

Determining the Minimum Uncertainty State of Nonclassical Light

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Squeezing experiments which are capable of creating a minimum uncertainty state during the nonlinear process, for example optical parametric amplification, are commonly used to produce light far below the quantum noise limit. This report presents a method with which one can characterize this minimum uncertainty state and gain valuable knowledge of the experimental setup.

INTRODUCTION

Experiments which produce light below the quantum noise limit (QNL) already exist since the mid 80s and several different technics, such as four wave mixing [1], the Kerr effect [2], second harmonic generation [3] or optical parametric amplification [4, 5], have been developed to produce many squeezing results. In this report we will focus on the popular method of optical parametric amplifiers (OPA), which is capable of generating high amounts of quadrature squeezing [6, 7, 8, 9], and theoretically can produce a minimum uncertainty state during the nonlinear process [10]. However, the minimum uncertainty state can never be measured directly as there will always be loss and noise sources which will make the detected state more noisy. A method with which one can determine this minimum uncertainty state precisely in a quick and easy way would be very useful as one could then characterize the system more closely by knowing how much squeezing is maximum possible and determining where the losses in the setup exactly are. A common way to describe a minimum uncertainty is by referring to the variance of the quadratures of the squeezed signal. This has been analyzed theoretically many times (for example see [10, 11]) and we will only focus and present the most important theoretical tools for this report. The variance of a generalized quadrature of the squeezed beam at the rotation angle θ is given by [10]

$$\Delta^2(\mathbf{X}(\theta)_{MU}) = \cosh(2r) - \sinh(2r) \cos(2(\theta - \theta_s)) \quad (1)$$

with r representing the degree of squeezing and θ_s the squeezing angle. The amplitude of this periodic function is determined by the degree of squeezing r , while its minimum and maximum are governed by the angle θ_s . This function describes a squeezed beam in its minimum uncertainty state. However, due to losses and different noise sources, one can never measure this minimum uncertainty state directly. In a typical squeezing experiment losses can occur at several points. It commences with the optical parametric amplifier where the squeezed light is produced and which normally has an escape efficiency below one $\eta_{esc} < 1$. Further losses can occur at lenses η_{lens} , beam splitters η_{BS} and finally at the photodetector η_{det} . The total efficiency of the system is then defined as

$$\eta = \eta_{esc} \cdot \eta_{lens} \cdot \eta_{BS} \cdot \eta_{det}. \quad (2)$$

The influence of the total efficiency η on the variance is given by [10]

$$\Delta^2(\mathbf{X}(\theta)_{Loss}) = 1 + \eta(\Delta^2(\mathbf{X}(\theta)_{MU}) - 1) \quad (3)$$

with $\Delta^2(\mathbf{X}(\theta)_{Loss})$ being the measured variance and $\Delta^2(\mathbf{X}(\theta)_{MU})$ the variance of the light in the minimum uncertainty state.

Another important parameter which can influence the measured signal is the dark noise. Even when there is no light shining on the photodetector a small signal can still be measured. This is due to various factors such as thermal effects, material impurities, stray fields etc. [10]. In most experiments the dark noise can be ignored as the optical

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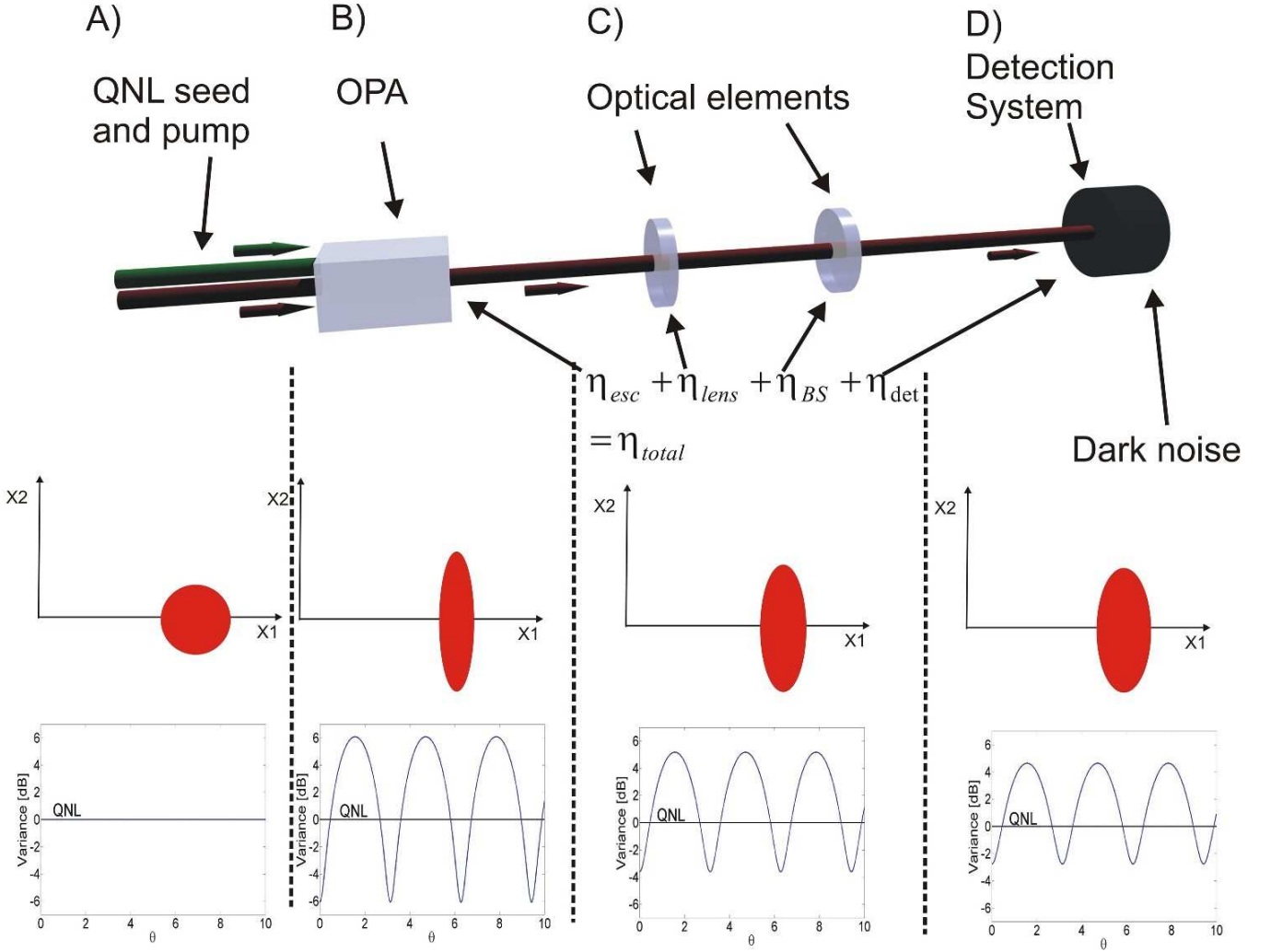


FIG. 1: A schematic of a typical squeezing experiment. Here an OPA, which is operated with a quantum noise limited pump and seed beam (part A)) generates a squeezed beam which is in its minimum uncertainty state (part B)). It leaves the OPA which has an escape efficiency η_{esc} , goes through optical components such as lenses η_{lens} and beam splitters η_{BS} and is then detected at a detection system with a detection efficiency η_{det} (part C)). When the signal is processed also the dark noise from the detection system influences the signal (part D)). In this Figure the commonly used ball on a stick picture and the output variance when varying θ are used to illustrate how the minimum uncertainty becomes more noisy during the course of the experiment.

power is much larger than the dark noise. However, if one measures squeezing the dark noise can have a big influence as only very small fluctuations are being measured.

This means that when simulating real measurements, one has to consider influences such as losses and dark noise to the variance in equation (1). The detected variance is the given by

$$\Delta^2(\mathbf{X}(\theta)_{Det}) = (1 + \eta(\Delta^2(\mathbf{X}(\theta)_{MU}) - 1))(\Delta^2(X_{QNL}) - \Delta^2(X_{dark})) + \Delta^2(X_{dark}) \quad (4)$$

with η being the efficiency, $\Delta^2(X_{dark})$ the amount of dark noise, $\Delta^2(X_{QNL})$ the quantum noise limit, $\Delta^2(\mathbf{X}(\theta)_{Det})$ the detected efficiency and $\Delta^2(\mathbf{X}(\theta)_{MU})$ the input state, which is assumed to be a minimum uncertainty state. It is very interesting to see what different effects, such as losses η , the dark noise $\Delta^2(X_{dark})$ and the amount of squeezing r , have on the output variance. This is shown in Figure 2 a), b) and c), respectively. In the graphs the zero-line represents the QNL. Notice that in Figure 2 a) and b) the zero-points stay the same for all cases and only the amount of squeezing and anti-squeezing changes. Only in Figure 2 c), where the degree of squeezing is changed, do the zero-points vary.

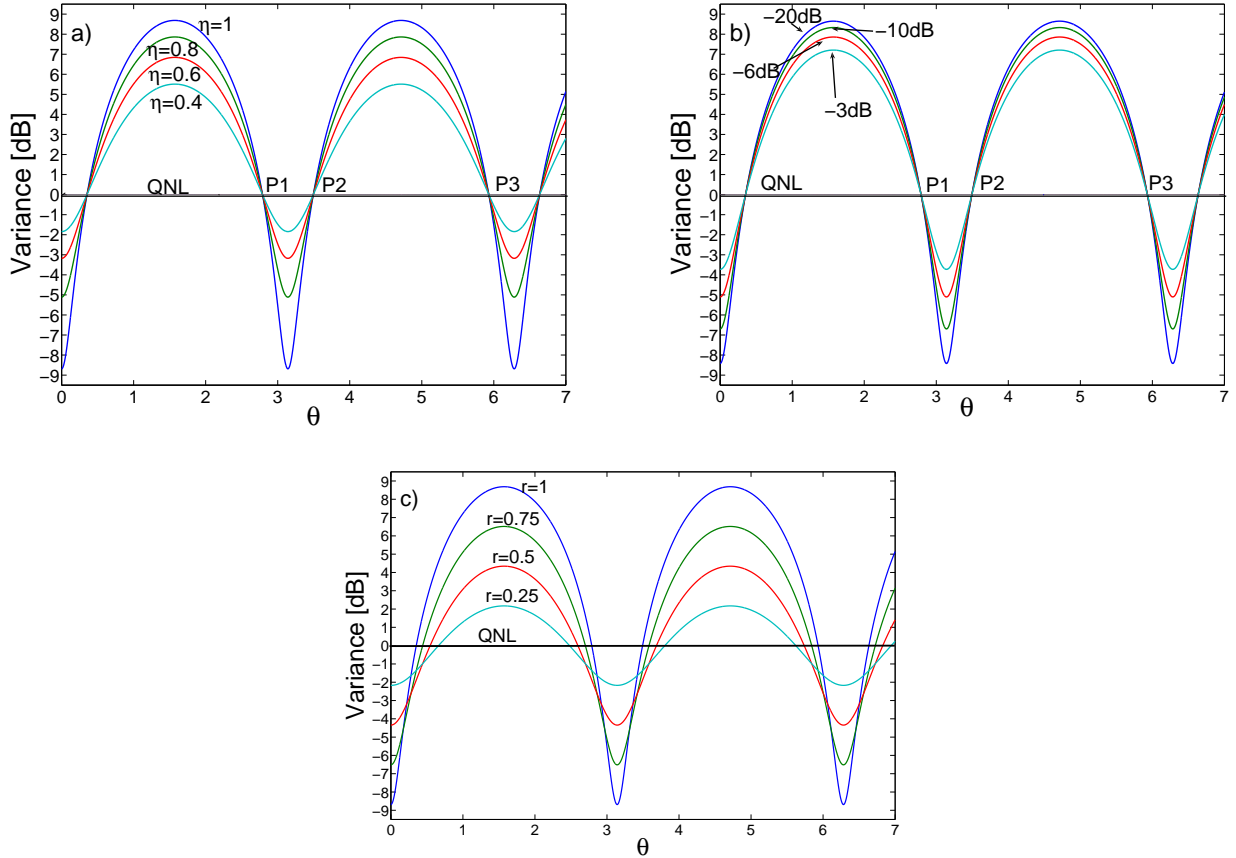


FIG. 2: Variance on a logarithmic plot. a) η is varied, there is no dark noise, $r = 1$, b) the dark noise is varied, $\eta = 1$, $r = 1$ and c) only the degree of squeezing is varied.

DETERMINING THE INITIAL DEGREE OF SQUEEZING

The observation that the zero-points vary only if the degree of squeezing r is changed is very interesting. The ratio seems to be independent of the dark noise and loss and due to this might be a very helpful tool to determine the minimum uncertainty state. Let us look at three consecutive points $P1$, $P2$ and $P3$ as marked in Figure 2 a) or b). When setting $\Delta^2(X_{QNL}) = 1$, $\theta_s = 0$ and combining it with equation (1), these consecutive points are given by:

$$x_{P1} = -\frac{1}{2} \arccos(l), \quad (5)$$

$$x_{P2} = \frac{1}{2} \arccos(l), \quad (6)$$

$$x_{P3} = -\frac{1}{2} \arccos(l) + \pi, \quad (7)$$

with l being

$$l = \frac{-1}{\sinh(2r)} + \frac{\cosh(2r)}{\sinh(2r)}. \quad (8)$$

This means that the only parameter that influences the zero-points is r . Consequently, the ratio $\frac{P1P2}{P2P3}$ only depends on r . It is given by

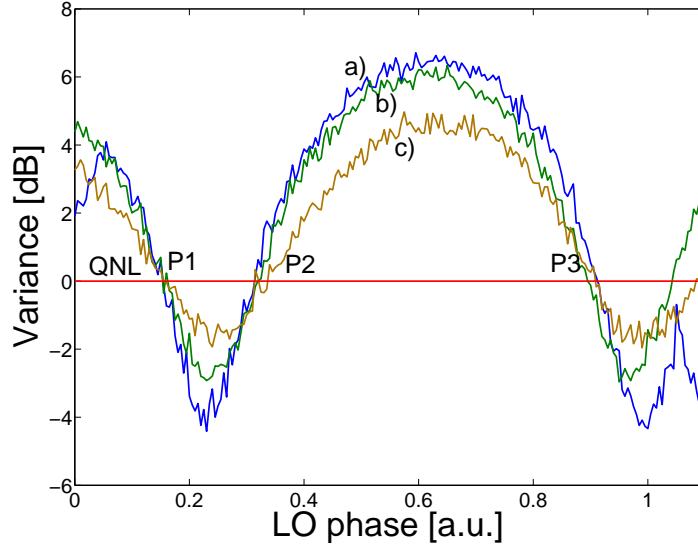


FIG. 3: Experimental measurements are displayed when the degree of squeezing is kept constant while the loss is varied. Measurement a) has no attenuation while b) and c) have 1dB and 3dB of attenuation respectively.

$$\frac{\overline{P1P2}}{\overline{P2P3}} = \frac{\arccos(l)}{-\arccos(l) + \pi}. \quad (9)$$

This is a rather powerful tool. It means that one only needs to measure the ratio $\frac{\overline{P1P2}}{\overline{P2P3}}$ to determine the initial degree of squeezing. The ratio can be measured directly from the graph on the spectrum analyzer. It is not necessary to correct the data for dark noise and similar influences anymore. This implies that it can be a much more accurate way of determining the minimum uncertainty state of the measured signal. Here only the error of the ratio will influence the accuracy of the minimum uncertainty state calculated. It should be noted again that this method is only valid if the initial state is a minimum uncertainty state.

Experimentally it can be shown that losses do not influence the ratio. This can be done simply by taking one squeezed beam and changing the attenuation of the measured signal, which means adding extra losses to the systems. This is displayed in Figure 3 a). The OPA is locked to de-amplification, meaning that squeezing will be measured in the amplitude quadrature. Three different measurements were taken: one with no attenuation, one with 1dB and one with 3dB attenuation. One can see that they all have different amounts of squeezing and anti-squeezing. However, the zero-points are all the same, meaning that the ratio is the same. This supports the argument that only the degree of squeezing influences the ratio and that dark noise and loss have no influence on the ratio whatsoever. It was assumed that θ , represented here by the time axis, is on a linear scale. However, there might be a nonlinear ramp of the PZT which can cause scaling problems such as can be seen on the right hand side of Figure 3 a). This can also cause small errors when determining the ratio exactly.

EXPERIMENTAL ANALYSIS

Let us have an even closer look at experimental data. In the experiments conducted a bow-tie cavity with a PPKTP crystal as the nonlinear medium was used as an OPA. The seed beam had a wavelength of 1064 nm and was produced inside an ultrastable, continuous-wave single frequency laser based on Nd:YAG laser material (Innolight GmbH, Diablo) [12]. The signal was detected by a homodyne detection system formed by two custom made InGaAs homodyne detectors. The efficiency of the detectors was $\eta_{det} = 95\%$, of the homodyne detector $\eta_{Vis} = 98\%$ and of the optical elements, such as mirrors and lenses, $\eta_{opt} = 97\%$. This gives a total efficiency of

$$\eta = \eta_{esc}\eta_{det}\eta_{Vis}\eta_{opt} = \eta_{esc} \cdot 0.95 \cdot 0.98 \cdot 0.97, \quad (10)$$

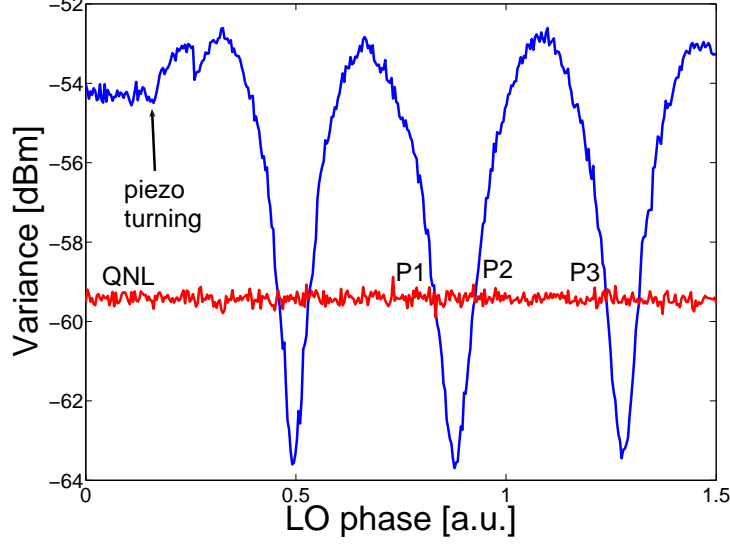


FIG. 4: Squeezing measurements from the bow-tie cavity. It was detected at zero-span at 4.25 MHz with a resolution bandwidth of 300 kHz and video bandwidth of 300 Hz.

with η_{esc} being the escape efficiency of the cavity which was unknown.

The measurements were done on a spectrum-analyzer at zero-span at the detection frequency of 4.25 MHz with a resolution bandwidth of 300 kHz and video bandwidth of 300 Hz. The OPA was locked to de-amplification, meaning that squeezing was observed in the amplitude quadrature. One measurement can be seen in Figure 4. The QNL was measured to be $\Delta^2(X_{Det}^{QNL}) = -59.4$ dBm, the maximum squeezing $\Delta^2(X_{Det}^+) = -63.4$ dBm and the anti-squeezing $\Delta^2(X_{Det}^-) = -52.5$ dBm. This gives relative values of $\Delta^2(X_{Det}^+) - \Delta^2(X_{Det}^{QNL}) = -4$ dB and $\Delta^2(X_{Det}^-) - \Delta^2(X_{Det}^{QNL}) = 6.9$ dB of squeezing and anti-squeezing respectively.

Let us apply the method to determine the minimum uncertainty state which was presented above. When applying this method it is important to note that the phase between the local oscillator and the signal beam must be scanned on a linear scale. The phase is scanned with a PZT located on a mirror of which the local oscillator beam is reflected. Measurements can be seen in Figure 4. Here it can be observed when the PZT is changing its direction and it was assumed that between two direction changes of the PZT the scan should be fairly linear. 18 different ratios were measured and a mean value of

$$\frac{\overline{P1P2}}{\overline{P2P3}} = 0.307 \pm 0.02 \quad (11)$$

calculated. From this ratio the degree of squeezing can be determined, by using equation (9), to be

$$r = 0.948 \pm 0.05. \quad (12)$$

By applying equation (1) a minimum uncertainty state with squeezing and anti-squeezing of

$$\Delta^2(X_{MU}^+) = 10 \log_{10}(\cosh(2 \cdot 0.948) - \sinh(2 \cdot 0.948)) = -8.23 \text{ dB} \pm 0.4 \text{ dBm} \quad (13)$$

$$\Delta^2(X_{MU}^-) = 10 \log_{10}(\cosh(2 \cdot 0.948) + \sinh(2 \cdot 0.948)) = 8.23 \text{ dB} \pm 0.4 \text{ dBm} \quad (14)$$

can be calculated. The minimum uncertainty state is displayed in Figure 5. When looking at this minimum uncertainty state the following important points should be noted:

- The results are only true for the frequency at which the signal was detected (in this case 4.25 MHz). Normally, it can be expected that at lower frequencies the squeezing increases, meaning that also the minimum uncertainty state will be different.

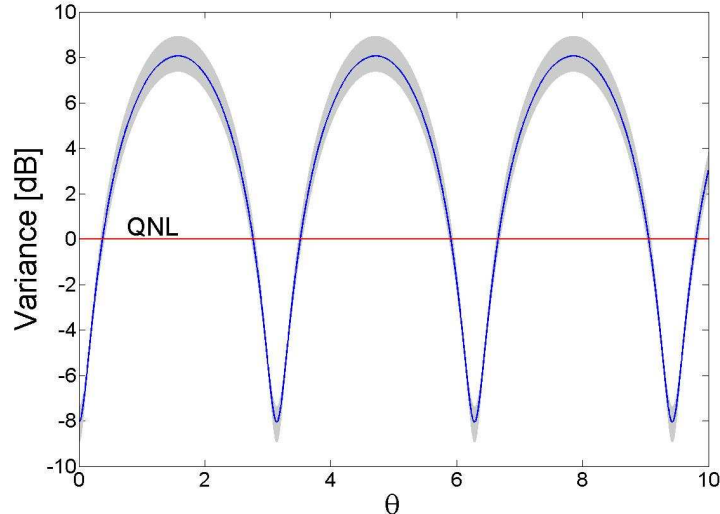


FIG. 5: The minimum uncertainty state which is produced inside the bow-tie cavity when calculated with the ratio-method. The grey-area represents the error margin.

- It is also only valid for the pump power used in the experiment. Different pump powers yield different gains which in turn automatically mean different minimum uncertainty states.
- Furthermore, the results are only true if the initial state was actually a minimum uncertainty state. If during the nonlinear process a minimum uncertainty was never created the results are obviously not accurate. A method on how to test the validity is presented below.

If the initial state was a minimum uncertainty state as the one calculated, a characterization of the loss and noise of the system should be possible. With the help of equation (4), one can derive the efficiency of the system to be

$$\eta = \frac{\Delta^2(\mathbf{X}(\theta)_{Det}) - \Delta^2(X_{QNL})}{(\Delta^2(\mathbf{X}(\theta)_{MU}) - 1)(\Delta^2(X_{QNL}) - \Delta^2(X_{dark}))}. \quad (15)$$

As we have measured $\Delta^2(\mathbf{X}(\theta)_{Det})$ and calculated $\Delta^2(\mathbf{X}(\theta)_{in})$ with the ratio method, we can calculate the total efficiency η . If one does this calculation twice with two distinctive values, such as squeezing and anti-squeezing, one can check if the minimum uncertainty calculated is correct (if the two calculated efficiencies are equal the minimum uncertainty state is correct). The following values were measured $\Delta^2(\mathbf{X}_{Det}^-) - \Delta^2(X_{Det}^{QNL}) = 6.9 \text{ dB} \pm 0.2 \text{ dB}$, $\Delta^2(\mathbf{X}_{Det}^+) - \Delta^2(X_{Det}^{QNL}) = -4 \text{ dB} \pm 0.2 \text{ dB}$, $\Delta^2(X_{Det}^{QNL}) - \Delta^2(X_{Det}^{dark}) = 10.6 \text{ dB} \pm 0.2 \text{ dB}$, and calculated (with the ratio method) $\Delta^2(X_{MU}^-) = 8.23 \text{ dB} \pm 0.4 \text{ dB}$, $\Delta^2(X_{MU}^+) = -8.23 \text{ dB} \pm 0.4 \text{ dB}$. After converting these values onto a linear scale, one then gets the following efficiencies:

$$\begin{aligned} \eta &= 0.756 \pm 0.05 && \text{(calculated with } \Delta^2(X_{MU}^-) \text{ and } \Delta^2(X_{Det}^-)) \\ \eta &= 0.774 \pm 0.05 && \text{(calculated with } \Delta^2(X_{MU}^+) \text{ and } \Delta^2(X_{Det}^+)) \end{aligned} \quad (16)$$

It can be seen that the values are nearly identical and well within the error margin. The dependence between the output variance, the efficiency and the ratio can be seen in Figure 6. If there would have been a big discrepancy between these two values, this could be explained due to the fact that a minimum uncertainty state was never created, for example due to excess noise on the seed or pump beam.

From the total efficiency calculated, we can now also determine what the escape efficiency of the OPA is. By using equation (10), a value of

$$\eta_{esc} = \frac{\eta}{\eta_{det}\eta_{vis}\eta_{opt}} = \frac{0.77}{0.95 \cdot 0.98 \cdot 0.97} = 0.85 \quad (17)$$

was calculated.

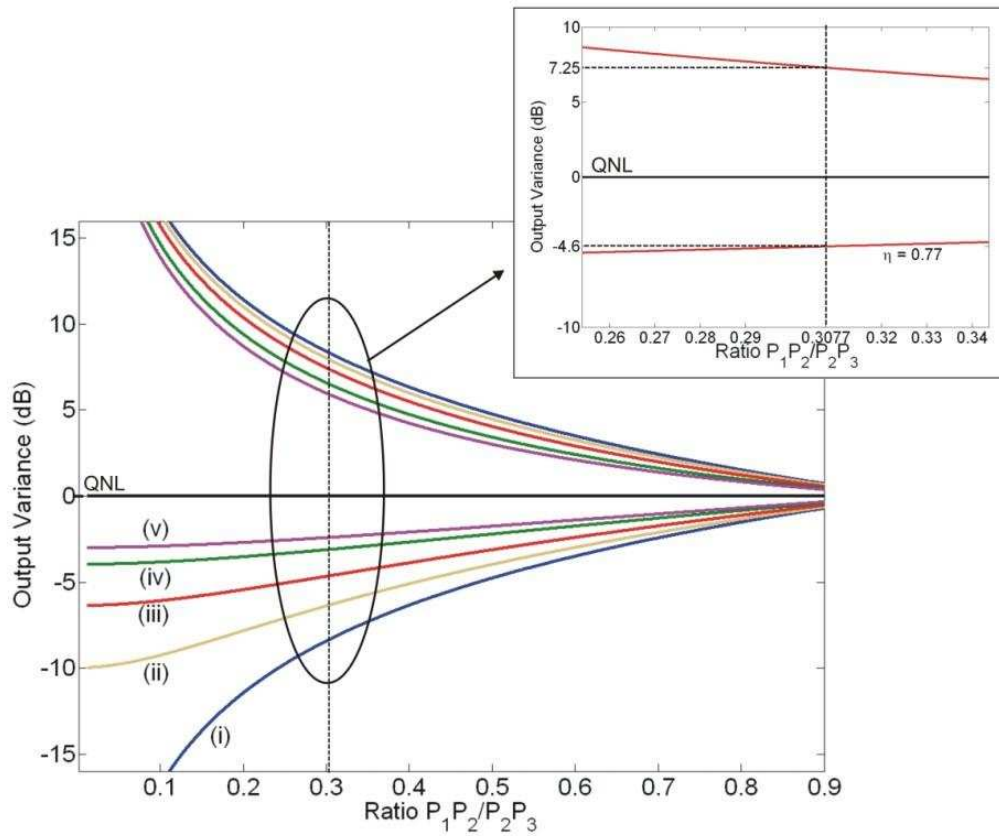


FIG. 6: The output variance as a function of the ratio for (i) $\eta = 1$, (ii) $\eta = 0.9$, (iii) $\eta = 0.77$, (iv) $\eta = 0.6$ and (v) (ii) $\eta = 0.5$. The inset shows the case for the experimental measurements where a ratio of 0.3077 was measured.

CONCLUSION

We can conclude that the ratio method is a quick and easy way to characterize an optical parametric amplifier. It allows us to determine what the initial minimum uncertainty state is and see how much squeezing is possible with the current experiment. From this it is also possible to quickly calculate the total loss of the system and determine other unknown variables such as the escape efficiency from the OPA. All this gained knowledge can be used to understand and improve the current setup. This method also allows one to determine whether a minimum uncertainty state was ever created or not. In systems such as an OPA this is theoretically the case but different effects, such as noisy seed or pump beams, can prohibit it. The gained knowledge can be used to further improve and characterize the experiment.

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