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Five Dimensional Anisotropic Homogeneous Cosmological Models in Second Self Creation Theory of Gravitation

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Abstract

Five dimensional homogeneous anisotropic cosmological models in Barber's second self creation theory are constructed when the gravitational field is generated by a perfect fluid. Under the power law dependence of the scale factors, the perfect fluid models degenerate Zeldovich fluid models. The physical and geometrical features of these models are discussed.

Key Words: Five dimensions, self creation theory, anisotropic spacetime.

1. Introduction

In recent years cosmologists have come to be interested in theories with more than four spacetime dimensions in which extra dimensions are contracted to a very small size, sizes beyond our present ability of experimental detection [1]. Marciano [2] suggested that the experimental detection of time variation of fundamental constants could provide strong evidence for the existence of extra dimensions. Chodos and Detweller [3] proposed cosmological dimensional reduction process such that the five dimensional universe naturally evolves into four-dimensional universe as a consequence of dimensional reduction. Various authors [4-12] constructed higher dimensional cosmological models containing variety of matter fields.

Various cosmological problems are being studied by cosmologists to reveal the evolution of the universe. Many authors have proposed various alternative theories by modifying Einstein's general theory of relativity. Barber [13] proposed two theories known as self-creation theories. His first theory is a modification of the Brans and Dicke [14] theory and the second theory is a modification of the general theory of relativity. His first theory is both inconsistent with experiment as well as internally inconsistent [15]. The second theory of Barber is a modification of general relativity to a variable G theory and predicts local effects within the observational limits. In view of the consistency of Barber's second self creation theory of gravitation many authors [16-26] investigated various aspects of different spacetime. Recently Venkateswarlu and Pavan Kumar [27] studied the role of higher dimensional FRW models in Barber's second self creation theory when the source of gravitation is a perfect fluid.

In this paper we have constructed five-dimensional anisotropic cosmological models in second self creation theory. The energy momentum tensor is assumed to be the simple extension of the usual four-dimensional case and the isotropy of pressure is assumed in all directions, including the extra one as has been usually done in the literature [5, 28, 29].

2. Field Equations and Their Consequences

It is believed that the universe is anisotropic in its early stage of evolution. Therefore we consider the five dimensional anisotropic and homogeneous spacetime described by a metric of the form

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2 - C^2 d\psi^2, \tag{1}$$

where A, B, C are functions of t only.

The field equations in Barber's second self creation theory are

$$G_{ij} \equiv R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi}{\phi} T_{ij} \tag{2}$$

and

$$\Box\phi = \frac{8\pi}{3} \lambda T, \tag{3}$$

where $\Box\phi = \phi^{k}_{;k}$ is the invariant D'Alembertian and T is the trace of energy momentum tensor which describes all non-gravitational and non-scalar field matter and energy. Here, λ is a coupling constant to be evaluated from experiment and ϕ is the Barber's scalar. The measurement of deflection of light restricts the value of coupling to $0 < |\lambda| < 10^{-1}$. In the limit $\lambda \to 0$, this theory approaches the standard general relativity theory in every respect. In this theory, the Newtonian gravitational parameter G is not a constant but a function of t and $G = \frac{1}{\phi}$.

The energy momentum tensor for perfect fluid distribution is given by

$$T_{ij} = (p + \rho) u_i u_j - p g_{ij} \tag{4}$$

together with

$$g^{ij} u_i u_j = 1, \tag{5}$$

where $\rho, p \; and \; u^i$ are energy density, isotropic pressure and velocity five vector of the fluid, respectively.

By adopting the co-moving coordinates system and using equations (4) and (5), field equations (2) and (3) for the metric (1) are obtained as

$$\left(\frac{A'}{A}\right)^2 + 2\frac{A'B'}{AB} + 2\frac{A'C'}{AC} + \frac{B'C'}{BC} = \frac{8\pi}{\phi}\rho \tag{6}$$

$$\frac{A''}{A} + \frac{B''}{B} + \frac{C''}{C} + \frac{A'B'}{AB} + \frac{A'C'}{AC} + \frac{B'C'}{BC} = \frac{-8\pi}{\phi} p \tag{7}$$

$$2\frac{A''}{A} + \frac{C''}{C} + \left(\frac{A'}{A}\right)^2 + 2\frac{A'C'}{AC} = \frac{-8\pi}{\phi} p \tag{8}$$

$$2\frac{A''}{A} + \frac{B''}{B} + \left(\frac{A'}{A}\right)^2 + 2\frac{A'B'}{AB} = \frac{-8\pi}{\phi} p \tag{9}$$

and

$$\phi'' + \left(2\frac{A'}{A} + \frac{B'}{B} + \frac{C'}{C}\right) \phi' = \frac{8}{3}\pi\lambda \left(\rho - 4p\right), \tag{10}$$

where prime symbol denotes ordinary differentiation with respect to time t. The system of field equations (6) − (10) is an under determined system having five equations involving six unknowns, viz. A, B, C, p, ρ and ϕ. In order to solve this system of equations we consider here the power law dependence of the scale factors

$$C = B^n, \tag{11}$$

where $n$is a real number.

Subtracting equation (9) from equation (8) we get

$$\frac{C''}{C} - \frac{B''}{B} + 2\frac{A'C'}{AC} - 2\frac{A'B'}{AB} = 0. \tag{12}$$

Substituting equation (11) in equation (12) we obtain

$$(n-1)\frac{B'}{B}\left(\frac{B''}{B'} + n\frac{B'}{B} + 2\frac{A'}{A}\right) = 0. \tag{13}$$

Equation (13) is satisfied for the following cases:

$$\text{Case I: } n = 1. \tag{14}$$

$$\text{Case II: } B = k \text{ (Constant)} \tag{15}$$

$$\text{Case III: } \frac{B''}{B'} + n\frac{B'}{B} + 2\frac{A'}{A} = 0. \tag{16}$$

3. Solutions

In this section we intend to derive explicit exact solutions of the field equations for each of the above cases.

3.1. Case I

Using equation (14) in equation (11) we find
$B = C$.

In this case the behavior of the scale factors of Z co-ordinates and fifth co-ordinates are same. But the physical situations demand that when the space co-ordinates expand the extra dimensions contract and become unobservable. So this case is not physically realistic.

3.2. Case II

Using equation (15) in equation (11), we obtain

$$C = k^n. \tag{17}$$

Substituting (15) and (17) in equation (6) − (10) we get

$$\left(\frac{A'}{A}\right)^2 = \frac{8\pi}{\phi}\rho \tag{18}$$

$$\frac{A''}{A} = -\frac{8\pi}{\phi} p \tag{19}$$

$$2\frac{A''}{A} + \left(\frac{A'}{A}\right)^2 = -\frac{8\pi}{\phi} p \tag{20}$$

and

$$\phi'' + 2\frac{A'}{A}\phi' = \frac{8}{3}\pi\lambda\,(\rho - 4p). \tag{21}$$

Subtracting (19) from (20) we find

$$\frac{A'}{A}\left(\frac{A''}{A'} + \frac{A'}{A}\right) = 0,$$

which yields following two sub cases:

$$\text{Sub case I: } A = l \text{ (Constant of Integration)} \tag{22}$$

$$\text{Sub case II: } \frac{A''}{A'} + \frac{A'}{A} = 0 \tag{23}$$

3.2.1. Sub case I

Substituting (22) in (18) and (19) we obtain

$$p = \rho = 0,$$

which reveals that the perfect fluid does not survive in this case and the spacetime becomes Minkowskian. The Barber scalar is obtained as

$$\phi = mt + m_0,$$

where $m$and m_0 are constants of integration.

3.2.2. Sub case II

Solving equation (23), we get

$$A = (a_1 t + a_2)^{\frac{1}{2}}, \tag{24}$$

where a_1 and a_2 are constants of integration.

Using (24) in (18) and (19) we find

$$p = \rho = \frac{\phi}{32\pi}\frac{a_1^2}{(a_1 t + a_2)^2}. \tag{25}$$

Thus in this case the general fluid degenerates the Zeldovich fluid.

With the help of equations (24) and (25), equation (21) reduces to

$$(a_1 t + a_2)^2 \phi'' + a_1 (a_1 t + a_2)\,\phi' + \frac{\lambda\, a_1^2}{4}\phi = 0. \tag{26}$$

Solving (26) we obtain

$$\phi = m_1 \cos\left\{\log(a_1t + a_2)^{\frac{\sqrt{\lambda}}{2}}\right\} + m_2 \sin\left\{\log(a_1t + a_2)^{\frac{\sqrt{\lambda}}{2}}\right\}. \tag{27}$$

Now equation (25) gives the physical parameter pressure with density as

$$p = \rho = \frac{a_1^2}{32\pi(a_1t + a_2)^2}\left[m_1 \cos\left\{\log(a_1t + a_2)^{\frac{\sqrt{\lambda}}{2}}\right\} + m_2 \sin\left\{\log(a_1t + a_2)^{\frac{\sqrt{\lambda}}{2}}\right\}\right]. \tag{28}$$

In this case the five dimensional cosmological model for the Zeldovich universe in Barber's second self creation theory is given by

$$ds^2 = dt^2 - (a_1t + a_2)(dx^2 + dy^2) - k^2dz^2 - k^{2n}d\psi^2. \tag{29}$$

3.2.3. Case III

Equation (16) yields

$$\frac{B''}{B'} + n\frac{B'}{B} = -2\frac{A'}{A}(= k_1)^{,} \tag{30}$$

where $k_1$is an arbitrary constant.

From (30) we obtain

$$A = e^{\frac{-(k_1t+k_2)}{2}} \tag{31}$$

and

$$B = (b_1e^{k_1t} + b_2)^{\frac{1}{n+1}}, \tag{32}$$

where k_2, b_1 and b_2 are constants of integration.

Using equation (32) in equation (11) we get

$$C = (b_1e^{k_1t} + b_2)^{\frac{n}{n+1}}. \tag{33}$$

With the help of equations (31)-(33), equations (9) and (6) yield

$$p = \frac{\phi}{8\pi}\left(\frac{nb_1k_1^2e^{2k_1t}}{(n+1)^2(b_1e^{k_1t} + b_2)^2} - \frac{3k_1^2}{4}\right) \tag{34}$$

and

$$\rho = \frac{\phi}{8\pi}\left(\frac{k_1^2}{4} - \frac{k_1^2b_1e^{k_1t}}{(b_1e^{k_1t} + b_2)} + \frac{nk_1^2b_1^2e^{2k_1t}}{(n+1)^2(b_1e^{k_1t} + b_2)^2}\right). \tag{35}$$

Substituting equations (31) − (35) in equation (10) we find

$$\phi'' - k_1b_2\phi' - \frac{\lambda}{3}\left(\frac{13k_1^2}{4} - \frac{k_1^2b_1e^{k_1t}}{(b_1e^{k_1t} + b_2)} - \frac{3nk_1^2b_1^2e^{2k_1t}}{(n+1)^2(b_1e^{k_1t} + b_2)^2}\right)\phi = 0. \tag{36}$$

From equation (36) it is difficult to obtain a general solution of ϕ. Therefore for a particular solution we take $k_1 = 0$ in equation (30).Thus we find

$$A = d_0 \tag{37}$$

$$B = (d_1t + d_2)^{\frac{1}{n+1}} \tag{38}$$

and

$$C = (d_1t + d_2)^{\frac{n}{n+1}}, \tag{39}$$

where d_0, d_1 and d_2 are constants of integration.

Thus equations (6) and (9) yield

$$\frac{8\pi}{\phi}p = \frac{8\pi}{\phi}\rho = \frac{nd_1^2}{(n+1)^2(d_1t + d_2)^2}. \tag{40}$$

The Barber scalar ϕ is obtained from equation (10) as

$$\phi = m_1 \cos\left\{\log(d_1t + d_2)^{\frac{\sqrt{n\lambda}}{n+1}}\right\} + m_2 \sin\left\{\log(d_1t + d_2)^{\frac{\sqrt{n\lambda}}{n+1}}\right\}. \tag{41}$$

Now from equation (40) the physical parameters pressure and density are given by

$$p = \rho = \frac{nd_1^2\left[m_1 \cos\left\{\log(d_1t + d_2)^{\frac{\sqrt{n\lambda}}{n+1}}\right\} + m_2 \sin\left\{\log(d_1t + d_2)^{\frac{\sqrt{n\lambda}}{n+1}}\right\}\right]}{8\pi(n+1)^2(d_1t + d_2)^2}. \tag{42}$$

In this case the five dimensional Zeldovich model in Barber's second self creation theory is obtained as

$$ds^2 = a_0^2(dx^2 + dy^2) - (d_1t + d_2)^{\frac{2}{n+1}}dz^2 - (d_1t + d_2)^{\frac{2n}{n+1}}d\psi^2. \tag{43}$$

4. Some physical and geometrical nature of the models

(a) Equation (29) represents five dimensional Zeldovich fluid universe in Barber's second self creation theory. The model is free from initial singularity i.e. at $t = \frac{-a_2}{a_1}$ but exhibits singularity at infinite time. The energy density with pressure $\rho(= p)$in the model (29) is given by equation (28). It is evident that $\rho(= p) \to 0$ as$t \to \infty$. The scalar of expansion θ calculated as

$$\theta = \frac{3a_1}{a_1t + a_2},$$

from which it is observed that the universe is expanding with increase of time but the rate of expansion becomes slow as time increases.

The shear scalar σ^2 for the model (29) is

$$\sigma^2 = \frac{a_1^2}{4(a_1t + a_2)^2} - \frac{a_1}{3(a_1t + a_2)} + \frac{2}{9}.$$

Since $\sigma^2 \to \infty$ as $t \to \frac{-a_2}{a_1}$and $\sigma^2 \to 0$ as $t \to \infty$, the shape of the model changes uniformly in x and y directions only and the rate of change of shape of the universe becomes slow with increase of time. It is observed that $\lim\limits_{t\to\infty} \frac{\sigma^2}{\theta^2} \neq 0$, which indicates that the universe remains anisotropic throughout the evolution.

The spatial volume is obtained as $V = k^{n+1}(a_1t + a_2)$. Here, $V \to 0$ as $t \to \frac{-a_2}{a_1}$ and $V \to \infty$ as $t \to \infty$. This indicates that the model is corresponding to open model. The deceleration parameter q vanishes in this model.

(b) Equation (43) represents five dimensional anisotropic homogeneous Zeldovich universe in second self creation theory. At the initial epoch $t = \frac{-d_2}{d_1}$ the model is free from singularity. As time $t$increases the model expands along z directions and the extra dimension contracts for *-1< n <0.* At infinite time the extra dimension becomes unobservable and reduces to the model obtained earlier by Mohanty et al. (2000).

In the model (43) the physical nature of the scalar expansionθ, spatial volume V and the deceleration parameter q coincides with the corresponding results of model (29) studied earlier in case (a). In this model the shear scalar σ^2 is calculated as

$$\sigma^2 = \frac{(n^2 + 1)d_1^2}{2(n+1)^2(d_1t + d_2)^2} - \frac{d_1}{3(d_1t + d_2)} + \frac{2}{9}.$$

Since $\sigma^2 \to \infty$ as $t \to \frac{-d_2}{d_1}$and $\sigma^2 \to 0$ as $t \to \infty$, the shape of the model changes uniformly in $z$direction only and the rate of change of shape of the universe becomes slow with increase of time.

4. Conclusion

In this paper we have constructed two five dimensional Zeldovich models given by (29) and (43) in Barber's second self creation theory for the spacetime (1). The models (29) and (43) start with the initial epoch $t = \frac{-a_2}{a_1}$and $t = \frac{-d_2}{d_1}$ respectively. In the model (29) the extra dimension remains constant throughout the evolution whereas in the model (43) it contracts for *-1*$<$ n $<$ *0*. Further it is interesting to note that when the Barber's coupling constant$\lambda \to 0$, the corresponding Barber's scalar ϕ tends to constant in both the models. Subsequently the model (29) and (43) degenerates Zeldovich fluid models in Einstein theory.

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