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Non-commutative and Non-anticommutative Supersymmetric U(1) Gauge Theories*

Kayhan ÜLKER
Feza Gürsey Institute, Çengelköy, 34684, İstanbul-TURKEY
e-mail: kulker@gursey.gov.tr

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Abstract

We introduce a parent action for component fields of N=1 supersymmetric U(1) gauge theory in four dimensions which generates both the original and the (s-) dual supersymmetric actions. By using the noncommutative and nonanticommutative deformations of this theory we propose the corresponding parent actions as generalizations of the ordinary case. We then show that duality symmetry under the interchange of fields with duals accompanied by the replacement of the noncommutativity and nonanticommutativity parameters in a suitable way.

Key Words: Supersymmetry, non-commutative theories, non-anti-commutative theories, duality.

1. Introduction

Duality in QFT in general means the equivalence of two or more descriptions of the same theory. In particular, (S–) duality transformations map strong coupling domains to weak coupling domains of gauge theories. For pure $U(1)$ gauge theory this duality can be shown, trivially, by rescaling its gauge fields. However, it can also be studied in terms of parent action formalism[1] that permits to study more complicated theories where this rescaling argument does not hold, such as noncommutative or nonanticommutative extensions of U(1) gauge theory.

Roughly speaking a non-commutative quantum field theory (NC-QFT) is a QFT that is defined on a space-time where the coordinates do not commute. The idea is quite old [2], however the subject became quite popular in recent times due to the fact that it can be related to the string theory (see for instance [3]). The above mentioned parent action formalism permits to introduce a dual formulation of the noncommutative $U(1)$ gauge theory[4].

On the other hand, when a supersymmetric gauge theory considered on a superspace, besides the ordinary bosonic coordinate noncommutativity one can also introduce a non-anti-commutativity where the fermionic coordinates do not anti-commute[1].

In this talk I will try to summarize how to obtain the (S-) duals of these noncommutative and nonanticommutative deformations of supersymmetric U(1) Gauge theories by using the aforementioned parent action formalism. The details can be found in the original papers [11] and [12].

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[1]Note that the formalism of superstring theory with pure spinors[5] in a graviphoton background[6] gives rise to a non-anticommutative superspace[7],[8] which was introduced also in other contexts [9], [10].

2. Duality and Parent Actions

Duality in QFT in general means the equivalence of two or more descriptions of the same theory. The simplest example is Electric-Magnetic (EM) duality. Consider the Maxwell action :

$$I_M = \int d^4x(-\frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu}) \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength.

We compute the physical quantities as a path integral over all gauge potential configurations,

$$Z \sim \int DA_\mu e^{i\,I_M}.$$

On the other hand, since $F_{\mu\nu}$ is the unique solution of the Bianchi identity, $\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0$ the above path integral can be rewritten as an integral over $F_{\mu\nu}$, if we insert the Bianchi identity as a constraint

$$Z_p \sim \int DF_{\mu\nu} DA_{D\mu} e^{i\,I_p}$$

where A_D is a Lagrange multiplier one form enforcing the Bianchi id. in the action,

$$I_p = \int d^4x(-\frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}A_{D\mu}\epsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho}). \tag{2}$$

Note that performing the functional integral over A_D gives the original action (1) and the one over F, gives the *"Dual Action"* I_D for A_D

$$I_D = \int d^4x(-\frac{g^2}{4}F_D^{\mu\nu}F_{D\mu\nu}) \tag{3}$$

that is equivalent to the original action (1) when one replaces A with A_D and g with $1/g$. In other words, U(1) gauge theory is equivalent to another such theory with coupling $1/g$ and this duality invariance can be formulated at the level of actions due to a *Parent Action* I_p which generates both the original and the dual actions.

3. Supersymmetry

Supersymmetry is, by definition, a symmetry between fermions and bosons.

$$\begin{aligned}\delta: \quad & Fermions \rightarrow Bosons \\ & Bosons \rightarrow Fermions\end{aligned}$$

A supersymmetric field theoretical model consists of a set of fields and of a Lagrangian such that the latter possesses this kind of symmetry with a rigid anticommuting parameter. Let us briefly mention N=1 SUSY U(1) gauge theory. For details we refer to [13].

To supersymmetrize U(1) gauge theory (1) one should introduce chiral and anti chiral Weyl spinors[2] $\lambda_\alpha\,\bar\lambda_{\dot\alpha}$ with a kinetic term $i\bar\lambda\sigma^\mu\partial_\mu\lambda$. However the off-shell degrees of freedom of the bosons A_μ and the fermions $\lambda_\alpha\,\bar\lambda_{\dot\alpha}$, are still not equal (i.e $3 \neq 4$). Therefore, we introduce another scalar real field D without a kinetic term (i.e an auxiliary field).

The off-shell N=1 SUSY algebra is ;

$$\begin{aligned}\{Q,\bar Q\} &= -2i\sigma^\mu\partial_\mu \\ \{Q,Q\} &= 0 = \{\bar Q,\bar Q\}.\end{aligned} \tag{4}$$

[2] Instead of introducing a chiral and a antichiral spinor field we could equivalently introduce a Majarona field.

where Q_α , $\bar{Q}_{\dot\alpha}$ are generators of chiral and antichiral SUSY transformations. It is clear that Q_α , $\bar{Q}_{\dot\alpha}$ maps mass dimension d fields to mass dimension d+1/2 fields. Therefore, we introduce anti-commutative parameters ξ_α , $\bar{\xi}_{\dot\alpha}$, to define $\delta = \xi Q + \bar{\xi}\bar{Q}$, and assume that the SUSY transformation acts linearly on the fields to get

$$\begin{aligned} \delta_\xi A_\mu &= i\xi\sigma^\mu\bar\lambda + i\bar\xi\bar\sigma^\mu\lambda, && (5)\\ \delta_\xi\lambda &= \sigma^{\mu\nu}\xi F_{\mu\nu} + i\xi D, && (6)\\ \delta_\xi D &= \bar\xi\bar\sigma^\mu D_\mu\lambda - \xi\sigma^\mu D_\mu\bar\lambda && (7)\end{aligned}$$

with the help of the algebra (4).

There is a unique action which is invariant under δ,

$$I = \frac{1}{g^2}\int d^4x[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - i\lambda\partial\!\!\!/\bar\lambda + \frac{1}{2}D^2]. \quad (8)$$

Note that these component fields form a vector multiplet $(A_\mu, \lambda_\alpha, \bar\lambda_{\dot\alpha}, D)$[3].

On the other hand, although it is possible to construct supersymmetric Lagrangeans directly from the components of a supermultiplet, there are advantages to construct these theories in superspace/superfield formalism. Superspace is a 4+4 dimensional coset space (superPoincare group/Lorentz group) that is parameterised by $(x_\mu, \theta_a, \bar\theta_{\dot\alpha})$. Here,$\theta_a, \bar\theta_{\dot\alpha}$ are constant anticommuting Weyl spinors. This coset space is commonly known as N=1 rigid superspace.

A superfield is defined as a function in superspace. It is obvious that a superfield, $\Phi(x, \theta, \bar\theta)$, has a finite series expansion in terms of θ_α , $\bar\theta_{\dot\alpha}$,

$$\Phi(x, \theta, \bar\theta) = \phi(x) + \theta\psi + \bar\theta\bar\psi + ... + \theta\theta\bar\theta\bar\theta F + 0$$

since $\theta^3 = \bar\theta^3 = 0$ identically. Note that, $(\phi, \psi, ..., F)$ are components of the superfield that belong to the same supermultiplet.

A motion in superspace (i.e the parameter space $(x_\mu, \theta_\alpha, \bar\theta_{\dot\alpha})$) is generated by the differential operators $Q_\alpha, \bar{Q}_{\dot\alpha}$;

$$iQ_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \quad , \quad i\bar{Q}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \quad (9)$$

These operators satisfy the SUSY algebra (4). The SUSY transformations (5-7) can then be found by applying these differential operators on the superfield $\Phi(x, \theta, \bar\theta)$.

To write an action functional $I[\Phi]$ that is invariant under SUSY, let us introduce superspace covariant derivatives,

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \quad , \quad \bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu. \quad (10)$$

that anticommutes with Q and $\bar{Q}$.

The members of the vector multiplet of N=1 U(1) gauge theory, $(A_\mu, \lambda_\alpha, \bar\lambda_{\dot\alpha}, D)$ can then be written in superspace as a vector superfield V in Wess-Zumino gauge as,

$$V = -(\theta\sigma^\mu\bar\theta)A_\mu + i\theta\theta\bar\theta\bar\lambda - i\bar\theta\bar\theta\theta\lambda + \frac{1}{2}\theta\theta\bar\theta\bar\theta D. \quad (11)$$

Note that V is a real superfield $V^\dagger = V$. Moreover, from V one can construct chiral and antichiral superfields[4] by using the covariant derivatives

$$W_\alpha = \frac{1}{2}\bar{D}^2 D_\alpha V. , \bar{W}_{\dot\alpha} = \frac{1}{2}D^2\bar{D}_{\dot\alpha}V.$$

W_α can be found in terms of the component fields as

$$W_\alpha(y) = -i\lambda_\alpha(y) + \theta_\alpha D(y) - i\sigma^{\mu\nu}_\alpha{}^\beta\theta_\beta F_{\mu\nu}(y) + \theta\theta\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar\psi^{\dot\alpha}(y)$$

[3]Note that, fields in the same supermultiplet always have the same mass and the coupling constant.

[4]i.e $\bar{D}_{\dot\alpha}\Phi = 0$, $D_\alpha\bar\Phi = 0$ respectively.

after a variable redefinition $y_\mu = x_\mu + i\theta\sigma_\mu\bar{\theta}$.

It is easy then to show that the action (8) can be written as :

$$I = \frac{1}{4g^2}\int d^4x(\int d^2\theta W^2 + \int d^2\bar{\theta}\bar{W}^2). \tag{12}$$

4. Parent Action For Supersymmetric U(1) Gauge Theory

Just as in the ordinary case for supersymmetric $U(1)$ gauge theory[5], duality can be formulated at the level of actions due to a parent action which generates both the original and the dual actions in terms of superfields [14].

Working in 4 dimensional Minkowski space–time and the $N = 1$ superspace $(x_\mu, \theta_\alpha, \bar{\theta}^{\dot{\alpha}})$ we consider a general chiral superfield (not a supersymmetric field strength) $\tilde{W}_\alpha$ and a real (dual) vector field V_D to write the parent action

$$I_p = \frac{1}{4g^2}\int d^4x(\int d^2\theta\tilde{W}^2 + \int d^2\bar{\theta}\bar{\tilde{W}}^2) + \frac{1}{2}\int d^4x d^4\theta(V_D D\tilde{W} - V_D\bar{D}\bar{\tilde{W}}) \tag{13}$$

where D_α is the supercovariant derivative (we use notations of [13]).

Equation of motion with respect to V_D leads to the supersymmetric Bianchi identity $D\tilde{W} - \bar{D}\bar{\tilde{W}}|_W = 0$ whose solution is the supersymmetric field strength $W_\alpha = \frac{1}{2}\bar{D}^2 D_\alpha V$. Replacement of $\tilde{W}, \bar{\tilde{W}}$ with this solution in the parent action[6], leads to

$$I = \frac{1}{4g^2}\int d^4x(\int d^2\theta W^2 + \int d^2\bar{\theta}\bar{W}^2). \tag{14}$$

This is the action of supersymmetric U(1) gauge theory.

On the other hand, when solutions of the equations of motion with respect to $\tilde{W}_\alpha$ and $\bar{\tilde{W}}^{\dot{\alpha}}$ following from I_p are plugged into we get the dual action

$$I_D = \frac{g^2}{4}\int d^4x(\int d^2\theta W_D^2 + \int d^2\bar{\theta}\bar{W}_D^2) \tag{15}$$

where W_D is the dual superfield strength $W_{D\alpha} = \frac{1}{2}\bar{D}^2 D_\alpha V_D$.

The original and the dual actions and are in the same form when g is replaced with $1/g$ as expected.

On the other hand, one is forced to deal with the *component field formalism* in particular when calculations on non-trivial backgrounds are considered. Therefore, instead of superfields, we would like to consider duality transformations in terms of component fields [11]. For this purpose we introduce a general chiral superfield[7] $\tilde{W}_\alpha$ that does not satisfy the condition $D\tilde{W} - \bar{D}\bar{\tilde{W}}|_W = 0$,

$$\tilde{W}_\alpha(y) = -i\lambda_\alpha(y) + \theta_\alpha\tilde{D}(y) - i\sigma_\alpha^{\mu\nu\ \beta}\theta_\beta\tilde{F}_{\mu\nu}(y) + \theta\theta\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\psi}^{\dot{\alpha}}(y)$$

where $y_\mu = x_\mu + i\theta\sigma_\mu\bar{\theta}$. Here, λ and $\bar{\psi}$ are two independent Weyl spinors, $\tilde{F}_{\mu\nu}$ is a antisymmetric field and $\tilde{D}$ is a complex scalar field.

Since the Lagrange multiplier V_D is a real vector superfield we have

$$V_D = -(\theta\sigma^\mu\bar{\theta})A_{D\mu} + i\theta\theta\bar{\theta}\bar{\lambda}_D - i\bar{\theta}\bar{\theta}\theta\lambda_D + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D_D \tag{16}$$

in terms of dual fields. Therefore, by using the action (6) and the above definitions we propose [11],

$$S_p = S_o[X] + S_l[X, X_D], \tag{17}$$

[5] See for instance the text book [13]

[6] Note that this is equivalent to perform the path integral over V_D in its partition function

[7] This choice is not unique, one can also choose F as a complex two form that also yields similar results. For details see [11].

as the parent action, where

$$S_o \equiv \frac{1}{4g^2}\int d^4x[-F^{\mu\nu}F_{\mu\nu} - 2i\bar{\lambda}\sigma^\mu\partial_\mu\psi - 2i\lambda\sigma^\mu\partial_\mu\bar{\psi} + \tilde{D}^2 + \tilde{D}^{\dagger 2}], \tag{18}$$

and the Legendre transformation part

$$S_l \equiv \frac{1}{2}\int d^4x[\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}\partial_\rho A_{D\sigma} + \lambda_D\sigma^\mu\partial_\mu\bar{\psi} + \bar{\lambda}_D\bar{\sigma}^\mu\partial_\mu\lambda - \lambda_D\sigma^\mu\partial_\mu\bar{\lambda} - \bar{\lambda}_D\bar{\sigma}^\mu\partial_\mu\psi + iD_D(\tilde{D} - \tilde{D}^\dagger)]. \tag{19}$$

We can now proceed as before to derive supersymmetric $U(1)$ gauge theory in terms of the component fields from the parent action (17): The solutions of the equations of motions with respect to the dual fields

$$\varepsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma} = 0 \quad , \quad \left(\tilde{D} - \tilde{D}^\dagger\right)_{\tilde{D}=D} = 0, \tag{20}$$

$$\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\psi}^{\dot{\alpha}} - \sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\lambda}^{\dot{\alpha}} = 0 \quad , \quad \bar{\sigma}^{\dot{\alpha}\alpha}_\mu\partial^\mu\lambda_\alpha - \bar{\sigma}^{\dot{\alpha}\alpha}_\mu\partial^\mu\psi_\alpha = 0, \tag{21}$$

yield $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ where A_μ is the usual $U(1)$ gauge field and $D = D^\dagger$, $\lambda_\alpha \simeq \psi_\alpha$ and $\bar{\lambda}^{\dot{\alpha}} \simeq \bar{\psi}^{\dot{\alpha}}$. When these solutions are plugged in the parent action (17) we obtain the supersymmetric $U(1)$ gauge theory action

$$I = \frac{1}{g^2}\int d^4x[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{i}{2}\lambda\partial\!\!\!/\bar{\lambda} - \frac{i}{2}\bar{\lambda}\bar{\partial\!\!\!/}\lambda + \frac{1}{2}D^2]. \tag{22}$$

in terms of component fields.

On the other hand, the equations of motions with respect to the fields $F_{\mu\nu}, \lambda, \psi, \bar{\lambda}, \tilde{D}, \bar{\psi}, \tilde{D}^\dagger$ are

$$-\frac{1}{g^2}F^{\mu\nu} + \varepsilon^{\mu\nu\rho\sigma}\partial_\rho A_{D\sigma} = 0 \quad , \quad \frac{1}{g^2}\tilde{D}^\dagger - iD_D = 0\, , \, \frac{1}{g^2}\tilde{D} + iD_D = 0, \tag{23}$$

$$\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu(-\frac{i}{g^2}\bar{\psi}^{\dot{\alpha}} + \bar{\lambda}^{\dot{\alpha}}_D) = 0 \quad , \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha}\partial_\mu(-\frac{i}{g^2}\psi_\alpha - \lambda_{D\alpha}) = 0, \tag{24}$$

$$\partial_\mu(-\frac{i}{g^2}\bar{\lambda}_{\dot{\alpha}} + \bar{\lambda}_{D\dot{\alpha}})\bar{\sigma}^{\mu\dot{\alpha}\alpha} = 0 \quad , \quad \partial_\mu(-\frac{i}{g^2}\lambda^\alpha + \lambda^\alpha_D)\sigma^\mu_{\alpha\dot{\alpha}} = 0. \tag{25}$$

Substituting solutions of these equations in the parent action yield the dual of action of N=1 supersymmetric U(1) gauge theory in terms of the component fields,

$$I_D = g^2\int d^4x[-\frac{1}{4}F^{\mu\nu}_D F_{D\mu\nu} - \frac{i}{2}\lambda_D\partial\!\!\!/\bar{\lambda}_D - \frac{i}{2}\bar{\lambda}_D\bar{\partial\!\!\!/}\lambda_D + \frac{1}{2}D^2_D]. \tag{26}$$

where $F_{D\mu\nu} = \partial_\mu A_{D\nu} - \partial_\nu A_{D\mu}$.

Therefore one may say that supersymmetric parent action (17) generates supersymmetric $U(1)$ gauge theory and its dual in terms of component fields [11].

5. Noncommutative Generalization of Supersymmetric $U(1)$ Gauge Theory

Noncommutativity is introduced through the star product

$$* \equiv \exp\frac{i\theta^{\mu\nu}}{2}\left(\overleftarrow{\partial}_\mu\overrightarrow{\partial}_\nu - \overleftarrow{\partial}_\nu\overrightarrow{\partial}_\mu\right), \tag{27}$$

for an antisymmetric and constant real parameter $\theta^{\mu\nu}$. Here, x_μ are space–time coordinates and satisfy the Moyal bracket

$$x^\mu * x^\nu - x^\nu * x^\mu = i\theta^{\mu\nu}. \tag{28}$$

We get the noncommutative (NC) supersymmetric theory[8] simply by replacing all the products with the *-product[9]:

$$S_{NC} = \frac{1}{2g^2}\int d^4x[-\frac{1}{2}\widehat{F}^{\mu\nu}\widehat{F}_{\mu\nu} - i\widehat{\bar{\lambda}}\bar{\sigma}^{\mu}\widehat{D}_{\mu} * \widehat{\lambda} - i\widehat{\lambda}\sigma^{\mu}\widehat{D}_{\mu} * \widehat{\bar{\lambda}} + \widehat{D}\widehat{D}], \tag{29}$$

where $\widehat{D}_\mu * \widehat{\lambda} = \partial_\mu\widehat{\lambda} + i(\widehat{A}_\mu * \widehat{\lambda} - \widehat{\lambda} * \widehat{A}_\mu)$ and $\widehat{F}_{\mu\nu} = \partial_\mu\widehat{A}_\nu - \partial_\nu\widehat{A}_\mu + i(\widehat{A}_\mu * \widehat{A}_\nu - \widehat{A}_\nu * \widehat{A}_\mu)$. is the NC field strength.

The action (29) is invariant under the (NC) supersymmetry transformations

$$\delta_\xi\widehat{A}_\mu = i\xi\sigma^\mu\widehat{\bar{\lambda}} + i\bar{\xi}\bar{\sigma}^\mu\widehat{\lambda}, \tag{30}$$

$$\delta_\xi\widehat{\lambda} = \sigma^{\mu\nu}\xi\widehat{F}_{\mu\nu} + i\xi\widehat{D}, \tag{31}$$

$$\delta_\xi\widehat{D} = \bar{\xi}\bar{\sigma}^\mu\widehat{D}_\mu * \widehat{\lambda} - \xi\sigma^\mu\widehat{D}_\mu * \widehat{\bar{\lambda}}. \tag{32}$$

where ξ is a fermionic constant spinor parameter.

It is possible to find an explicit map (Seiberg-Witten map) from the NC vector potential $\widehat{A}_\mu$ and to a conventional vector potential A_μ,[3]

$$\widehat{A}_\mu(A) + \widehat{\delta}_\Lambda\widehat{A}_\mu(A) = \widehat{A}_\mu(A + \delta_\Lambda A)$$

where $\widehat{\Lambda} = \widehat{\Lambda}(A, \Lambda)$ is the gauge parameter.

Similarly for the supersymmetric $U(1)$ case we can find a supersymmetric Seiberg-Witten Map[10] [11] :

$$\widehat{A}_\mu = A_\mu - \frac{1}{2}\Theta^{\nu\rho}(A_\nu\partial_\rho A_\mu + A_\nu F_{\rho\mu}), \tag{33}$$

$$\widehat{\lambda} = \lambda - \Theta^{\nu\rho}\partial_\nu\lambda A_\rho \quad , \quad \widehat{\bar{\lambda}} = \bar{\lambda} - \Theta^{\nu\rho}\partial_\nu\bar{\lambda}A_\rho, \quad , \quad \widehat{D} = D - \Theta^{\nu\rho}\partial_\nu D A_\rho. \tag{34}$$

where we get the NC fields in terms of the ordinary ones at the first order in $\Theta^{\mu\nu}$.

With help of SW map (33) we can write the action (29) in terms of the ordinary component fields

$$\begin{aligned} S_{NC}[F,\lambda,D,\Theta] = &\int d^4x\{-\frac{1}{4g^2}(F^{\mu\nu}F_{\mu\nu} + 2\Theta^{\mu\nu}F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu} - \frac{1}{2}\Theta^{\mu\nu}F_{\nu\mu}F_{\rho\sigma}F^{\sigma\rho}) \\ &-\frac{i}{g^2}[\frac{1}{2}\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda + \Theta^{\mu\nu}(\frac{1}{4}\bar{\lambda}\bar{\sigma}^\rho\partial_\rho\lambda F_{\mu\nu} + \frac{1}{2}\bar{\lambda}\bar{\sigma}^\rho\partial_\mu\lambda F_{\nu\rho}) + \frac{1}{2}\lambda\sigma^\mu\partial_\mu\bar{\lambda} \\ &+\Theta^{\mu\nu}(\frac{1}{4}\lambda\sigma^\rho\partial_\rho\bar{\lambda}F_{\mu\nu} + \frac{1}{2}\lambda\sigma^\rho\partial_\mu\bar{\lambda}F_{\nu\rho})] + \frac{1}{2g^2}(D^2 + \frac{1}{2}\Theta^{\mu\nu}D^2F_{\mu\nu})\} \end{aligned} \tag{35}$$

The SUSY transformations that are found after performing the SW map (33),

$$\delta_\xi A_\mu = i\xi\sigma_\mu\bar{\lambda} + i\bar{\xi}\bar{\sigma}_\mu\lambda - i\Theta^{\rho\kappa}(\xi\sigma_\rho\bar{\lambda} + \bar{\xi}\bar{\sigma}_\rho\lambda)(\frac{1}{2}F_{\kappa\mu} + \frac{1}{2}\partial_\kappa A_\mu) - i\Theta^{\rho\kappa}\frac{1}{2}(\xi\sigma_\rho\partial_\mu\bar{\lambda} + \bar{\xi}\bar{\sigma}_\rho\partial_\mu\lambda)A_\kappa, \tag{36}$$

$$\delta_\xi\lambda = \sigma^{\mu\nu}\xi F_{\mu\nu} + i\xi D + \Theta^{\rho\kappa}\partial_\rho\lambda(i\xi\sigma_\kappa\bar{\lambda} + i\bar{\xi}\bar{\sigma}_\kappa\lambda) + \Theta^{\rho\kappa}i\sigma^{\mu\nu}\xi F_{\mu\rho}F_{\nu\kappa}, \tag{37}$$

$$\delta_\xi D = \bar{\xi}\bar{\sigma}^\mu\partial_\mu\lambda - \xi\sigma^\mu\partial_\mu\bar{\lambda} - i\Theta^{\rho\kappa}(\xi\sigma_\rho\bar{\lambda} + \bar{\xi}\bar{\sigma}_\rho\lambda)\partial_\kappa D + \Theta^{\rho\kappa}\xi\sigma^\mu F_{\rho\mu}\partial_\kappa\bar{\lambda} - \Theta^{\rho\kappa}\bar{\xi}\bar{\sigma}^\mu F_{\rho\mu}\partial_\kappa\lambda. \tag{38}$$

leaves the action (35) invariant.

6. Dual of NC Supersymmetric Gauge Theory

We generalize the parent action (17) of the ordinary supersymmetric gauge theory to the noncommutative case

$$S_{oNC} = \int d^4x[-\frac{1}{4g^2}\widehat{F}^{\mu\nu}\widehat{F}_{\mu\nu} - \frac{i}{2g^2}\widehat{\bar{\lambda}}\bar{\sigma}^\mu\widehat{D}_\mu * \widehat{\psi} - \frac{i}{2g^2}\widehat{\lambda}\sigma^\mu\widehat{D}_\mu * \widehat{\bar{\psi}} + \frac{1}{2g^2}\widehat{D}\widehat{D}^\dagger]. \tag{39}$$

[8]Although we deal with Euclidean $\mathbb{R}^4$, we use Minkowski space notation and follow the conventions of [13]. For detailed, discussion of we refer to [10]

[9]We assume that surface terms are vanishing, so that $\int d^4xf(x) * g(x) = \int d^4xf(x)g(x)$ and $\int d^4xf(x) * g(x) * h(x) = \int d^4x(f(x) * g(x))h(x) = \int d^4xf(x)(g(x) * h(x))$.

[10]For other approaches see Ref.[15, 16].

and through the supersymmetric Seiberg–Witten map (33) we write,

$$S_{oNC}[F,\lambda,\psi,D] = \int d^4x\{-\frac{1}{4g^2}(F^{\mu\nu}F_{\mu\nu} + 2\Theta^{\mu\nu}F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu} - \frac{1}{2}\Theta^{\mu\nu}F_{\nu\mu}F_{\rho\sigma}F^{\sigma\rho})$$
$$-\frac{i}{2g^2}(\bar\lambda\bar\sigma^\mu\partial_\mu\psi + \Theta^{\mu\nu}\bar\lambda\bar\sigma^\rho\partial_\mu\psi F_{\nu\rho} + \frac{1}{2}\Theta^{\mu\nu}\bar\lambda\bar\sigma^\rho\partial_\rho\psi F_{\mu\nu})$$
$$-\frac{i}{2g^2}(\lambda\sigma^\mu\partial_\mu\bar\psi + \Theta^{\mu\nu}\lambda\sigma^\rho\partial_\mu\bar\psi F_{\nu\rho} + \frac{1}{2}\Theta^{\mu\nu}\lambda\sigma^\rho\partial_\rho\bar\psi F_{\mu\nu})$$
$$+\frac{1}{4g^2}[D^2 + D^{\dagger 2} + \frac{1}{2}\Theta^{\mu\nu}(D^2 + D^{\dagger 2})F_{\mu\nu})]\}. \tag{40}$$

Similarly, we can define the parent action as,

$$S_P = S_{oNC}[X] + S_l[X, X_D] \tag{41}$$

where S_l is the Legendre transformation term (19) as before.

The equations of motions with respect to the dual fields are the same for the commuting case (20) and when the solutions are used in the parent action we get the one of the NC supersymmetric $U(1)$ gauge theory (35).

On the other hand equations of motion with respect to the other fields are more complicated,

$$-\frac{1}{g^2}F^{\mu\nu} - \frac{1}{g^2}\Theta^{\rho[\mu}F^{\nu]\sigma}F_{\sigma\rho} - \frac{1}{2g^2}\Theta^{\rho\sigma}F_{\sigma[\mu}F_{\nu]\rho} + \frac{1}{4g^2}\Theta^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + \frac{1}{2g^2}\Theta^{\rho\sigma}F_{\rho\sigma}F^{\mu\nu}$$
$$-\frac{i}{2g^2}(\Theta^{\rho\mu}\bar\lambda\bar\sigma^\nu - \Theta^{\rho\nu}\bar\lambda\bar\sigma^\mu)\partial_\rho\psi - \frac{i}{2g^2}\Theta^{\mu\nu}(\bar\lambda\bar\sigma^\rho\partial_\rho\psi) - \frac{i}{2g^2}(\Theta^{\rho\mu}\lambda\sigma^\nu - \Theta^{\rho\nu}\lambda\sigma^\mu)\partial_\rho\bar\psi$$
$$-\frac{i}{2g^2}\Theta^{\mu\nu}\lambda\sigma^\rho\partial_\rho\bar\psi + \frac{1}{4g^2}\Theta^{\mu\nu}(\tilde D^2 + \tilde D^{\dagger 2}) - \varepsilon^{\mu\nu\rho\sigma}\partial_\rho A_{D\sigma} = 0, \tag{42}$$

$$-\frac{i}{2g^2}\sigma^\mu\partial_\mu\bar\psi - \frac{i}{4g^2}\Theta^{\mu\nu}\sigma^\rho\partial_\rho\bar\psi F_{\mu\nu} - \frac{i}{2g^2}\Theta^{\mu\nu}\sigma^\rho\partial_\mu\bar\psi F_{\nu\rho} + \frac{1}{2}\sigma^\mu\partial_\mu\bar\lambda_D = 0, \tag{43}$$

$$-\frac{i}{2g^2}\bar\sigma^\mu\partial_\mu\psi - \frac{i}{4g^2}\Theta^{\mu\nu}\bar\sigma^\rho\partial_\rho\psi F_{\mu\nu} - \frac{i}{2g^2}\Theta^{\mu\nu}\bar\sigma^\rho\partial_\mu\psi F_{\nu\rho} - \frac{1}{2}\bar\sigma^\mu\partial_\mu\lambda_D = 0, \tag{44}$$

$$-\partial_\mu(\frac{i}{2g^2}\bar\lambda\bar\sigma^\mu - \frac{i}{4g^2}\Theta^{\rho\nu}\bar\lambda\bar\sigma^\mu F_{\rho\nu} - \frac{i}{2g^2}\Theta^{\mu\nu}\bar\lambda\bar\sigma^\rho F_{\nu\rho} - \frac{1}{2}\bar\lambda_D\bar\sigma_\mu) = 0, \tag{45}$$

$$\partial_\mu(-\frac{i}{2g^2}\lambda\sigma^\mu - \frac{i}{4g^2}\Theta^{\rho\nu}\lambda\sigma^\mu F_{\rho\nu} - \frac{i}{2g^2}\Theta^{\mu\nu}\lambda\sigma^\rho F_{\nu\rho} + \frac{1}{2}\lambda_D\sigma^\mu) = 0, \tag{46}$$

$$\frac{1}{2g^2}\tilde D + \frac{1}{4g^2}\Theta^{\mu\nu}\tilde D F_{\mu\nu} + \frac{i}{4}D_D = 0, \tag{47}$$

$$\frac{1}{2g^2}\tilde D^\dagger + \frac{1}{4g^2}\Theta^{\mu\nu}\tilde D^\dagger F_{\mu\nu} - \frac{i}{4}D_D = 0. \tag{48}$$

We can solve these equations for $F, \psi, \lambda, \tilde D$ and plug the solutions into the parent action to obtain the dual action [11]

$$S_{NCD} = \int d^4x[-\frac{g^2}{4}(F_D^{\mu\nu}F_{D\mu\nu} + 2\tilde\Theta^{\mu\nu}F_{D\nu\rho}F_D^{\rho\sigma}F_{D\sigma\mu} - \frac{1}{2}\tilde\Theta^{\mu\nu}F_{D\nu\mu}F_{D\rho\sigma}F^{D\sigma\rho})$$
$$-ig^2(\frac{1}{2}\lambda_D\sigma^\mu\partial_\mu\bar\lambda_D + \frac{1}{2}\bar\lambda_D\bar\sigma^\mu\partial_\mu\lambda_D + \frac{1}{4}\tilde\Theta^{\mu\nu}\lambda_D\sigma_\mu\partial^\rho\bar\lambda_D F_{D\rho\nu})$$
$$+\frac{1}{4}\tilde\Theta^{\mu\nu}\bar\lambda_D\bar\sigma_\mu\partial^\rho\lambda_D F_{D\rho\nu}) + \frac{1}{2}(D_D^2 + \frac{1}{2}\tilde\Theta^{\mu\nu}D_D^2 F_{D\mu\nu})],$$

where

$$\tilde\Theta^{\mu\nu} \equiv g^2\epsilon^{\mu\nu\rho\sigma}\Theta_{\rho\sigma}, \tag{49}$$

When the fermionic and auxiliary fields λ_D, D_D set equal to zero one obtains the result of [4]: There is a duality symmetry under the replacement of A^μ with A_D^μ and $\Theta_{\mu\nu}$ with $\tilde\Theta_{\mu\nu}$.

Unfortunately, this symmetry accompanied by the replacement of λ, D with λ_D, D_D, cease to exist between the noncommutative supersymmetric action and its dual. However, by inspecting the terms which obstruct the duality symmetry it is still possible to define an action

$$\Sigma(\Theta, F, \lambda, \bar{\lambda}, D) = S_{NC} - \frac{i}{g^2}\int d^4x\Theta^{\mu\nu}\left(\lambda\sigma_\mu\partial^\rho\bar{\lambda} + \bar{\lambda}\bar{\sigma}_\mu\partial^\rho\lambda\right)F_{\rho\nu},$$

which can be obtained from the parent action

$$\Sigma_P = S_P - \frac{i}{2g^2}\int d^4x\Theta^{\mu\nu}(\psi\sigma_\mu\partial^\rho\bar{\lambda} + \bar{\psi}\bar{\sigma}_\mu\partial^\rho\lambda + \lambda\sigma_\mu\partial^\rho\bar{\psi} + \bar{\lambda}\bar{\sigma}_\mu\partial^\rho\psi)F_{R\rho\nu}, \tag{50}$$

when the solutions of equations of motion with respect to dual fields A_D, λ_D, D_D are plugged into it. Now, the dual theory can be shown to have the same form with the action (50):

$$\Sigma_D = g^4\Sigma(\tilde{\Theta}, F_D, \lambda_D, \bar{\lambda}_D, D_D). \tag{51}$$

The action (50) is gauge invariant and possesses the duality symmetry when the original fields are substituted by the dual ones and the noncommutativity parameter Θ is replaced with $\tilde{\Theta}$. However, whether the action Σ is supersymmetric or not is an open question [11].

7. Dual of Non-anticommutative $\mathcal{N} = \frac{1}{2}$ Supersymmetric U(1) Gauge Theory

One can introduce nonanticommutativity in 4 dimensional superspace $N = 1$ superspace $(x, \theta, \bar{\theta})$ by taking the Grassmann odd coordinates of one-chirality, i.e. the chiral one θ_α, not to anticommute with itself [7] :

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}, \tag{52}$$

where $C^{\alpha\beta}$ are constant deformation parameters. It is clear that this theory should be defined in Euclidean space since in $\mathbb{R}^4$ chiral and antichiral fermions are not related with complex conjugation.The deformation (52) breaks half of the supersymmetry. As in the noncommutative case, where the space-time coordinates x_μ do not commute, Moyal products are employed to interpose non-anticommutativity between the coordinates. Vector superfields taking values in this deformed superspace utilized to define a non–anticommutative supersymmetric Yang-Mills gauge theory. However, due to a change of variables one deals with the standard gauge transformations and component fields[7].

After aforementioned change of variables, the $N = \frac{1}{2}$ supersymmetric Yang-Mills theory action[7] is found to be

$$I_{1/2} = \frac{1}{g^2}\int d^4xTr(-\frac{1}{4}G^{\mu\nu}G_{\mu\nu} - i\lambda\not{\mathcal{D}}\bar{\lambda} + \frac{1}{2}D^2 - \frac{i}{2}C^{\mu\nu}G_{\mu\nu}(\bar{\lambda}\bar{\lambda}) + \frac{|C|^2}{8}(\bar{\lambda}\bar{\lambda})^2), \tag{53}$$

where $C^{\mu\nu} = C^{\alpha\beta}\epsilon_{\beta\gamma}\sigma_\alpha^{\mu\nu\ \gamma}$ and $\mathcal{D}_\mu$ is the covariant derivative. Gauge transformations possess the usual form. $G_{\mu\nu}$ is the non–Abelian field strength related to the gauge field A_μ. λ , $\bar{\lambda}$ are independent fermionic fields and D is auxiliary bosonic field.

The surviving part of the $N = 1$ supersymmetry acts on the standard component fields as

$$\begin{aligned}\delta\lambda &= i\epsilon D + \sigma^{\mu\nu}\epsilon(G_{\mu\nu} + \frac{i}{2}C_{\mu\nu}\bar{\lambda}\bar{\lambda})\\ \delta A_\mu &= -i\bar{\lambda}\bar{\sigma}_\mu\epsilon\,,\ \delta D = -\epsilon\sigma^\mu\mathcal{D}_\mu\bar{\lambda}\,,\ \delta\bar{\lambda} = 0\,,\end{aligned} \tag{54}$$

where ϵ is a constant Grassmann parameter.

Note that the action (53) can also be obtained by applying the supersymmetry generator Q defined by $\delta = \epsilon Q$, to the lower dimensional field monomial $Tr\lambda\lambda$ as [12]

$$I_{1/2} = \frac{1}{8g^2}Q^2\int d^4xTr(\lambda\lambda), \tag{55}$$

up to total derivatives, similar to the usual $N=1$ super Yang-Mills theory [17].

We propose the following parent action in terms of component fields $\mathrm{X}=(F_{\mu\nu},\ \lambda_\alpha,\ \bar\lambda^{\dot\alpha},\ \psi_\alpha,\ \bar\psi^{\dot\alpha},\ D_1,\ D_2)$ and $\mathrm{X}_D=(A_{D\mu},\ \lambda_{D\alpha},\ \bar\lambda^{\dot\alpha}_D,\ D_D)$ [12],

$$I_p = I_0[\mathrm{X}] + I_l[\mathrm{X},\mathrm{X}_D] \tag{56}$$

where I_0 is

$$I_0 = \frac{1}{g^2}\int d^4x\Bigg\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{i}{2}\lambda\partial\!\!\!/\bar\lambda - \frac{i}{2}\bar\psi\bar\partial\!\!\!/\psi + \frac{1}{4}D_1^2 + \frac{1}{4}D_2^2 - \frac{i}{4}C^{\mu\nu}F_{\mu\nu}(\bar\lambda\bar\lambda+\bar\psi\bar\psi)\Bigg\} \tag{57}$$

and I_l is defined as in (19).

When one plugs the solutions of the equations of motion of dual fields in terms of $\lambda,\ \bar\lambda,\ D$, into the parent action, the NAC $N=\frac{1}{2}$ supersymmetric $U(1)$ gauge theory action follows[11]:

$$I = \frac{1}{g^2}\int d^4x\Bigg\{-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - i\lambda\partial\!\!\!/\bar\lambda + \frac{1}{2}D^2 - \frac{i}{2}C^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu)\bar\lambda\bar\lambda\Bigg\}. \tag{58}$$

On the other hand, the equations of motion with respect to other fields are

$$\begin{aligned}
&\frac{1}{2g^2}F^{\mu\nu} + \frac{i}{4g^2}C^{\mu\nu}(\bar\lambda\bar\lambda+\bar\psi\bar\psi) - \frac{1}{2}\epsilon^{\mu\nu\lambda\kappa}\partial_\lambda A_{D_\kappa} = 0\ ,\\
&\partial\!\!\!/\bar\lambda + ig^2\partial\!\!\!/\bar\lambda_D = 0 \quad , \quad \partial\!\!\!/\bar\psi - ig^2\partial\!\!\!/\bar\lambda_D = 0\ ,\\
&\bar\partial\!\!\!/\lambda + C^{\mu\nu}F_{\mu\nu}\bar\lambda + ig^2\bar\partial\!\!\!/\lambda_D = 0 \quad , \quad \bar\partial\!\!\!/\psi + C^{\mu\nu}F_{\mu\nu}\bar\psi - ig^2\bar\partial\!\!\!/\lambda_D = 0\ ,\\
&D_1 + ig^2 D_D = 0 \quad , \quad D_2 - ig^2 D_D = 0
\end{aligned} \tag{59}$$

When we substitute the solutions of these equations in the parent action we obtain the dual non–anticommutative $N=\frac{1}{2}$ supersymmetric $U(1)$ gauge theory action :

$$I_D = g^2\int d^4x\Bigg\{-\frac{1}{4}F_D^{\mu\nu}F_{D\mu\nu} - i\lambda_D\partial\!\!\!/\bar\lambda_D + \frac{1}{2}D_D^2 + \frac{i}{4}g^2\epsilon^{\mu\nu\lambda\kappa}C_{\mu\nu}F_{D\lambda\kappa}\bar\lambda_D\bar\lambda_D\Bigg\}. \tag{60}$$

One can observe that the non–anticommutative $N=\frac{1}{2}$ supersymmetric $U(1)$ gauge theory action and its dual possess the same form and

$$\begin{aligned}
g &\rightarrow \frac{1}{g}\\
C^{\mu\nu} &\rightarrow C_D^{\mu\nu} = -\frac{1}{2}g^2\epsilon^{\mu\nu\lambda\kappa}C_{\lambda\kappa} = ig^2C^{\mu\nu}
\end{aligned} \tag{61}$$

is the duality transformation [12].

8. Appendix

We use Wess-Bagger conventions [13]:

$$\eta_{\mu\nu} = -diag(-1,1,1,1)$$

$$\lambda\psi = \lambda^\alpha\psi_\alpha\ ;\ \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta\ ,\ \psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta$$

$$\bar\lambda\bar\psi = \bar\lambda_{\dot\alpha}\bar\psi^{\dot\alpha}\ ;\ \psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\psi^{\dot\beta}\ ,\ \psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\psi_{\dot\beta}$$

[11]Note that, since we deal with $U(1)$ gauge group, the term quadratic in the deformation parameter, $\frac{|C|^2}{8}(\bar\lambda\bar\lambda)^2$, of the action vanishes.

$$\epsilon_{21} = -\epsilon_{12} = \epsilon^{12} = -\epsilon^{21} = 1$$

$$\bar{\sigma}^0 = \sigma^0 \qquad , \qquad \bar{\sigma}^{1,2,3} = -\sigma^{1,2,3} \tag{62}$$

$$\sigma^{\mu}_{\alpha\dot{\alpha}} = \epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}^{\mu\,\dot{\beta}\beta} \qquad , \qquad \bar{\sigma}^{\mu\,\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma^{\mu}_{\beta\dot{\beta}} \tag{63}$$

$$\sigma^{\mu\nu\ \beta}_{\alpha} = \frac{1}{4}(\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\beta\,\nu} - \sigma^{\nu}_{\alpha\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\beta\,\mu})$$

$$\bar{\sigma}^{\mu\nu\ \dot{\alpha}}_{\dot{\beta}} = \frac{1}{4}(\bar{\sigma}^{\dot{\alpha}\alpha\,\mu}\sigma^{\nu}_{\alpha\dot{\beta}} - \bar{\sigma}^{\dot{\alpha}\alpha\,\nu}\sigma^{\mu}_{\alpha\dot{\beta}})$$

where $\mu, \nu, ... = 0, 1, 2, 3, 4$ are Lorentz indices $\alpha..., \dot{\alpha}... = 1, 2$ are spinor indices; λ_α are Chiral Weyl spinors; $\bar{\lambda}_{\dot{\alpha}}$ are Anti-chiral Weyl spinors.

The integration over anticommuting coordinates $\theta, \bar{\theta}$ are defined as follows:

$$\int d\theta_\alpha \theta^\beta \equiv \frac{d\theta^\beta}{d\theta^\alpha} = \delta^\beta_\alpha\,, \int d\bar{\theta}^{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} \equiv \frac{d\bar{\theta}_{\dot{\beta}}}{d\bar{\theta}_{\dot{\alpha}}} = \delta^{\dot{\alpha}}_{\dot{\beta}}.$$

Note also that then a integral of a integrated superfield can be written as

$$\int d^4x \int d\theta_\alpha \Phi = \int d^4x \frac{\partial \Phi}{\partial \theta^\alpha} = \int d^4x D_\alpha \Phi,$$

up to a boundary term since the superspace covariant derivative D_α contains a x-space derivative explicitly (10).

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