Tensor Polarization and Quadrupole Form Factor of the Deuteron

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Abstract

A new experimental value, $M = 0.06925 \pm 0.00281$, of the peak of the quadrupole form factor $F_Q(q)$ is obtained by fitting the straight line relation found between the peak values of both $T_{21}(q)$ and $F_Q(q)$.

Key Words: e-d scattering, tensor polarization, quadrupole form factors.

1. Introduction

Electron scattering from nuclei has a long and rich history. In impulse approximation, the charge form factor probed in such experiments is the Fourier transformation of the nuclear charge distribution, and so these measurements have often been regarded as independent tests of models of nuclear structure [1,2]. In particular, the structures of nuclei with $A \leq 10$ can now be calculated from a given two- and three-nucleon interaction [3]. Calculations of electromagnetic form factors of these nuclei then reveal very good agreement with experimental data [4,5]. Herein, we focus on the simplest non-trivial nucleus: deuteron. Deuteron is a spin-one nucleus and so has three independent form factors. These are magnetic dipole $F_M(q)$, charge quadrupole $F_Q(q)$, and charge monopole $F_C(q)$ form factors [6]. The quadrupole form factor $F_Q(q)$ of the deuteron is given by [7]

$$F_Q(q) = 2 \int_0^\infty [u(r)w(r) - \frac{w^2(r)}{\sqrt{8}}] j_2(\frac{qr}{2}) dr \tag{1}$$

where $u = \psi_0 r$ and $w = \psi_2\, r$ are the s- and d- radial wave functions of the deuteron. With the advent of tensor polarimeters and tensor polarized internal targets, polarization observables have been measured as well, which allow the separation of the two charge form factors The tensor polarization observables t_{2k}, or equivalently, the analyzing powers T_{2k}, have been measured as well. Their expression is a function of the three form factors, and the tensor polarization $T_{21}(q)$ is given by [8, 9]:

$$T_{21} = \frac{2}{\sqrt{3}S \cos\frac{\vartheta}{2}} \eta [\eta + \eta^2 \sin^2 \frac{\vartheta}{2}]^{1/2} F_M(q) F_Q(q), \tag{2}$$

where $S = A(q) + B(q)\tan^2\frac{\vartheta}{2}$, where ϑ is the scattering angle at laboratory frame, and q in units of fm^{--1} is the momentum transfer to the deuteron. $A(q)$ and $B(q)$ are the usual electric and magnetic structure functions [10].

2. The Deuteron Quadrupole Form Factor $\boldsymbol{F_Q(q)}$

The slope of the deuteron quadrupole form factor for small momentum transfers is proportional to the deuteron quadrupole moment Q_D [7]. Additionally, $F_Q(q)$, at the peak, is related to the deuteron D-state probability, P_D [7]. The deuteron tensor polarization, $T_{21}(q)$, depends on the combination of $F_M(q).F_Q(q)$, *i.e.*, depends on the magnetic and quadrupole form factors. This allows for the extraction of the quadrupole form factor, which is closely related to the tensor force strength and the D- state probability, P_D [11]. Azzam *et al.* [10], concluded that the peak values of the tensor polarization, $T_{21}(q)$, for several local potential models, are related to some deuteron properties. They used these relations to extract an experimental value for the deuteron D- state probability, P_D. In this paper, a new relation between the peak values, $T_{21}^{Max}(q)$, of the deuteron tensor polarization $T_{21}(q)$ and the peak values of the deuteron quadrupole form factor $F_Q(q)$ will be deduced.

There exist a number of models for the *NN* interaction potential. The deuteron wave functions for these potentials give the results for deuteron electromagnetic properties, which differ essentially from one another. It is a difficult task to give a preference to one of them. Thirty-three local potential models are used here to study the deuteron quadrupole form factor. They are denoted by the following notations: GK1, GK3, and GK8 of Glendenning and Kramer [12]; PARIS of Lacomb *et al.* [13]; RHC, RSC, RSCA of Reid [14]; TSB and TSC of de Tourreil and Sprung [15]; HJ of Hamada and Johnston [16]; TRS of de Tourreil *et al.* [17]; L1, L2, 2, 4-6 of Mustafa [18]; r1, r3- r7 of Mustafa *et al.* [19]; MHKZ of Mustafa *et al.* [20].

We found that when $q > 1.4\ fm^{-1}$ and, especially, at $q > 2.0\ fm^{-1}$, the quadrupole form factor, $F_Q(q)$, starts to become model-dependent, and in the region between $2.0\ fm^{-1} < q < 3.0 fm^{-1}$, it appears to scale with the potential models properties. Figures 1 and 2 illustrate $F_Q(q)$ versus q for fourteen of the local potential models used. It is clear from the figures that each potential has its own peak value.

The peak values of $F_Q(q)$ for each potential model are extracted and are denoted by M. We found that $T_{21}^{Max}(q)$ is related to M. Figure 3 shows that for all potential models, this relation tends to be a straight-line. The experimental value, $T_{21}^{Max}(q) = 0.44513 \pm 0.00478$ [10], is substituted in the straight-line relation between the peak values of both $F_Q(q)$ and $T_{21}^{Max}(q)$ to extract an experimental value of M. The deduced value is:

$$M = 0.06925 \pm 0.00281$$

This experimental value is very important in theoretical calculations. It may be used in the future to extract a new experimental value of the deuteron D- state probability, P_D. This can be done by substituting this value in the empirical relation found between M and P_D [7].
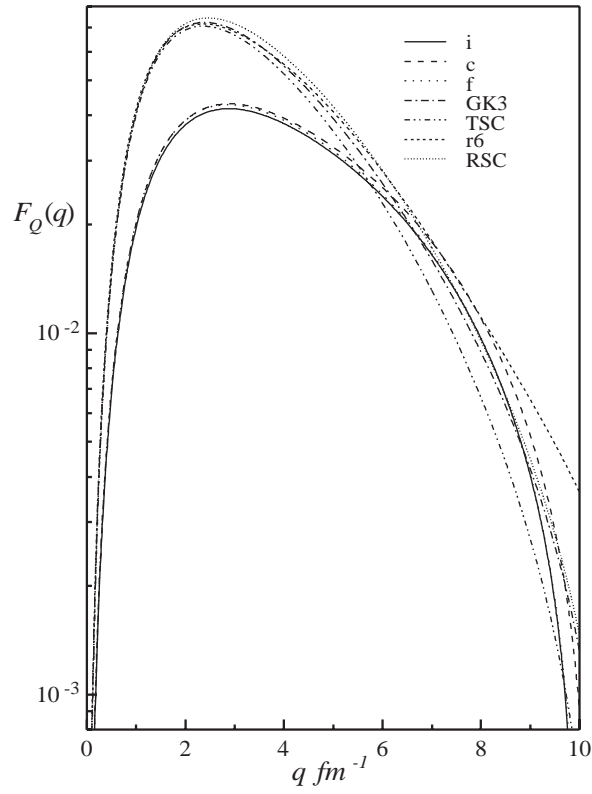
Figure 1. The quadrupole form factor, $F_Q(q)$, versus the momentum transfer, $q\ fm^{-1}$, for the local potential models indicated in the graph.
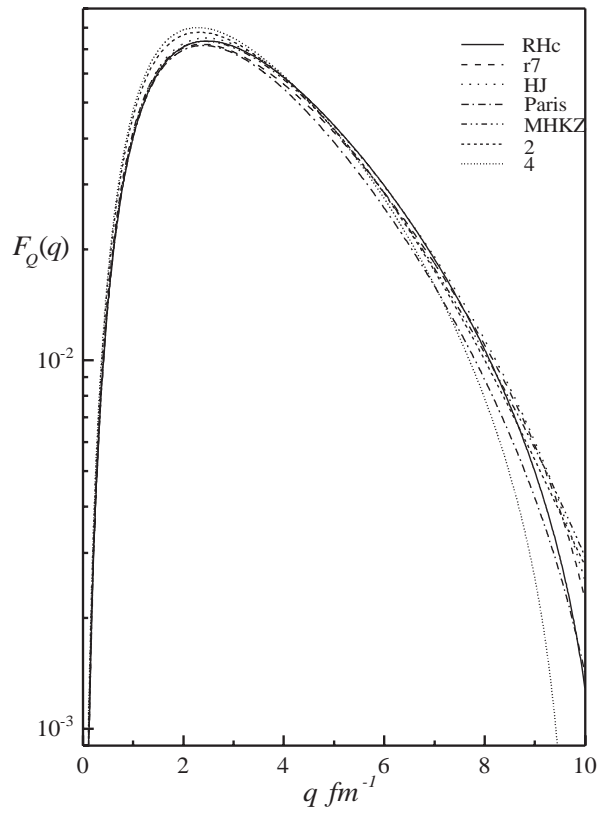
Figure 2. The quadrupole form factor, $F_Q(q)$, versus the momentum transfer, $q\ fm^{-1}$, for the local potential models indicated in the graph.
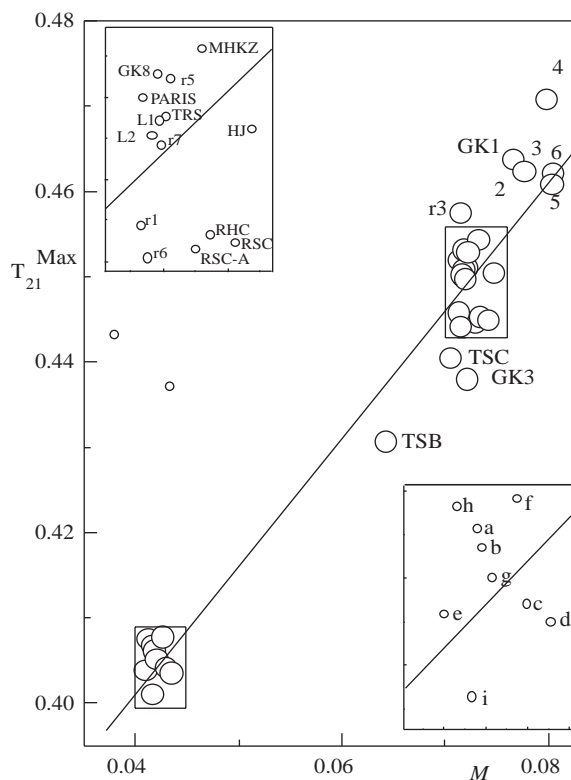
Figure 3. The relation between $T_{21}^{Max}(q)$ and the peak of the quadrupole form factor, M, for the 33 potential models.

3.   Conclusion

The calculations of the deuteron quadrupole form factor, $F_Q(q)$, for the potential models used, demonstrate that $F_Q(q)$ can distinguish between the competing potential models. A new linear relation between the peak values of both $T_{21}(q)$ and $F_Q(q)$ is found. This relation is used to extract an experimental value of M. Therefore, a measurement of $T_{21}(q)$ at the peak can greatly improve our knowledge of the deuteron quadrupole form factor, $F_Q(q)$, and vice versa.

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