

# IBM-2 Calculations of Selected Even-Even Palladium Nuclei

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## Abstract

In this study, we have employed the Interacting Boson Model-2 (IBM-2) to determine the most appropriate Hamiltonian for the study of palladium nuclei. Using the best fit values of parameters to construct the Hamiltonian, we have estimated energy levels and multipole mixing ratios ( $\delta(E2/M1)$ ) for some doubly-even Pd nuclei. The results are compared with previous experimental and theoretical data and it is observed that they are in good agreement.

**Key Words:** Palladium, electromagnetic transition, multipolarity, Interacting Boson Model-2 (IBM-2).

## 1. Introduction

There have been many attempts to explore the factors responsible for the onset of large deformation in nuclei of the mass region  $A \cong 100$ . The Interacting Boson Model (IBM) is one of those attempts that has been successful in describing the low-lying nuclear collective motion in medium and heavy mass nuclei [1–3].

The purpose of this paper is to set up some even even nuclei around the mass region  $A \cong 100$ . The neutron rich even even Pd isotopes around the mass region  $A \cong 100$  are very important for understanding the gradual change from spherical to a deformed state via transitional phase[4]. These nuclei lie between strongly deformed <sup>100</sup>Zr and doubly magic <sup>132</sup>Sn, near which structural changes are rather rapid with changes in the proton and neutron numbers.

The outline of the remaining part of this paper is as follows. Starting from an approximate IBM-2 formulation for the Hamiltonian in section 2, we review the theoretical background of the study. Previous experimental and theoretical [5–11] data are compared with estimated values and the general features of Pd isotopes in the range  $A = 102–110$  are reviewed in section 3. There are three tables in this section; Table 1 gives the best fitted parameters used in the present work, while Table 2 gives a comparison of estimated and experimental energy levels for <sup>102–110</sup>Pd. Table 3 shows a comparison of estimated and experimental multipole mixing ratios ( $\delta(E2/M1)$ ) of some transitions in <sup>102–110</sup>Pd nuclei. The last section contains some concluding remarks.

**Table 1.** Best fit values of the Hamiltonian parameters for  $^{102-110}\text{Pd}$ .

$\frac{A}{Z}X_N$	$N_\pi$	$N_\nu$	$N$	$\varepsilon$	$\kappa$	$\chi_\nu$	$\chi_\pi$	$C_{L\nu}$ (L=0,2,4)	$C_{L\pi}$ (L=0,2,4)
$^{102}_{46}\text{Pd}_{56}$	2	3	5	0.780	-0.080	-1.20	0.60	0.50, 0.0, 0.0	0.20, 0.0, 0.0
$^{104}_{46}\text{Pd}_{58}$	2	4	6	0.760	-0.082	-1.00	0.60	0.00, 0.0, 0.0	0.20, 0.0, 0.0
$^{106}_{46}\text{Pd}_{60}$	2	5	7	0.740	-0.085	-0.80	0.60	0.20, 0.0, 0.0	0.20, 0.0, 0.0
$^{108}_{46}\text{Pd}_{62}$	2	6	8	0.690	-0.090	-0.60	0.60	-0.12, 0.0, 0.0	0.00, 0.0, 0.0
$^{110}_{46}\text{Pd}_{64}$	2	7	9	0.650	-0.095	-0.40	0.60	-0.10, 0.0, 0.0	0.00, 0.0, 0.0

**Table 2.** Comparison of estimated energy levels with experiment for  $^{102-110}\text{Pd}$ .

Isotope	Spin Parity ( $I^\pi$ )	This Work (MeV)	Experiment [5]
$^{102}_{46}\text{Pd}_{56}$	$2_1^+$	0.607	0.557
	$4_1^+$	1.322	1.276
	$6_1^+$	2.140	2.112
	$8_1^+$	3.058	3.013
	$10_1^+$	4.075	3.993
	$2_2^+$	1.312	1.535
	$3_1^+$	2.109	2.249
	$4_2^+$	2.122	2.138
	$0_2^+$	1.565	1.593
	$2_3^+$	2.444	1.944
$^{104}_{46}\text{Pd}_{58}$	$2_1^+$	0.561	0.556
	$4_1^+$	1.225	1.324
	$6_1^+$	1.984	2.250
	$8_1^+$	2.832	3.221
	$10_1^+$	3.767	4.023
	$12_1^+$	4.786	4.635
	$2_2^+$	1.216	1.342
	$3_1^+$	1.960	1.821
	$4_2^+$	1.968	2.082
	$0_2^+$	1.331	1.334
$^{106}_{46}\text{Pd}_{60}$	$2_1^+$	0.517	0.512
	$4_1^+$	1.129	1.229
	$6_1^+$	1.829	2.076
	$8_1^+$	2.612	2.963
	$10_1^+$	3.476	3.533
	$12_1^+$	4.416	4.088
	$14_1^+$	5.433	4.893
	$2_2^+$	1.121	1.128
	$3_1^+$	1.810	1.558
	$4_2^+$	1.816	1.932

**Table 2.** Continued.

Isotope	Spin Parity	This Work	Experiment	
	$0_2^+$	1.333	1.334	
	$2_3^+$	2.068	1.562	
	$4_3^+$	2.578	2.077	
$^{108}_{46}Pd_{62}$	$2_1^+$	0.441	0.434	
	$4_1^+$	0.979	1.048	
	$6_1^+$	1.604	1.771	
	$8_1^+$	2.310	2.548	
	$10_1^+$	3.090	3.350	
	$12_1^+$	3.958	-	
	$14_1^+$	4.864	-	
	$2_2^+$	0.972	0.931	
	$3_1^+$	1.589	1.335[7]	
	$4_2^+$	1.593	1.625[7]	
	$0_2^+$	1.050	1.053	
	$2_3^+$	1.644	1.441	
	$4_3^+$	2.284	2.864	
	Isotope	Spin Parity	This Work	Experiment
	$^{110}_{46}Pd_{64}$	$2_1^+$	0.376	0.374
	$4_1^+$	0.849	0.921	
	$6_1^+$	1.405	1.574	
	$8_1^+$	2.044	2.296	
	$10_1^+$	2.749	-	
	$12_1^+$	3.570	-	
	$14_1^+$	4.385	-	
	$2_2^+$	0.847	0.814	
	$3_1^+$	1.399	1.214	
	$4_2^+$	1.402	1.398	
	$0_2^+$	0.956	0.947	
	$2_3^+$	1.482	1.214[7]	
	$4_3^+$	2.031	-	

**Table 3.** Comparison of estimated  $\delta(E2/M1)$  multipole mixing ratios of some transitions for  $^{102-110}Pd$  isotopes.

Isotope	Transition $I_i^+ \rightarrow I_f^+$	Energy (MeV)	$\delta(E2/M1)$ ( $eb/\mu_N$ )			
			Experimental	This Work	Theoretical	
$^{102}_{46}Pd_{56}$	$2_2^+ \rightarrow 2_1^+$	0.705	2.8 <sup>(b)</sup> 10.4 (+12.1,-3.7) <sup>(c)</sup>	3.55	-72 <sup>(b)</sup>	
	$2_3^+ \rightarrow 2_1^+$	1.837	8.1 (+7.3,-2.6) <sup>(c)</sup>	0.45	-	
	$2_3^+ \rightarrow 2_2^+$	1.132	-	4.71	-	
	$4_2^+ \rightarrow 4_1^+$	0.800	-	1.89	-	
	$4_3^+ \rightarrow 4_2^+$	0.878	-	2.37	-	
			2.095	-	0.32	-
		$4_4^+ \rightarrow 4_2^+$	1.295	-	2.65	-

Table 3. Continued.

	Transition	Energy	$\delta(E2/M1)$ ( $eb/\mu_N$ )		
$^{104}_{46}Pd_{58}$	$2_2^+ \rightarrow 2_1^+$	0.701	$ \delta  \geq 5^{(c)}$ $ \delta  \geq 8^{(c)}$ 11 (+10,-3) <sup>(c)</sup>	7.04	-
	$2_3^+ \rightarrow 2_1^+$	1.528	-	0.29	-
	$2_3^+ \rightarrow 2_2^+$	0.873	-	9.51	-
	$4_2^+ \rightarrow 4_1^+$	0.743	-0.84 <sup>(b,c)</sup> -0.4 (+0.10,-0.14) <sup>(c)</sup>	3.08	-8 <sup>(b)</sup>
	$4_3^+ \rightarrow 4_2^+$	0.820	-	4.38	-
	$4_4^+ \rightarrow 4_1^+$	1.703	-0.64 <sup>(b)</sup>	0.21	0.22 <sup>(b)</sup>
	$4_4^+ \rightarrow 4_2^+$	0.960	-	3.82	-
$^{106}_{46}Pd_{60}$	$2_2^+ \rightarrow 2_1^+$	0.604	-9.4 <sup>(b)</sup> -8.3 (+0.5,-0.6) <sup>(c)</sup> -10 (+2,-4) <sup>(c)</sup> $ \delta  \geq 10^{(c)}$	10.19	-13 <sup>(b)</sup>  -12 (+5,-15) <sup>(a)</sup>
	$2_3^+ \rightarrow 2_1^+$	1.551	0.24 <sup>(b)</sup> 0.21 <sup>(c)</sup> 0.19 <sup>(c)</sup> $0.1 \leq \delta \leq 0.5$ <sup>(c)</sup> 0.30 <sup>(c)</sup> 0.24 <sup>(c)</sup> 0.23 <sup>(c)</sup> 0.16 <sup>(c)</sup>	0.25	0.17 <sup>(b)</sup>
	$2_3^+ \rightarrow 2_2^+$	0.947	-	12.03	-
	$4_2^+ \rightarrow 4_1^+$	0.687	-2.30 <sup>(b,c)</sup> -1.1 <sup>(c)</sup>	4.70	-9.9 <sup>(b)</sup>
	$4_3^+ \rightarrow 4_2^+$	0.762	-	6.69	-
	$4_4^+ \rightarrow 4_1^+$	1.751	M1,E2 <sup>(b)</sup>	0.18	0.3 <sup>(b)</sup>
	$4_4^+ \rightarrow 4_2^+$	1.064	-	8.76	-
$^{108}_{46}Pd_{62}$	$2_2^+ \rightarrow 2_1^+$	0.531	-3.1 <sup>(b,c)</sup> -5.2 (+2.5,-1.4) <sup>(c)</sup>	3.80	-8.3 <sup>(b)</sup> -5.2 (+1.4,-2.5) <sup>(a)</sup>
	$2_3^+ \rightarrow 2_2^+$	0.672	-	4.42	-
	$3_1^+ \rightarrow 2_2^+$	0.617	M1,E2 <sup>(b)</sup>	2.97	-2 <sup>(b)</sup>
	$3_2^+ \rightarrow 2_4^+$	0.755	-	0.76	-
	$4_2^+ \rightarrow 4_1^+$	0.614	-	2.94	-
	$4_3^+ \rightarrow 4_2^+$	0.691	-	12.05	-
	$4_4^+ \rightarrow 4_2^+$	0.719	-	0.72	-
	$4_4^+ \rightarrow 4_3^+$	0.028	-	0.14	-

**Table 3.** Continued.

Isotope	Transition $I_i^+ \rightarrow I_f^+$	Energy (MeV)	$\delta(E2/M1)$ ( $eb/\mu_N$ )		
			Experimental	This Work	Theoretical
$^{110}_{46}Pd_{64}$	$2_2^+ \rightarrow 2_1^+$	0.471	-4.6 (+1.9,-1.2) <sup>(b,c)</sup> -4.6 (+1.2,-1.9) <sup>(a)</sup>	4.39	-8.6 <sup>(b)</sup>
	$2_3^+ \rightarrow 2_1^+$	1.106	-	0.32	-
	$2_3^+ \rightarrow 2_2^+$	0.635	-	6.20	-
	$2_4^+ \rightarrow 2_2^+$	1.181	-	0.73	-
	$2_4^+ \rightarrow 2_3^+$	0.546	-	0.12	-
	$3_1^+ \rightarrow 2_2^+$	0.552	-	4.82	-
	$3_2^+ \rightarrow 2_4^+$	0.711	-	1.34	-
	$4_2^+ \rightarrow 4_1^+$	0.553	-	2.65	-
	$4_3^+ \rightarrow 4_1^+$	1.182	-	0.12	-
	$4_3^+ \rightarrow 4_2^+$	0.629	-	11.46	-
	$4_4^+ \rightarrow 4_2^+$	0.686	-	0.96	-

<sup>(a)</sup> Ref.[9], <sup>(b)</sup>Ref.[10], <sup>(c)</sup>Ref.[11]

## 2. Theoretical Background

It is proposed that the change from spherical to deformed structure is related to an exceptionally strong neutron-proton interaction. It is also suggested that the neutron-proton effective interactions have a deformation producing tendency, while the neutron-neutron and proton-proton interactions are of spheriphying nature [12, 13].

Within the region of medium-heavy and heavy nuclei, a large of nuclei exhibit properties that are neither close to anharmonic quadrupole vibrational spectra nor to deformed rotors [14]. While defining such nuclei in a geometric description [15], these phenomena will have a standard description that is given in terms of nuclear triaxiality [16], going from rigid triaxial shapes to softer potential energy surfaces. In the first version of the interacting boson model (IBM-1) [17], no distinction is made between proton and neutron variables while describing triaxiality explicitly. This can be done by introducing the cubic terms in the boson operators [18, 19]. This is a contrast to the recent work of Dieperink and Bijker [20, 21] who showed that triaxiality also occurs in particular dynamic symmetries of the IBM-2 that does distinguish between protons and neutrons.

According to A. Arima et al. [22], IBM Hamiltonian takes on different forms, depending on the regions (SU(5), SU(3), O(6)) of the traditional IBA triangle. The Hamiltonian that we consider is in the form [18]

$$H = H_{sd} + \Sigma\theta_L[d^+d^+d^+]^{(L)}[d^\sim d^\sim d^\sim]^{(L)}, \quad (1)$$

where  $H_{sd}$  is the standard Hamiltonian of the IBM [23, 24],

$$H_{sd} = \epsilon_d \eta_d + \kappa \mathbf{Q} \cdot \mathbf{Q} + \kappa' \mathbf{L} \cdot \mathbf{L} + \kappa \mathbf{P}^+ \cdot \mathbf{P} + q_3 \mathbf{T}_3 \cdot \mathbf{T}_3 + q_4 \mathbf{T}_4 \cdot \mathbf{T}_4. \quad (2)$$

In the Hamiltonian,  $\epsilon_d \eta_d$  and  $\mathbf{P}^+ \cdot \mathbf{P}$  terms produce the characteristics of U(5) and O(6) structures, respectively. So the Hamiltonian is a mixture of the U(5) and SO(6) chains, but not diagonal in any of the IBM chains. In the IBA-2 model the neutrons' and protons' degrees of freedom are taken into account explicitly. Thus the Hamiltonian [25] can be written as

$$H = \varepsilon_v n_{dv} + \varepsilon_\pi n_{d\pi} + \kappa \mathbf{Q}_\pi \cdot \mathbf{Q}_v + V_{\pi\pi} + V_{vv} + M_{\pi v}, \quad (3)$$

where  $n_{d\nu(\pi)}$  is the neutron (proton) d-boson number operator.

$$\begin{aligned} n_{d\rho} &= d^+ d^\sim, \rho = v, \pi \\ d_{\rho m}^\sim &= (-1)^m d_{\rho -m} \end{aligned} \quad (4)$$

where  $s_\rho^+$ ,  $d_{\rho m}^+$  and  $s_\rho$ ,  $d_{\rho m}$  represent the s- and d-boson creation and annihilation operators. The rest of the operators in equation (3) are defined as

$$\begin{aligned} Q_\rho &= (s_\rho^+ d_\rho^\sim + d_\rho^+ s_\rho) + \chi_\rho (d_\rho^+ d_\rho^\sim) \\ V_{\rho\rho} &= \sum_{L=0,2,4} C_{L\rho} \left( (d_\rho^+ d_\rho^+)^{(L)} (d_\rho^+ d_\rho^\sim)^{(L)} \right)^{(0)}; \rho = \nu, \pi \end{aligned} \quad (5)$$

and

$$M_{\pi\nu}; \sum_{L=1,3} \xi_L (d_\nu^+ d_\pi^+)^{(L)} (d_\nu d_\pi)^{(L)} + \xi_2 (s_\nu d_\pi^\sim - s_\pi d_\nu^\sim)^{(2)} \cdot (s_\nu^+ d_\pi^+ - s_\pi^+ d_\nu^+)^{(2)}. \quad (6)$$

In the present case,  $M_{\pi\nu}$  affects only the position of the non-fully symmetric states relative to the symmetric states. For this reason  $M_{\pi\nu}$  is often referred to as the Majorana force [25].

The rule of choice for the total angular momentum is given as

$$|J_i - J_f| \leq L\gamma \leq |J_i + J_f| \quad (7)$$

The mixing ratio E2/M1, where  $T(E2; J_i \rightarrow J_f)$  is the number of E2 transitions per second and  $T(M1; J_i \rightarrow J_f)$  is the number of M1 transitions per second, is given by

$$\delta(E2/M1; J_i \rightarrow J_f) = \frac{\sqrt{T(E2; J_i \rightarrow J_f)}}{\sqrt{T(M1; J_i \rightarrow J_f)}}. \quad (8)$$

The ratio of  $\delta(E2/M1)$  can be written in terms of matrix elements as follows

$$\delta(E2/M1) = 0.836 E\gamma (MeV) \frac{\langle J_f || M(E2) || J_i \rangle}{\langle J_f || M(M1) || J_i \rangle}. \quad (9)$$

The electric quadrupole (E2) transitions are one of the important factors within the collective nuclear structure. In IBM, the general linear E2 operator with  $L = 2$  is given

by

$$\begin{aligned} T(E2) &= \alpha_2 (s^+ d + d^+ s) + \beta_2 [d^+ d]_2 \\ &= \alpha_2 (s^+ d + d^+ s) + \xi [d^+ d]_2. \end{aligned} \quad (10)$$

In this form,  $\alpha_2$ ,  $\beta_2$  and  $\chi$  are free parameters.  $B(E2; J_i \rightarrow J_f)$  is given in the following formulation:

$$\begin{aligned} B(E2; J_i \rightarrow J_f) &= \sum_{mM'} | \langle J_f M' | T(E2) m | J_i M \rangle |^2 \\ B(E2; J_i \rightarrow J_f) &= \frac{1}{2J_i + 1} | \langle J_f || T(E2) || J_i \rangle |^2. \end{aligned} \quad (11)$$

### 3. Calculation Details

The parameters  $\varepsilon$ ,  $\kappa$  and  $C_{L\rho}$  are free parameters that have been determined so as to reproduce as closely as possible the excitation-energy of all positive parity levels for which a clear indication of the spin value exists, following the same procedure described in [26]. The value of  $\chi_\pi$  has been kept fixed along the isotopic chain as suggested by microscopic considerations which predict that this parameter depends only on the proton number [27]. This parameter is extremely important because it is closely related to the nuclear shape (prolate or oblate) [5]. The full set of adopted parameters is reported in Table 1. Altogether, six parameters are appearing in the Hamiltonian. The  $^{102-110}\text{Pd}$  isotopes have  $N_\pi = 2$  (relative to  $Z = 50$ ) and  $N_\nu$  varies from 3 to 7 (relative to  $N = 50$ ), while the parameters  $\varepsilon$ ,  $\chi_\rho$  and  $\kappa$ , as well as  $C_{L\rho}$ , with  $L = 0, 2, 4$  were treated as free parameters and their values were estimated by fitting to the measured level energies. This procedure was made by selecting the “traditional” values of parameters and then allowing one parameter to vary while keeping the others constant until a best fit was obtained. This was carried out iteratively until an overall fit was achieved. Having obtained wave functions for the states in  $^{102-110}\text{Pd}$  after fitting the experimental energy levels in IBM-2, we can estimate the electromagnetic transition rates between states using the program PHINT [25]. As it is pointed out by Bijker et al. [28], nuclei with  $\chi_\pi + \chi_\nu = 0$  have properties close to those of the O(6) limit. This is not in agreement with earlier IBM [29, 30] calculations for the Sm isotopes. In this study, we take  $\chi_\pi = 0.6$  for all Pd isotopes. In particular, the spectrum of the SU(5) nuclei is dominated by the value of  $\varepsilon$ , which is large in comparison with the other parameters, whereas O(6) nuclei are characterized by their value of  $\kappa$ , which is large compared to  $\varepsilon$  [31].

Using these parameters, the estimated energy levels are shown in Table 2 along with experimental energy levels. As can be seen, the agreement between experiment and theory is quite good and the general features are reproduced well. We observe the discrepancy between theory and experiment for high spin states. But one must be careful in comparing theory with experiment, since all calculated states have a collective nature, whereas some of the experimental states may have a particle-like structure.

Behavior of the ratio ( $R_{4/2} = E(4_1^+)/E(2_1^+)$ ) of the energies of the first  $4^+$  and  $2^+$  states are good criteria for the shape transition. The value of  $R_{4/2}$  ratio has the limiting value 2.0 for a quadrupole vibrator, 2.5 for a non-axial gamma-soft rotor and 3.33 for an ideally symmetric rotor.  $R_{4/2}$  remain nearly constant at  $N = 60$  and then increase with neutron number. The estimated values change from about 2.18 to about 2.26, meaning that their structure seems to be varying from quadrupole vibrator to non-axial gamma soft.

We have also estimated multipole mixing ratios  $\delta(\text{E2/M1})$  of some transitions for  $^{102-110}\text{Pd}$  isotopes and then compared them with some previous experimental and theoretical results in Table 3, where one can see good agreement with estimated and experimental values. The variations in sign of the E2/M1 mixing ratios from nucleus to nucleus for the same class of transitions, and within a given nucleus for transitions from different spin states, suggest that a microscopic approach is needed to explain the data theoretically. For that reason, we did not take into consideration the sign of mixing ratios. Sign convention of mixing ratios has been explained in detail by J. Lange et al. [32] and A. M. Demidov et al. [33].

### 4. Conclusion

The shape transition has been investigated in detail via the IBM framework on even-even Pd isotopes ( $^{102-110}\text{Pd}$ ) and those properties predicted by this study is consistent with the spectroscopic data for these nuclei.  $^{102-110}\text{Pd}$  are typical examples of isotopes that exhibit a smooth phase transition from vibrational nuclei to soft triaxial rotors. As it is seen from Table 2 and Table 3  $^{102-110}\text{Pd}$  isotopes are lined up along the SU(5)-O(6) side of the IBM triangle. Calculated and experimental energies, and multipole mixing ratios ( $\delta(\text{E2/M1})$ ) are mostly in agreement with each other.

In view of the growing pursuit in this kind of theoretical interest, it is expected new studies investigating the properties of neutron rich full isotopic mass chains around  $A \cong 100$  mass region will be carried out.

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