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Magnetic Levitation of a Small Magnetic Ring Above Cylindrical Superconductor Sample in the Meissner State

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Abstract

The levitation force between a cylindrical superconductor in the Meissner state and a small magnetic ring was calculated using the dipole-dipole interaction model under the assumption that the magnetic ring is small compared with the physical dimensions of the system. We obtained analytical expressions for the levitation forces as a function of the geometrical parameters of the ring and the superconductor sample as well as the height of magnetic ring. We analyzed the levitation force in two configurations of the magnetic moment of the small ring with respect to the surface of the superconductor: one horizontal in which the magnetic moment is parallel to the surface of superconductor and the other vertical in which the magnetic moment is perpendicular to the surface of superconductor. The levitation force for vertical configuration is always higher than that for horizontal configuration. Also, the force in vertical configuration does not depend on the geometry of the magnetic ring.

Key Words: Levitation force; Meissner state, superconducting cylinder.

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1. Introduction

The discovery of magnetic levitation phenomenon in superconductor magnetic systems has attracted the attention of researchers for new technological applications. Recently, with advances in material processing techniques, magnetic levitation becomes more interesting for both practical applications as well as basic scientific research. The levitation force between a superconductor and a magnet has been measured or calculated through a series of research papers [1–8]. The theoretical investigations of these levitation forces give some qualitative agreement with experimental results [3, 9–11]. Various applications of the superconducting materials, besides magnetic levitation systems, have been tested, such as magnetic levitated bearings, flywheels, and motors/generators.

The levitation force on a magnet above a superconductor has been studied whether the superconductor is in the Meissner state or in the mixed state. Even though the levitation force can be calculated in different ways, most works in this field took into account the effect of the geometrical characteristic of both the superconductor and the magnet on the levitation force. The ability of using a commercial finite element program to calculate the levitation forces between a cylindrical permanent magnet and a cylinder superconductor has been explored by Camacho et al [12]. Xu et al [13] calculated the levitation forces between a magnet and a type II superconductor in both Meissner and mixed states using the London model. They also found a simple relationship between the levitation force and London penetration depth. In a very interesting work by Teshima et al [8], they describe numerically and experimentally the magnetic levitation force on a cylindrical magnet above cylindrical superconductor. They found that this force is at maximum when the radius of the magnet is slightly smaller than the superconductor radius. Badia and Freydardt [14] showed how the use of integral transforms provides a very convenient tool for studying finite size effects in the screening of the magnetic fields. They calculated the levitation force and the induced super-current density inside the superconductor when a cylindrical permanent magnet brought over an ideal superconductor. Yang and Hull [5] derived an analytical expression of the levitation force between a long magnetic dipole line and an infinitely long superconducting strip in the Meissner state. Lugo and Sosa [3] calculated the effect of the thickness and the radius of a cylindrical superconductor on the levitation force between a small magnet and a superconducting cylinder in the Meissner state using dipole-dipole interaction model. Recently, Alzoubi et al [4] studied the same system of Lugo and Sosa, and they generalized their results by studying the angular dependence of the levitation force for the same system. Also, in reference [11], Alzoubi et al, studied the vibrations in magnet/superconductor system due to the importance of the stability of such systems in the technological applications.

In this article, we study a system consisting of a small magnet ring placed symmetrically above a finite cylindrical superconductor in the Meissner state. The Meissner state represents the simplest model to deal with in superconductors in which the magnetic flux is totally expelled from its interior. This is not quite true because in real experiments lines of magnetic field penetrate the superconductor. To neglect this penetration we have to choose very small magnetic ring that produce weak magnetic field.

In the weak field limit, it is possible to reduce the flux penetration into the superconducting material and hence the Meissner state is a good approximation that leads to an accurate estimation of the actual force acts on the magnet. To achieve this limit, our calculations are based on the assumption the magnet is very small. To test the model experimentally, different approaches may help in getting levitation in the Meissner state. First, cooling the superconductor below its critical temperature without an applied magnetic field and then approaching the magnet very carefully. Second, preparing high quality samples of small dimensions as those prepared by melt-powder melt-growth process helps in achieving complete flux exclusion.

Using the dipole-dipole interaction model, we obtained an analytical formula of the levitation force acting on the ring. In our calculations, we assumed that the magnetic ring as well as the superconducting sample was equivalent to continuous array of point dipoles. Integrating the dipole-dipole interaction over the length of the small ring and the volume of the superconductor sample enabled us to derive analytical expressions for the levitation force. These expressions depend on the geometry of magnet/superconductor system as well as the height between the magnet and the superconducting sample.

2. Theoretical Model

The dipole-dipole interaction model is an easy and simple model to handle the problem, among other techniques such as the image method, the finite element method and the method of scalar potential. In this article, we use the dipole-dipole interaction model to derive analytical expression for the force on a small magnetic ring above cylindrical superconductor. In our system, as illustrated in Figure 1, a small magnetic ring with radius a and net magnetic moment $\vec{\mu}$ (which is assumed uniformly distributed over its circumference) is levitating at height h above a cylindrical superconductor of radius R and thickness l. The magnetic ring and the superconducting sample have the same axial symmetry about z-axis. In our calculations, both the thickness and the radial width of the magnetic ring are negligible compared with its radius. The magnetic field $\vec{H}$ produced by the magnetic ring at a distance r within the volume of the superconductor is given by

$$\vec{H}(\vec{r}) = \int \left(\frac{3\vec{r}(\vec{r} \cdot d\vec{\mu})}{r^5} - \frac{d\vec{\mu}}{r^3} \right), \tag{1}$$

where the $d\vec{\mu}$ is the magnetic moment for an arc length ds along the circumference of the ring. In this case, it is convenient to define a linear magnetizationλ, as a magnetization per unit length, i.e. $d\vec{\mu} = \lambda ds\ \hat{\mu}$. Therefore, the net magnetic moment of the ring can be written as $\vec{\mu} = 2\pi a \lambda\ \hat{\mu}$.

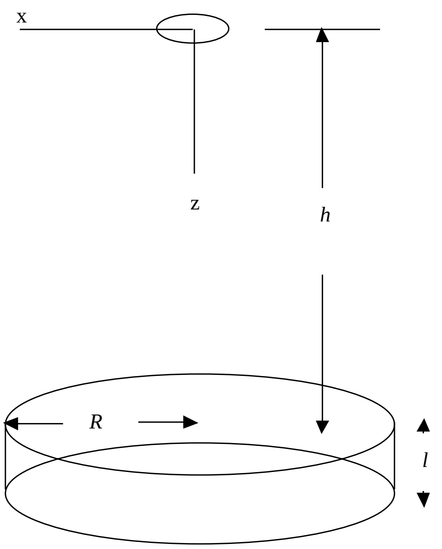

Figure 1. Schematic diagram of a small magnet ring/superconductor system. The ring has a total permanent magnetic moment $\vec{\mu}$ levitated over superconducting cylinder of radius R and thickness l. The ring of radius a is lying in xy-plane and at a distance h from the superconductor surface.

In the Meissner state, we assume a total expulsion of the magnetic field from the interior of the superconductor ($\vec{B} = 0$) and therefore $\vec{M} = -\vec{H}/4\pi$. Under the assumption that the superconductor is a continuous array of magnetic dipoles distributed uniformly within its volume, the magnetization is defined as the magnetic moment per unit volume $\vec{M}(\vec{r}) = d\vec{\mu}'/dV$, where $d\vec{\mu}'$ is the magnetic moment of each point dipole in the superconductor. The magnetic ring acts by a small force $d\vec{F}$on each dipole$d\vec{\mu}'$, and this force is given by [3]

$$d\vec{F} = \vec{\nabla}(d\vec{\mu}' \cdot \vec{H}). \tag{2}$$

To find the force acting on the whole superconductor due to the magnet, we should integrate equation (2) over the volume of the superconductor. Using Newton's third law, we can write the force on the magnet due to the superconductor as

$$\vec{F} = -\int_V \vec{\nabla}(d\vec{\mu}' \cdot \vec{H}) = \frac{\vec{\nabla}}{4\pi} \int_V \vec{H}^2 dV. \tag{3}$$

To simplify our problem without affecting the physical results, it is important to mention that in our calculations of the levitation force we neglected the demagnetization effect and focus on the qualitative behavior of the force for the systems. This is valid because the analytical simulation of demagnetization effect is complicated, and it will affect the magnitude of the force rather than its behavior [7, 15].

3. Results and Discussion

Generally, performing the integral in equation (3) will result in an analytical expression for the force acts on the magnet as a function of the geometrical parameters of our problem. In this section, we will evaluate the integral in equation (3) for two cases of the net magnetic moment $\vec{\mu}$of the ring: $\vec{\mu}$ parallel to the superconductor surface (horizontal configuration) and the other perpendicular to the superconductor surface (vertical configuration). Due to the axial symmetry of our system, the x and y-components of $\vec{F}$ vanish in both cases. The integration in equation (3) results in a complicated expression for the z-component of the force$\vec{F}$(the levitation force$\vec{F}_z$). To simplify this expression, we assume that the radius of the magnetic ring is too small compared with the dimensions of the superconductor as well as the levitation height of the ring. The levitation force $F_z$can be calculated in terms of a, R, l, and h.

3.1. Horizontal configuration

In this configuration as shown in Figure 1, $\vec{\mu} = \mu\,\hat{r}$, and the levitation force is given by

$$F_z(z,a) = \frac{3\mu^2}{16}\left\{\, f(h,a) - \; f(h+l,a)\,\right\}, \tag{4}$$

where

$$f(z,a) = \frac{1}{z^4} - \frac{8a^2}{3z^6} - \frac{24a^2z^2}{3(R^2+z^2)^4} + \frac{32a^2 - 5R^2 - 3z^2}{3(R^2+z^2)^3}. \tag{5}$$

Note that for fixed values of the radius and the thickness of superconductor sample, F_z depends on the radius of the ring and its height from the superconductor surface. As a check to our calculations, it obvious to show that when $a \to 0$(our system reduced to a point magnetic dipole moment above cylindrical superconductor), the levitation force is identical to that of Lugo et al [3] for parallel configuration.

3.2. Vertical configuration

In this configuration as shown in Figure 1, $\vec{\mu} = \mu\,\hat{k}$, and the levitation force is given by

$$F_z(z) = \frac{3\mu^2}{8}\left\{ g(h) - \; g(h+l)\,\right\}, \tag{6}$$

where

$$g(z) = \frac{1}{z^4} - \frac{R^2 + 3z^2}{3(R^2+z^2)^3}. \tag{7}$$

It is important to note that the levitation force in this configuration does not depend on the radius of the ring. Also, this result is identical to Lugo's et al [3] result for a point magnetic moment above cylindrical superconductor in the vertical configuration.

3.3. Dependence of levitation force on geometrical dimensions

As clear from the analytical expression for the levitation force, in general, the levitation force depends on the geometrical parameters of our system, such as R, a, h, and l. Figure 2 shows the levitation force $F_z$as a function of the thickness of the superconductor, l for both configurations and for three different heights of the magnetic ring (at $h = 0.30,\ 0.35,$ and 0.45 cm). For all curves in Figure 2, we used: $\mu = 0.40$ G·cm^3, $R = 2.0$ cm, and $a = 0.06$ cm. In each curve, as the thickness of the superconductor is increased, the levitation force increases and levels off at certain thickness. In both configurations, the saturation force is decreased by increasing the height h. For a given value of h, the ratio of the levitation force in the vertical configuration to that in the horizontal configuration is approximately constant. For different values of the height h, by increasing the height of the ring this ratio decreased very slowly. For example, the ratio increases from about 2.10 to 2.25 by decreasing h from 0.45 to 0.30 cm.

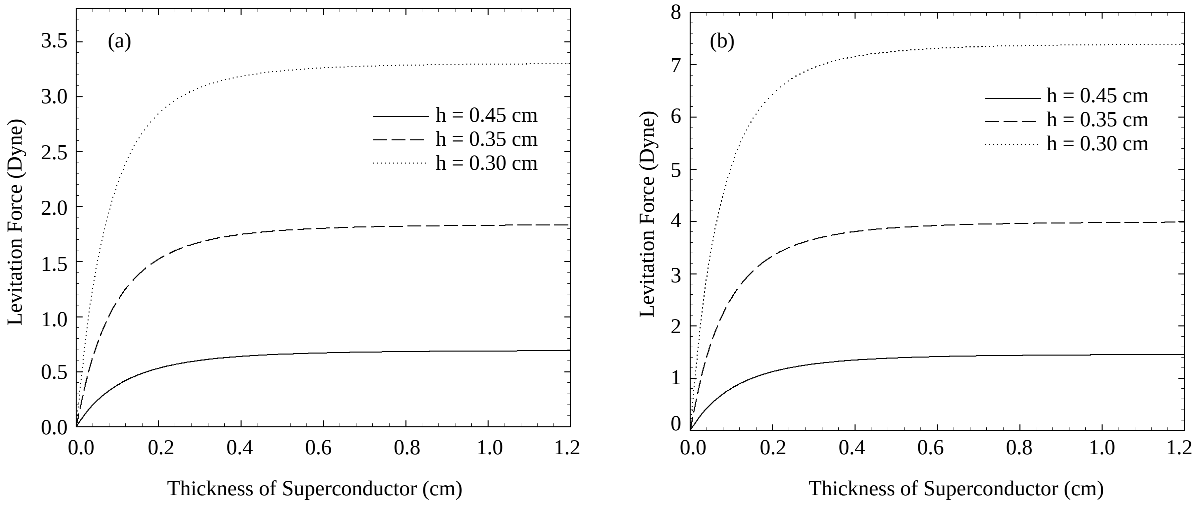

Figure 2. Levitation force as a function of the thickness of the superconductor for: (a) horizontal configuration; (b) vertical configuration. The curves for the height $h = 0.30$, 0.35 and 0.45 cm are represented. For all curves the following parameters have been used: $\mu = 0.4\text{G}\cdot\text{cm}^3$, $R = 2$ cm, and $a = 0.06$ cm.

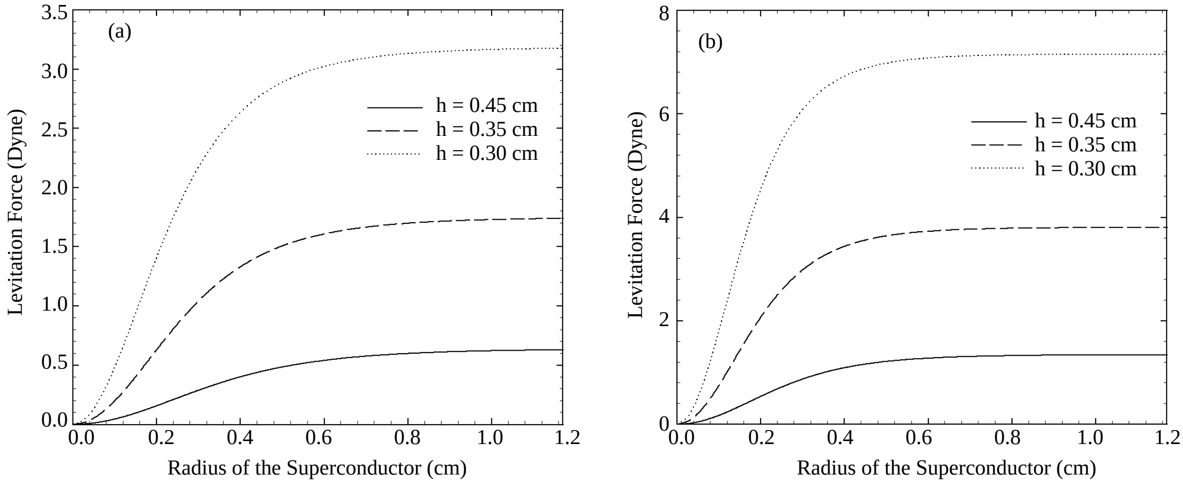

Figure 3. Levitation force as a function of the radius of the superconductor for: (a) horizontal configuration; (b) vertical configuration. The curves for the height $h = 0.30$, 0.35 and 0.45cm are represented. For all curves the following parameters have been used: $\mu = 0.4\text{G}\cdot\text{cm}^3$, $l = 0.4$ cm, and $a = 0.06$ cm.

In Figure 3, we have plotted the variation of the levitation force F_z against the radius of the superconductor sample R for both configurations and for three values of the height (at $h = 0.30,\ 0.35,$ and 0.45 cm). A monotonic growth of each curve can be observed up to the saturation of the levitation force. For each configuration, the saturation force is decreased by increasing the height h. For a given value h, the ratio of the levitation force in the vertical configuration to that in the horizontal configuration is changing significantly. For example, this ratio decrease from 4 in the region of small R (at $R = 0.03 - 0.09$ cm) to about 2 in a large region of R (at $R = 0.9 - 1.2$ cm). The ratio is approximately the same for all different values of the height h. It is important to mention that for all curves in figure 3, we have assumed that $\mu = 0.40$ G·cm^3, $R = 2.0$ cm, and $a = 0.06$ cm. From figures 2 and 3, it is obvious to see that the behavior of the levitation force on a small ring with respect of the thickness and radius of the superconductor is similar to the behavior of the levitation force on a small magnetic dipole above the same superconductor (see reference 3).

Figure 4 displays the decrease of the levitation force F_z as the height of the magnetic ring increases for different values of the radius R(at $R = 0.15$, 0.25, and 2 cm). For a given value R, the ratio of the levitation force in the vertical configuration to that in the horizontal configuration decreases as the height of the magnetic ring increases. Also, this ratio increases by increasing the radius of the superconductor R. Finally, in this figure we assumed that $\mu = 0.40$G.cm^3, $l = 0.4$cm, and $a = 0.06$cm.

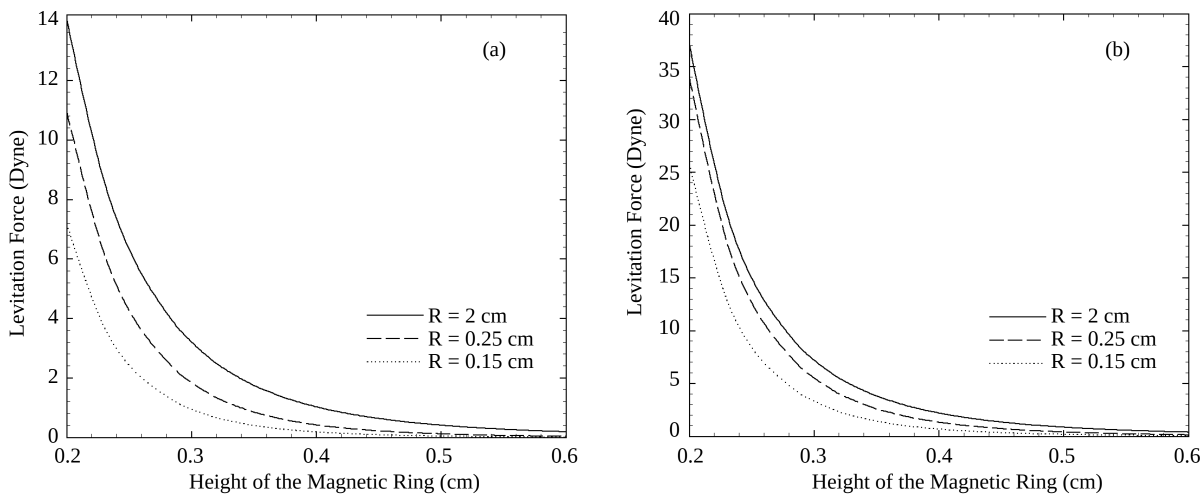

Figure 4. Levitation force as a function of the height of the magnetic ring above the surface of the superconductor for: (a) horizontal configuration; (b) vertical configuration. The curves for the radius of the superconductor $R = 0.15$, 0.25 and 2 cm are represented. For all curves the following parameters have been used: $\mu = 0.4$G.cm^3, $l = 0.4$ cm, and $a = 0.06$ cm.

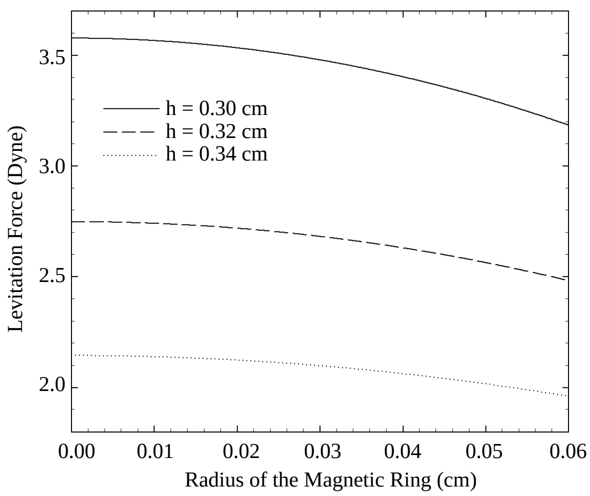

Figure 5. Levitation force as a function of the radius of the magnetic ring for horizontal configuration. The curves for the height above of the superconductor surface $h = 0.32$, 0.34 and 0.36 cm are represented. For all curves the following parameters have been used: $\mu = 0.4$G·cm^3, $l = 0.4$ cm, and $R = 2$ cm.

Figure 5 shows the variation of the levitation force F_z in the horizontal configuration with respect to the radius $a$of the magnetic ring. The curves are shown at different values of its height h (at $h = 0.30$, 0.32, and 0.34 cm) and keeping the other parameters fixed: $\mu = 0.40$ G·cm^3, $l = 0.4$cm, and $R = 2$cm. It is clear that the levitation force decreases as the radius of the magnetic ring increases. By increasing the height of the ring above the superconductor, the variation in the levitation force as a function of magnetic ring radius becomes much smaller and the curves becomes a flat line. This indicates that the levitation force becomes independent of the radius magnetic ring. It is important to mention that the levitation force in the vertical configuration is independent of the radius of magnetic ring.

4. Conclusion

We calculated the levitation force between a small ring and a superconducting cylinder in the Meissner state using dipole-dipole interaction model. We obtained analytical expressions for the force as a function of the geometry of ring/superconductor system. The results showed that the levitation force in vertical configuration does not depend on the geometry of the small ring. The radius of the superconductor is more effective than its thickness in calculating the ratio between the vertical and horizontal configuration. In the horizontal configuration, the levitation force depends in the radius of the ring. By increasing the height of the ring above the superconductor, the levitation force becomes independent of the radius magnetic ring.

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