W_1 \Delta W_2\rangle \\ &= M|B_{12}|^2 = M(B_1^2 + B_2^2 + 2|\bar{D}_{12}|^2) \\ &= M(B_1 + B_2)^2, \\ \langle \Delta W_0 \Delta W_{12}\rangle &= M|D_{0,12}|^2 = M|f_0|^2(|g_0|^2 + |h_0|^2). \quad (8) \end{aligned}$$

Real value of γ_1 has been again assumed.

Note that the above-presented analysis can be generalized to stimulated processes [31]. However, single as well as compound fields have to be stimulated in order to support nonclassical effects by interference terms. As for the compound field, stimulation of one of its components (i.e. field a_1 or a_2) is sufficient to observe increased nonclassical effects. Stimulation of only one field (i.e. field a_0 or a_1 or a_2) results in increased values of noise only.

III. PHOTON-NUMBER DISTRIBUTIONS AND QUASI-DISTRIBUTIONS OF INTEGRATED INTENSITIES

The experimental scheme used to generate two nonlinear interactions is depicted in Fig. 1. The harmonics of a continuous-wave mode-locked Nd:YLF laser regeneratively amplified at a repetition rate of 500 Hz (High Q Laser Production, Hohenems, Austria) provided two pump fields. In particular, the third harmonic pulse at 349 nm ($\sim$ 4.45 ps pulse-duration) was exploited as the pump field a_{p0} in frequency down-conversion, whereas the fundamental pulse at 1047 nm ($\sim$ 7.7 ps pulse-duration) was used as the pump field a_{p1} in parametric amplification. The two interactions simultaneously satisfied energy-matching ($\omega_{p0} = \omega_0 + \omega_2$ and $\omega_1 = \omega_2 + \omega_{p1}$) and type I phase-matching ($\mathbf{k}_{p0}^e = \mathbf{k}_0^o + \mathbf{k}_2^o$, $\mathbf{k}_1^e = \mathbf{k}_2^o + \mathbf{k}_{p1}^o$)

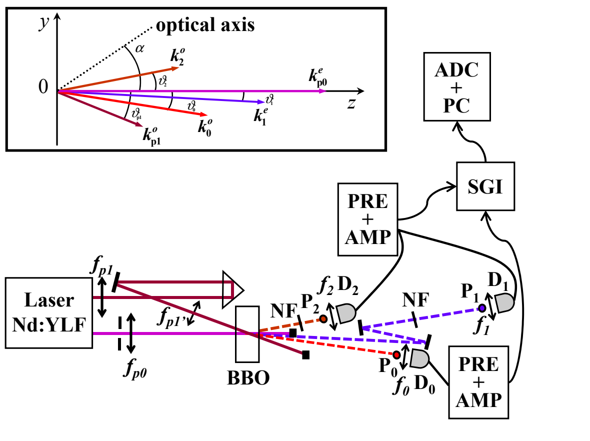


FIG. 1: Scheme of the experimental setup: BBO, nonlinear crystal; NF, variable neutral-density filter; P_i, pinholes and D_i, p-i-n photodiodes, $i = 0, 1, 2$; f_i, lenses, $i = 0, 1, 2, p0, p1, p1'$; PRE+AMP, low-noise charge-sensitive pre-amplifiers followed by amplifiers; SGI, dual-channel synchronous gated-integrator; ADC+PC, computer integrated digitizer.

conditions, in which ω_j are the angular frequencies, $\mathbf{k}_j$ denote the wave vectors and suffixes o and e indicate ordinary and extraordinary field polarizations. As depicted in the inset of Fig. 1, we set the pump-field a_{p0} direction so that the wave vector $\mathbf{k}_{p0}$ was normal to the crystal entrance face and propagated along the z-axis of the medium. We also aligned the wave vector $\mathbf{k}_{p1}$ of the other pump field a_{p1} in the plane $(y,\ z)$ containing the optical axis (OA) of the crystal and the wave vector $\mathbf{k}_{p0}$. As the nonlinear medium we used a β-BaB_2O_4 crystal (BBO, Fujian Castech Crystals, China, 10 mm × 10 mm cross section, 4 mm thickness) cut for type-I interaction ($\vartheta_{\rm cut} = 38.4$ deg), into which both pumps were strongly focused. Typical intensity values of the pump fields were ~ 5 GW/cm^2 for a_{p0} and ~ 2 GW/cm^2 for a_{p1}. The required superposition in time of the two pump fields was obtained by a variable delay line.

As we have already shown in Ref. [16], we decided to generate three fields at non-degenerate frequencies by choosing a phase-matching condition in the plane $(y,\ z)$ [32]. In order to investigate the nature of the state obtained by the interlinked interactions, we selected a triplet of coherence areas by means of pin-holes, whose sizes and distances from the crystal were chosen by searching for the condition of maximum intensity correlations between the generated fields [33]. In fact, we expect strong correlations not only between the number of photons in the field a_0 and the sum of the other two fields (compound field), but also singularly among the numbers of photons in all pairs of fields. By applying this criterion, we put two pin-holes of 30 μm diameter at distances $d_0 = 60$ cm and $d_2 = 49$ cm from the BBO along the path of the signal beam at 632.8 nm and of the idler beam at 778.2 nm, respectively. Moreover, as the beam at 446.4 nm has smaller divergence compared to the other two fields, we selected it by means of a 50 μm diameter pin-hole placed at a distance $d_1 = 141.5$ cm from the crystal. The light was suitably filtered by means of band-pass filters and focused on each detector. In particular, as we performed measurements in the macroscopic intensity regime (more than 1000 photons per coherence area), we used three p-i-n photodiodes (two, D_0 and D_1 in Fig. 1, S5973-02 and one, D_2, S3883, Hamamatsu, Japan) as the detectors. We obtained the same overall detection efficiency ($\eta = 0.28$) on the three arms by inserting two adjustable neutral-density filters in the pathways of fields a_1 and a_2. The current output of the detectors was amplified by means of two low-noise charge-sensitive pre-amplifiers (CR-110, Cremat, Watertown, MA) followed by two amplifiers (CR-200-4 μs, Cremat): to this aim, we connected the detectors D_1 and D_2 to the same amplifier device by means of a T-adapter. The two amplified outputs were then integrated by synchronous gated-integrators (SGI in Fig. 1, SR250, Stanford Research Systems, Palo Alto, CA) sampled, digitized by a 12-bit converter (AT-MIO-16E-1, DAQ National Instruments) and recorded by a computer.

From the collected experimental data we obtained the first ($\langle m_0\rangle$, $\langle m_{12}\rangle$) and the second ($\langle m_0^2\rangle$, $\langle m_{12}^2\rangle$) moments of the photocount (photoelectron) distribution of the single and compound fields, respectively. Moreover, the correlation between the number of photoelectrons in the single field and that in the compound field was measured. The unavoidable contributions of additive noise present in the detection chain can be quantified in an independent measurement and then subtracted from the experimental data. By correcting the moments of the photoelectron distribution for the quantum detection efficiency, we can obtain the moments for photons. Here we present the first ($\langle n_0\rangle$, $\langle n_{12}\rangle$) and the second ($\langle n_0^2\rangle$, $\langle n_{12}^2\rangle$, $\langle n_0 n_{12}\rangle$) moments of the photon-number distribution:

$$\begin{aligned} \langle n_i\rangle &= \langle m_i\rangle/\eta, \\ \langle n_i^2\rangle &= \langle m_i^2\rangle/\eta^2 - (1-\eta)\langle m_i\rangle/\eta^2,\ \ i = 0, 12, \\ \langle n_0 n_{12}\rangle &= \langle m_0 m_{12}\rangle/\eta^2. \end{aligned} \tag{9}$$

In addition, the moments for the integrated intensities can be determined as follows:

$$\begin{aligned} \langle W_i\rangle &= \langle n_i\rangle, \\ \langle W_i^2\rangle &= \langle n_i^2\rangle - \langle n_i\rangle, i = 0, 12, \\ \langle W_0 W_{12}\rangle &= \langle n_0 n_{12}\rangle. \end{aligned} \tag{10}$$

In a theory based on the generalized superposition of signal and noise properties of the fields can be quantified using coefficients B_0, B_{12}, $D_{0,12}$ and number M of temporal modes. These quantities can be determined from the moments of integrated intensities [28]:

$$\begin{aligned} B_i &= \langle(\Delta W_i)^2\rangle/\langle W_i\rangle, \\ M_i &= \langle W_i\rangle^2/\langle(\Delta W_i)^2\rangle,\ \ i = 0, 12, \\ |D_{0,12}| &= \sqrt{\langle \Delta W_0 \Delta W_{12}\rangle/M}. \end{aligned} \tag{11}$$

The mean number of photons in mode i is given by B_i ($i = 0, 12$) whereas $D_{0,12}$ quantifies the correlations between the single field and the compound one. The number of modes M can be determined either from the experimental data measured in the single or in the compound field (see Eqs. (11)). Ideally, M_0 should be equal to M_{12} [15]. This cannot be reached in a real experiment because of non-perfect alignment and detection noise. However, a correct alignment of the experimental setup allows to have $M_0 \approx M_{12}$ so that we can define $M = (M_0+M_{12})/2$. A more detailed analysis concerning the determination of M can be found in [34].

The JSCF photon-number distribution $p(n_0, n_{12})$, which can be derived from the normal characteristic function C in Eq. (5), is written as follows [28]:

$$p(n_0, n_{12}) = \frac{1}{\Gamma(M)} \frac{(B_0 + K_{0,12})^{n_0} (B_{12} + K_{0,12})^{n_{12}}}{(1 + B_0 + B_{12} + K_{0,12})^{n_0+n_{12}+M}} \times \sum_{r=0}^{\min(n_0,n_{12})} \frac{\Gamma(n_0 + n_{12} + M - r)}{r!(n_0 - r)!(n_{12} - r)!} \times \frac{(-K_{0,12})^r (1 + B_0 + B_{12} + K_{0,12})^r}{[(B_0 + K_{0,12})(B_{12} + K_{0,12})]^r}, \qquad (12)$$

where Γ is the gamma function. If determinant $K_{0,12}$ defined in Eqs. (6) is negative, the given field cannot be described classically. The quantities $B_0 + K_{0,12}$ and $B_{12} + K_{0,12}$ occurring in Eq. (12) cannot be negative and can be interpreted as components of fictitious noise. In the ideal lossless case, $K_{0,12} = -B_0 = -B_{12}$ and then the joint photon-number distribution $p(n_0, n_{12})$ takes the form of the diagonal Mandel-Rice distribution. As frequency down-conversion produces couples of photons, we expect that the highest values of the elements of $p(n_0, n_{12})$ are near the diagonal $n_0 = n_{12}$. In addition certain classical inequalities can be violated in this region [24].

If $K_{0,12} = 0$, *i.e.* at the boundary between the classical and nonclassical behaviors, the compound Mandel-Rice formula for JSCF photon-number distribution $p(n_0, n_{12})$ can be simplified as follows

$$p(n_0, n_{12}) = \frac{\Gamma(n_0 + n_{12} + M) B_0^{n_0} B_{12}^{n_{12}}}{\Gamma(M) n_0!\, n_{12}!\, (1 + B_0 + B_{12})^{n_0+n_{12}+M}}. \qquad (13)$$

In the ideal case only the diagonal elements of the distribution $p(n_0, n_{12})$ are different from zero: in this case the post-selection scheme for the preparation of a state entangled in the number of photons perfectly works. However, losses present in any experimental implementation cause discrepancy from this ideal situation. In order to quantify this discrepancy, we determine the conditional compound-field photon-number distribution $p_{c,12}(n_{12}; n_0)$ provided that n_0 photons are detected in the single field:

$$p_{c,12}(n_{12}; n_0) = p(n_0, n_{12}) / \sum_{k=0}^{\infty} p(n_0, k). \qquad (14)$$

Fano factor $F_{c,12}$ of photon-number distribution $p_{c,12}(n_{12}; n_0)$ can be expressed as follows:

$$F_{c,12}(n_0) = 1 + \frac{(1 + M/n_0)[(B_{12} + K_{0,12})/(1 + B_0)]^2 - (K_{0,12}/B_0)^2}{(1 + M/n_0)(B_{12} + K_{0,12})/(1 + B_0) - K_{0,12}/B_0} \approx 1 + K_{0,12}/B_0. \qquad (15)$$

Note that the last approximation in Eq. (15) holds for $K_{0,12} \approx -B_{12}$ and highlights that negative values of determinant $K_{0,12}$ cause sub-Poissonian conditional photon-number distribution. For the ideal lossless case, $K_{0,12} = -B_0 = -B_{12}$ and Fano factor $F_{c,12}$ is equal to 0.

The correlations in the number of photons can also be quantified by the distribution p_- of the difference photon number $n_0 - n_1 - n_2$ defined as:

$$p_-(n) = \sum_{n_0,n_1,n_2=0}^{\infty} \delta_{n,n_0-n_1-n_2} p(n_0, n_1, n_2), \qquad (16)$$

where δ is the Kronecker symbol. The variance of the difference $n_0 - n_1 - n_2$ can be lower than the sum $\langle n_0 + n_1 + n_2 \rangle$ of the mean photon numbers in all fields. This happens for nonclassical fields and we obtain sub-shot-noise correlations in this case [16, 26]. Under this condition the so-called noise reduction factor [26],

$$R = \frac{\langle [\Delta(n_0 - n_1 - n_2)]^2 \rangle}{\langle n_0 \rangle + \langle n_1 \rangle + \langle n_2 \rangle}, \qquad (17)$$

is lower than 1.

The JSCF photon-number distribution $p(n_0, n_{12})$ is given by Mandel's photo-detection formula [3, 28, 35], which is defined in terms of the s-ordered quasi-distribution $P_s(W_0, W_{12})$ of integrated intensities. This formula can be inverted in such a way that the quasi-distributions of integrated intensities can be written in terms of the distribution $p(n_0, n_{12})$ in Eq. (12) with the following results. If we have the s-ordered determinant $K_{0,12s} > 0$ ($K_{0,12s} = B_{0s} B_{12s} - |D_{0,12}|^2$, $B_{i,s} = B_i + (1 - s)/2$, $i = 0, 12$), the s-ordered JSCF quasi-distribution $P_s(W_0, W_{12})$ of integrated intensities is a non-negative ordinary function [28]:

$$P_s(W_0, W_{12}) = \frac{1}{\Gamma(M) K_{0,12s}^M} \left(\frac{K_{0,12s}^2 W_0 W_{12}}{|D_{0,12}|^2} \right)^{(M-1)/2} \times \exp\left[-\frac{(B_{12s} W_0/B_{0s} + W_{12}) B_{0s}}{K_{0,12s}} \right] \times I_{M-1} \left(2 \sqrt{\frac{|D_{0,12}|^2 W_0 W_{12}}{K_{0,12s}^2}} \right), \qquad (18)$$

where I_M is the modified Bessel function.

On the other hand, for $K_{0,12s} < 0$, the JSCF quasi-distribution $P_s(W_0, W_{12})$ of integrated intensities takes the form of a generalized function. It can be approximated using the following expression [28]:

$$P_s(W_0, W_{12}) \approx \frac{A(W_0W_{12})^{(M-1)/2}}{\pi\Gamma(M)(B_{0s}B_{12s})^{M/2}} \times \exp\left(-\frac{W_0}{2B_{0s}} - \frac{W_{12}}{2B_{12s}}\right) \times \mathrm{sinc}\left[\mathrm{A}\left(\sqrt{\frac{\mathrm{B_{12s}}}{\mathrm{B_{0s}}}}\mathrm{W_0} - \sqrt{\frac{\mathrm{B_{0s}}}{\mathrm{B_{12s}}}}\mathrm{W_{12}}\right)\right], \quad (19)$$

in which $\mathrm{sinc}(x) = \sin(x)/x$ and $A = (-K_{0,12s})^{-1/2}$. The quasi-distribution derived in Eq. (19) typically oscillates and has negative values in some regions. The threshold value s_{th} giving the boundary between the expressions in Eqs. (18) and (19) can be determined from the condition $K_{0,12s} = 0$:

$$s_{\mathrm{th}} = 1 + B_0 + B_{12} - \sqrt{(B_0 + B_{12})^2 - 4K_{0,12}}, \quad (20)$$

in which $-1 \leq s_{\mathrm{th}} \leq 1$.

The variances of the difference $n_0 - n_{12}$ of single- and compound-field photon numbers ($n_{12} = n_1 + n_2$) and the difference $W_0 - W_{12}$ of the single- and compound-field integrated intensities ($W_{12} = W_1 + W_2$) are linked by the following formula:

$$\langle[\Delta(n_0 - n_{12})]^2\rangle = \langle n_0\rangle + \langle n_{12}\rangle + \langle[\Delta(W_0 - W_{12})]^2\rangle. \quad (21)$$

Equation (21) shows that negative values of the quasi-distribution $P_s(W_0, W_{12})$ of integrated intensities are needed to observe sub-shot-noise correlations in single- and compound-field photon numbers, i.e. $R < 1$ in Eq. (17).

IV. EXPERIMENTAL RESULTS AND THEIR INTERPRETATION

From the intensity measurements of the single and compound fields we calculated the following moments for photoelectrons: $\langle m_0\rangle = 1225.183$, $\langle m_{12}\rangle = 1186.138$, $\langle m_0^2\rangle = 1609827$, $\langle m_{12}^2\rangle = 1518257$, $\langle m_0m_{12}\rangle = 1562402$. Moreover, the knowledge of the quantum detection efficiencies $\eta_0 = \eta_1 = \eta_2 = 0.28$ allowed us to derive the corresponding moments for photons: $\langle n_0\rangle = 4375.654$, $\langle n_{12}\rangle = 4236209$, $\langle n_0^2\rangle = 20522260$, $\langle n_{12}^2\rangle = 19354630$, $\langle n_0n_{12}\rangle = 19928600$.

In the case of photoelectrons the calculated values of the coefficients in Eqs. (11) are:

$$B_0 = 87.765104, B_{12} = 92.861384, \quad |D_{0,12}| = 90.371968, M = 13.3665. \quad (22)$$

We also note that the number of modes $M_0 = 13.95980$ and $M_{12} = 12.77321$ relative to the single and compound fields, respectively, are almost the same. In the case of photons the values of coefficients B_0, B_{12} and $|D_{0,12}|$ are the following:

$$B_0 = 313.447, B_{12} = 331.648, \quad |D_{0,12}| = 322.757. \quad (23)$$

The number of modes M is the same for photocount and photon-number distributions.

Determinant $K_{0,12}$ is negative both for photoelectron- ($K_{0,12} = -17.104$) and photon-number ($K_{0,12} = -218.158$) distributions. At the same time, the fluctuations in the difference between the single and the compound fields both for photoelectrons ($R = 0.954$) and for photons ($R = 0.837$) are below the shot-noise level. In accordance with Eq. (21), this means that the wave variance $\langle[\Delta(W_0 - W_{12})]^2\rangle$ of the difference between the single and compound field integrated intensities is negative: in fact we have the wave variance equal to -110.572 for photoelectrons and -1406.699 for photons. These negative values are due to the fact that frequency down-conversion emits the same number of photons into the single field a_0 and in the compound field formed by fields a_1 and a_2. For the same reason we obtain that the covariance $C = \langle\Delta m_0\Delta m_{12}\rangle/\sqrt{\langle(\Delta m_0)^2\rangle\langle(\Delta m_{12})^2\rangle}$ for photoelectrons and the analogous expression for photons assume values very close to 1: 0.991 for photoelectrons and 0.990 for photons. Moreover, we note that even the principal squeezing parameter $\lambda = 1 + B_0 + B_{12} - 2|D_{0,12}|$ [36] indicates a nonclassical behavior: we obtained $\lambda = 0.882$ for photoelectrons and 0.581 for photons; $\lambda = 1$ holds for coherent states. On the basis of the above mentioned quantities, we want to emphasize that the quantum nature of the state produced by the nonlinear process is more evident in terms of photons than in terms of photoelectrons.

The JSCF photocount, $p(m_0, m_{12})$, and photon-number, $p(n_0, n_{12})$, distributions calculated along the formula in Eq. (12) with values of parameters appearing in Eqs. (22) and (23) are shown in Fig. 2 and Fig. 3, respectively. There are strong photon-number correlations between the single field and the compound one. In particular, we note that the elements of $p(m_0, m_{12})$ and $p(n_0, n_{12})$ assume the highest values in the vicinity of the diagonal, *i.e.* for $m_0 \approx m_{12}$ and $n_0 \approx n_{12}$ (see contour plots in Figs. 2 and 3).

Values of Fano factors $F_{c,12}$ of the conditional photocount and photon-number distributions are important from the point of view of the conditional state-preparation scheme. We remind that a nonclassical state requires $F_{c,12} < 1$, i.e. sub-Poissonian statistics of the conditional distributions. In our case this requirement is not fulfilled by the photocount distribution, as shown in Fig. 4. This behavior is due to the relatively low quantum detection efficiency ($\eta = 0.28$). On the other hand, if the experimental data are corrected for the quantum efficiency, we can obtain a sub-Poissonian photon-number distribution for $n_0 > 5$ (see Fig. 5). The greater the number n_0 of detected single-field photons the smaller the value of Fano factor $F_{c,12}$. Actually, the values of

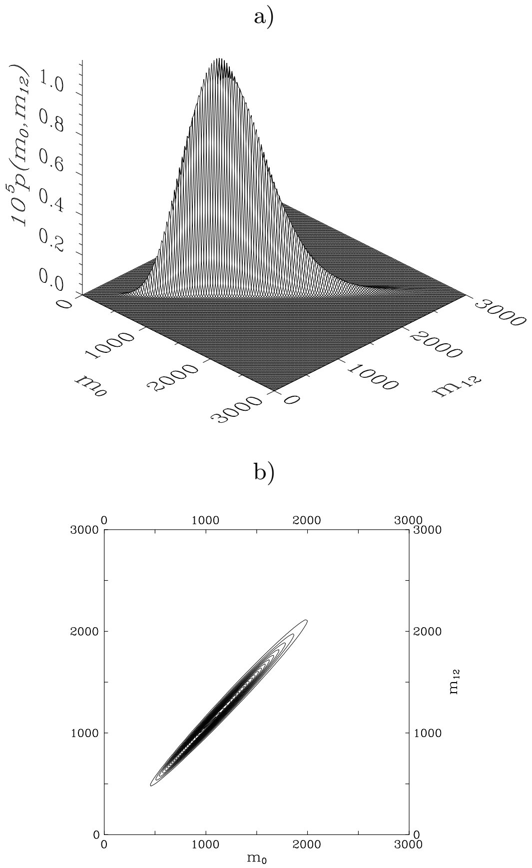


FIG. 2: JSCF photocount distribution $p(m_0, m_{12})$ (a) and its contour plot (b)

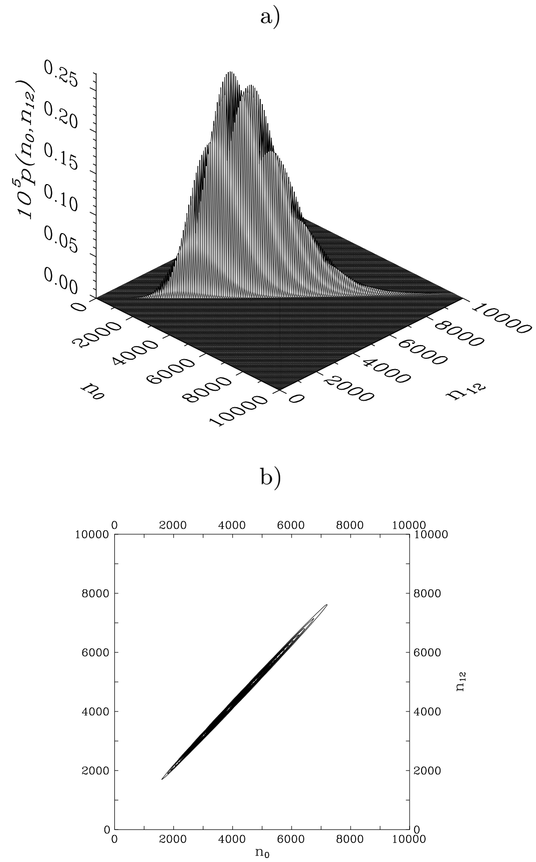


FIG. 3: JSCF photon-number distribution $p(n_0, n_{12})$ (a) and its contour plot (b).

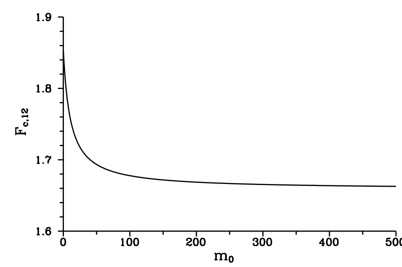


FIG. 4: Fano factor $F_{c,12}$ of the compound-field conditional photocount distribution $p_{c,12}$ as a function of the number m_0 of detected single-field photoelectrons.

Fano factor $F_{c,12}$ do not change for numbers of m_0 and n_0 greater than 100.

The behavior of the JSCF quasi-distribution $P_s(W_0, W_{12})$ of integrated intensities depends on the value of the ordering parameter s. In our case the threshold value of the ordering parameter, $s_{\rm th}$, is equal to 0.811 for photoelectrons. This means that the nonclassical behavior of the quasi-distribution $P_s(W_0, W_{12})$ is expected only for values of s greater than $s_{\rm th}$. Indeed, in Fig. 6 we show that oscillations and negative values of $P_s(W_0, W_{12})$ occur for $s = 0.9$ ($K_{0,12s} < 0$); on the other hand, the nonclassical features do not appear for $s = 0.4$ ($K_{0,12s} > 0$). This means that the quantum noise present in the detection chain covers the nonclassical behavior. In the case of photons, the JSCF quasi-distribution $P_s(W_0, W_{12})$ has a lower threshold value of the ordering parameter, namely $s_{\rm th} = 0.324$. For this reason, the JSCF quasi-distribution $P_s(W_0, W_{12})$ shown in Fig. 7 already shows nonclassical features like oscillations and negative values for $s = 0.4$.

It is clear from all these considerations that to appreciate the quantum features of the three-mode state and its usefulness for the preparation of a conditional state, the quantum detection efficiencies must be high enough. A typical dependence of the conditional Fano factor $F_{c,12}(n_0)$ on the quantum detection efficiency η ($\eta = \eta_0 = \eta_1 = \eta_2$) is plotted in Fig. 8 a) for $n_0 = 3000$. By decreasing the value of η we obtain an increase in the value of $F_{c,12}(n_0)$. Moreover, it can be demonstrated that there is a threshold value of the efficiency $\eta_{\rm crit}$ below which the conditional compound-field photon-number distribution is no more sub-Poissonian. As shown in Fig. 8 b), the critical value $\eta_{\rm crit}$ decreases by increasing the number n_0 of the photons in the single

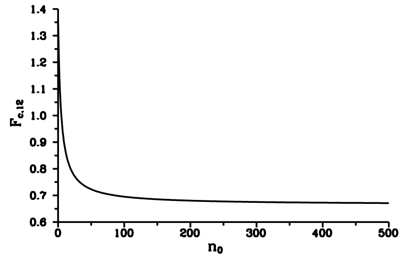
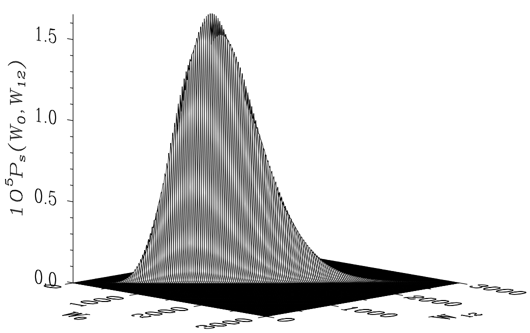


FIG. 5: Fano factor $F_{c,12}$ of the compound-field conditional photon-number distribution $p_{c,12}$ as a function of the number n_0 of detected single-field photons.

a)



b)

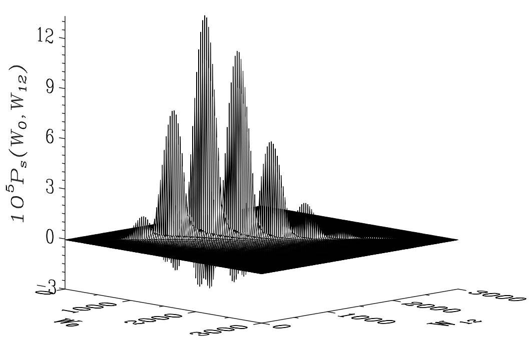
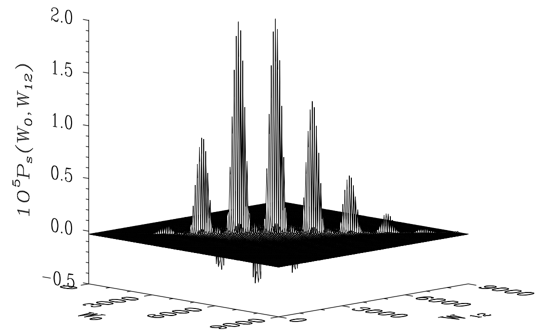


FIG. 6: JSCF quasi-distributions $P_s(W_0, W_{12})$ of the single-field (W_0) and compound-field (W_{12}) integrated intensities corresponding to photoelectrons for $s = 0.4$ (a) and $s = 0.9$ (b).

a)



b)

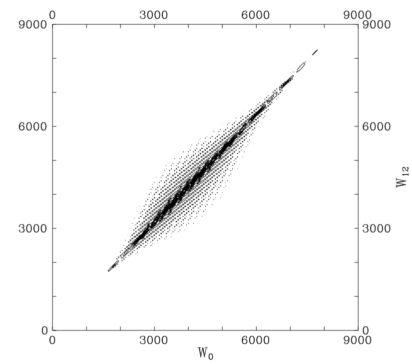


FIG. 7: JSCF quasi-distribution $P_s(W_0, W_{12})$ of the single-field (W_0) and compound-field (W_{12}) integrated intensities of photons for $s = 0.4$ (a) and the corresponding contour plot (b).

field. We note that graphs in Figs. 8 and 9 have been obtained using the formula for Fano factor $F_{c,12}$ in Eq. (15) assuming substitution $B_0 \rightarrow \eta_0 B_0$, $B_{12} \rightarrow \eta_1 B_{12}$, and $D_{0,12}^2 \rightarrow \eta_0 \eta_1 D_{0,12}^2$ and coefficients B_0, B_{12}, and $D_{0,12}$ for photons [Eqs. (23)]

In more detail, only the value of the quantum detection efficiency η_0 is crucial for the sub-Poissonian behavior of the conditional compound-field photon-number distribution, as shown in Fig. 9, where the contour plot of Fano factor $F_{c,12}$ is depicted as a function of detection efficiencies η_0 and η_{12} for $n_0 \rightarrow \infty$. In fact, for $\eta_0 > \eta_{0,\mathrm{crit,min}} = 0.7585$ we have a sub-Poissonian behavior, which does not depend on the value of η_{12} (we suppose that $\eta_{12} = \eta_1 = \eta_2$). This means that when η_0 exceeds its critical value, the quantum detection efficiency corresponding to the conditionally prepared state will not affect the nonclassical behavior of the state itself.

Note that, from the experimental point of view, the nonclassical behavior of an optical state is usually exhibited by the condition $R < 1$. On the other hand, in accordance with the theoretical study presented in [28], nonclassical fields satisfy the condition $K_{0,12} < 0$. Actually, the two conditions can be linked together. In fact, if $B_0 = B_{12} = B$, negative values of the determinant $K_{0,12}$, which testify a nonclassical character, lead to $\langle [\Delta(W_0 - W_{12})]^2 \rangle = 2M(B^2 - |D_{0,12}|^2) < 0$. By using Eq. (17) we thus obtain that $R < 1$, i.e. the sub-shot-noise reduction of fluctuations in the difference between the photons in the single and compound fields. However, it might happen that $R \geq 1$ for $B_0 \neq B_{12}$ and $K_{0,12} < 0$: in this case the noise reduction factor cannot be used to check the nonclassical nature of the measured state. However, when this condition occurs other quantities, such as the conditional Fano factor $F_{c,12}$, can provide evidence of the nonclassical character of the fields.

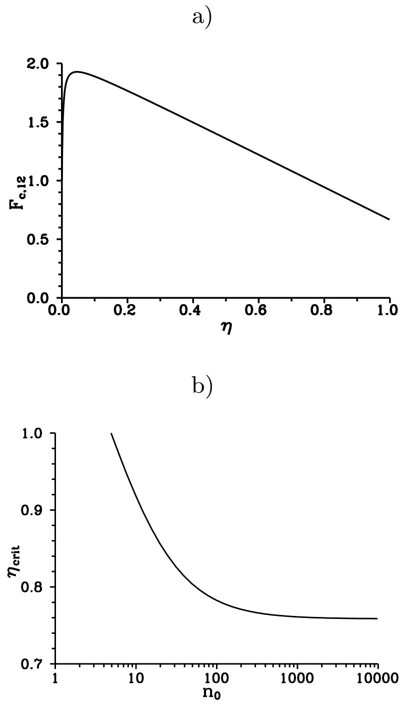


FIG. 8: Conditional Fano factor $F_{c,12}$ as a function of the quantum efficiency $\eta = \eta_0 = \eta_1 = \eta_2$ for the fixed number $n_0 = 3000$ of the single-field photons (a) and critical quantum detection efficiency $\eta_{\rm crit}$ as a function of the number n_0 of the single-field photons (b). If $\eta > \eta_{\rm crit}$ then $F_{c,12} < 1$ and viceversa. In graph (b), logarithmic scale is used on the x-axis.

V. CONCLUSIONS

The quantum properties of two interlinked nonlinear interactions generating a tripartite entangled state have been analyzed with special attention to the mutual correlations in the number of photons. In particular, we have taken into account the correlation between the photons in the first field and the sum of photons in the other two. The joint photon-number distribution, its conditional photon-number distribution as well as the joint quasi-distribution of integrated intensities have been determined to study the nonclassical properties of the measured fields. It has been shown that states entangled in photon numbers can occur in the second and third fields provided that a given number of photons is detected in the first field. In this entangled state, sum of photon numbers in the second and third fields equals the given number of photons in the first field and photon number in the second field (as well as in the third field) is not determined. The crucial role played by the quantum detection efficiencies in the production of the conditional state is widely discussed.

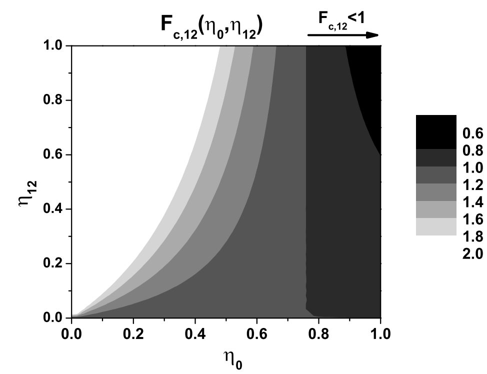


FIG. 9: Contour plot of the conditional Fano factor $F_{c,12}$ as a function of the quantum efficiencies η_0 and η_{12}, $\eta_{12} = \eta_1 = \eta_2$ for high values of n_0 ($n_0 \to \infty$). The values in the white area in the upper left corner have not been determined. $F_{c,12} < 1$ for $\eta_0 > \eta_{0,{\rm crit,min}}$, $\eta_{0,{\rm crit,min}} = 0.7585$.

Acknowledgments

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