# Information loss in local dissipation environments

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**Abstract**

The sensitivity of entanglement to the thermal and squeezed reservoirs' parameters is investigated regarding entanglement decay and what is called sudden-death of entanglement, ESD, for a system of two qubit pairs. The dynamics of information is investigated by means of the information disturbance and exchange information. We show that for squeezed reservoir, we can keep both of the entanglement and information survival for a long time. The sudden death of information is seen in the case of thermal reservoir.

## 1   Introduction

Entangled qubits are one of the most promising candidates for quantum communication and computation. There are many interesting applications based on these entangled systems. Among these applications, dense coding [1],quantum teleportation [2], quantum cryptography [3] etc. These entangled systems can not be isolated from their surrounding environments. cause deterioration of entanglement. So, investigating and quantifying the amount of entanglement contained in entangled system interacting with open systems is very important in the context of quantum information [4]. As an example, Yu and Eberly [5, 6], has investigated the dynamics of entanglement for entangled qubit pairs undergoing various modes of decoherence. They showed that the dynamics of entanglement between a two qubit system interacting independently with classical or quantum noise, displays two different types of behavior; the phenomena of entanglement decay and entanglement sudden death, ESD [6, 7]. Since then, different systems have been investigated. For some systems, the ESD appears whenever the system is open or closed [8]. The effect of the local squeezed reservoir on initially entangled two qubit system is investigated, where it is shown that the squeezing causes different behavior of entanglement decay on different time scales [9]. Recently in [10], it has been shown that the ESD always exists with thermal and squeezed reservoirs, where the authors presented explicit expression for the ESD for some entangled states. Also, it has been recently shown [11], that the ESD and entanglement decay phenomena appear for qubit system passed through Bloch channel [12, 13]. The ESD under the effect of the individual environments has been experimentally seen [14, 15].

The purpose of this paper is to continue investigating questions of this sort: by how much are the entanglement and the information of entangled two qubits state degrade when it passes through a thermal or squeezed reservoir.

The paper is organized as follows: In sec.2, we present the model and its solution. The dynamics of entanglement is investigated in Sec.3, where we consider two classes of entangled states as an initial states, maximum and partial entangled states. Sec.4 is devoted to study the dynamics of quantum information, where we quantify the amount of disturbance and exchange information between the entangled state and the local reservoirs. Finally we give a conclusion of our results in Sec.5.

## 2    The model and its solution

Assume that we have a source to generate entangled qubit pairs. One qubit is sent to Alice and the other to Bob. The general two qubit state is defined by

$$\rho_{ab}(0) = \frac{1}{4}(1 + \vec{s} \cdot \sigma_1^{\downarrow} + \vec{t} \cdot \sigma_2^{\downarrow} + \vec{\sigma}_1 \cdot \overrightarrow{C} \cdot \sigma_2^{\downarrow}), \tag{1}$$

where the vectors $\vec{s}$ and $\vec{t}$ are the Bloch vectors for Alice and Bob's qubits respectively, $\overrightarrow{C}$ is a $3 \times 3$ matrix represents the cross dyadic and $\vec{\sigma}_1$ and $\vec{\sigma}_2$ are the spin Pauli vectors [16]. Now assume that each qubit interacts individually with its squeezed vacuum environment[9]. Within Markov approximation, we can write the master equation in the Schrödinger form as,

$$\frac{\partial}{\partial t}\rho_{ab} = L_a(\rho_{ab}) + L_b(\rho_{ab}), \tag{2}$$

with,

$$\begin{aligned}
L_i(\rho_{ab}) &= -\frac{\Gamma_i}{2}(1 + \mathcal{N}_i)(\sigma_i^+ \sigma_i^- \rho_{ab} - 2\sigma_i^- \rho_{12}\sigma_i^+ + \rho_{ab}\sigma_i^+ \sigma_i^-) \\
&\quad -\frac{\Gamma_i}{2}\mathcal{N}_i(\sigma_i^- \sigma_i^+ \rho_{ab} - 2\sigma_i^+ \rho_{12}\sigma_i^- + \rho_{ab}\sigma_i^- \sigma_i^+) \\
&\quad -\frac{\Gamma_i}{2}\mathcal{M}_i(\sigma_i^+ \sigma_i^+ \rho_{ab} - 2\sigma_i^+ \rho_{12}\sigma_i^+ + \rho_{ab}\sigma_i^+ \sigma_i^+) \\
&\quad -\frac{\Gamma_i}{2}\mathcal{M}_i^*(\sigma_i^- \sigma_i^- \rho_{ab} - 2\sigma_i^- \rho_{12}\sigma_i^- + \rho_{ab}\sigma_i^- \sigma_i^-),
\end{aligned} \tag{3}$$

where $i = 1, 2$ refers to the first (Alice's qubit) and the second for (Bob's qubit), $\Gamma_i$ is the atomic spontaneous emission rate in local squeezed field. The parameter $\mathcal{M} = |\mathcal{M}|e^{i\theta}$, describes the strength of the two photon correlation, where $|\mathcal{M}_i| \leq \sqrt{\mathcal{N}_i(1 + \mathcal{N}_i)}$. Finally, $\sigma_i^{\pm} = \sigma_{ix} \pm i\sigma_{iy}$.

To investigate the dynamical behavior of the initial entangled state $\rho_{ab}(0)$, we solve the Schrödinger equation (2). In this context, we use the Kraus representation described in [9]. The time-evolution of the input state (1) is given by

$$\rho_{ab}(t) = \sum_{j}^{4} \kappa_j^a \otimes \kappa_j^b \rho_{ab}(0)\kappa_j^{a\dagger} \kappa_j^{b\dagger}. \tag{4}$$

For our analysis, we describe the Kraus operators in the computational basis $|0\rangle$ and $|1\rangle$ as,

$$
\begin{aligned}
\kappa_1^i &= \alpha_1^i|0\rangle\langle 1| + \beta_1^i|1\rangle\langle 1|, \quad \kappa_2^i = \beta_2^i|1\rangle\langle 1|, \\
\kappa_3^i &= \alpha_3^i|0\rangle\langle 1| + \beta_3^i|1\rangle\langle 0|, \quad \kappa_4^i = \alpha_4^i|1\rangle\langle 0|,
\end{aligned}
\tag{5}
$$

where

$$
\begin{aligned}
\alpha_1^i &= e^{-\frac{\zeta t}{2}}\sqrt{\cosh\zeta t + \frac{\Gamma_i}{2\zeta}\sinh\zeta t}, \quad \beta_1^i = e^{-\frac{\zeta t}{2}}\frac{\cosh\eta t}{\alpha_1^i}, \\
\beta_2^i &= e^{-\frac{\zeta t}{2}}\sqrt{\frac{[1-(\frac{\Gamma_i}{2\zeta})^2]\sinh^2\zeta t - \sinh^2\eta t}{\cosh\zeta t + \frac{\Gamma_i}{2\zeta}\sinh\zeta t}}, \\
\alpha_3^i &= e^{-\frac{\zeta t}{2}}\frac{\sinh(\eta t)}{\sqrt{(1+\frac{\Gamma_i}{2\zeta})\sinh(\eta t)}}, \quad \beta_3^i = e^{-\frac{\zeta t}{2}}\sqrt{(1+\frac{\Gamma_i}{2\eta})\sinh(\eta t)}\, e^{-i\theta}, \\
\alpha_4^i &= e^{-\frac{\zeta t}{2}}\sqrt{\frac{[1-(\frac{\Gamma_i}{2\zeta})^2]\sinh^2(\zeta t) - \sinh^2(\eta t)}{(1+\frac{\Gamma_i}{2\eta})\sinh(\eta t)}},
\end{aligned}
\tag{6}
$$

$\zeta = \frac{\Gamma_i}{2}(2\mathcal{N}_i + 1)$ and $\eta = \Gamma_i|\mathcal{M}_i|$.

To show our idea, let us consider a class of entangled states with zero Bloch vectors, $(\vec{s} = \vec{t} = 0)$. This simplification leads us to what is called a generalized Werner state [17, 18],

$$
\rho(0) = \frac{1}{4}(1 + c_1\sigma_x^{(1)}\sigma_x^{(2)} + c_2\sigma_y^{(1)}\sigma_y^{(2)} + c_3\sigma_z^{(1)}\sigma_z^{(2)}).
\tag{7}
$$

By means of Bell states $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, we can rewrite this initial state (7) as,

$$
\begin{aligned}
\rho(0) &= \frac{1 + c_1 + c_2 + c_3}{4}|\phi^+\rangle\langle\phi^+| + \frac{1 - c_1 + c_2 + c_3}{4}|\phi^-\rangle\langle\phi^-| \\
&+ \frac{1 - c_1 - c_2 - c_3}{4}|\psi^-\rangle\langle\psi^-| + \frac{1 + c_1 + c_2 - c_3}{4}|\psi^+\rangle\langle\psi^+|.
\end{aligned}
\tag{8}
$$

From this class of states we can get the singlet state $(|\psi^-\rangle\langle\psi^-|)$ if we set $c_1 = c_2 = c_3 = -1$, and if $c_1 = c_2 = c_3 = 1$, one gets $|\phi^+\rangle\langle\phi^+|$ and so on. Also, if we assume that $c_1 = c_2 = c_3 = x$, one gets Werner state [17],

$$
\rho_w(0) = \frac{3x+1}{4}|\psi^-\rangle\langle\psi^-| + \frac{1-x}{4}\left(|\phi^+\rangle\langle\phi^+| + |\phi^-\rangle\langle\phi^-| + |\psi^+\rangle\langle\psi^+|\right).
\tag{9}
$$

Now, let us assume that we have a source which supplies us with an entangled qubit state of form (8). The qubits leave each other and then interact with their local reservoir. With the Kraus operators, the time evolvement of the density operator (8) is,

$$
\begin{aligned}
\rho(t) &= e^{-i\Gamma t}\Bigg[\frac{1+c_3}{8}\Big\{(s_1 + s_4)(|\phi^+\rangle\langle\phi^+| + |\phi^-\rangle\langle\phi^-|) + (s_1 - s_4)(|\phi^+\rangle\langle\phi^-| + |\phi^-\rangle\langle\phi^+|)\Big\} \\
&+ \frac{c_1 - c_2}{8}\Big\{(s_2 + s_3)(|\phi^+\rangle\langle\phi^+| - |\phi^-\rangle\langle\phi^-|) + (s_2 - s_3)(|\phi^-\rangle\langle\phi^+| - |\phi^+\rangle\langle\phi^-|)\Big\}
\end{aligned}
$$

$$+ \quad \frac{1-c_3}{8}\Big\{(s_5+s_8)(|\psi^+\rangle\langle\psi^+|+|\psi^-\rangle\langle\psi^-|)+(s_5-s_8)(|\psi^+\rangle\langle\psi^-|+|\psi^-\rangle\langle\psi^+|)\Big\}$$

$$+ \quad \frac{c_1+c_2}{8}\Big\{(s_6+s_7)(|\psi^+\rangle\langle\psi^+|-|\psi^-\rangle\langle\psi^-|)+(s_6-s_7)(|\psi^-\rangle\langle\psi^+|-|\psi^+\rangle\langle\psi^-|)\Big\}\Big],$$

$$(10)$$

where,

$$
\begin{aligned}
s_1 &= \sum_{i=1,3}|\alpha_i^a|^2|\alpha_i^b|^2, \quad s_2 = \sum_{i=1,3}\alpha_i^a\alpha_i^b\beta_i^{*a}\beta_i^{*b}, \qquad s_3 = \sum_{i=1,3}\beta_i^a\beta_i^b\alpha_i^{*a}\alpha_i^{*b}, \\
s_4 &= \sum_{i=2,4}|\alpha_i^a|^2|\alpha_i^b|^2 + \sum_{i=1,3}|\beta_i^a|^2|\beta_i^b|^2, \qquad s_5 = \sum_{i=1,3}|\alpha_i^a|^2|\beta_i^b|^2, \\
s_6 &= \sum_{i=1,3}\alpha_i^a\beta_i^b\beta_i^{*a}\alpha_i^{*b}, \quad s_7 = \sum_{i=1,3}\beta_i^a\alpha_i^b\alpha_i^{*a}\beta_i^{*b}, \quad s_8 = \sum_{i=1,3}|\beta_i^a|^2|\alpha_i^b|^2, \\
\Gamma &= \Gamma_1 + \Gamma_2.
\end{aligned}
$$

$$(11)$$

## 3    Entanglement Dynamics

In this section, we investigate the robustness of the entangled state when each qubit interacts with its own reservoir individually, where we study the effect of reservoir parameters. In our *first example*, we assume that the source supplies the user Alice and Bob with maximum entangled state say, $|\phi^+\rangle\langle\phi^+|$ with $c_1 = c_2 = c_3 = 1$ or partially entangled state with $c_1 = c_2 = c_3 = 0.85$. Unfortunately, each qubit forced to passes through individual reservoir for some time. During this time there is non desirable interactions between the qubits and the reservoir. These interactions cause deteriorate of the amount entanglement contained in the entangled two qubits state and consequently the efficiency of performing quantum information tasks decreases. In our treatment we consider the local reservoirs to be thermal or squeezed.

To quantify the degree of entanglement we use the negativity, where it is easily calculable measure. It is given in terms of the eigenvalues of the partial transpose of the density operator [19],

$$DoE = \sum_i |\lambda_i| - 1, \tag{12}$$

where $\lambda_i$ are the eigenvalues of the partial transpose of the output density operator (10).

Fig.1, shows the dynamics of entanglement against the normalized time $\Gamma t$, where we assume that the two qubits pass through a thermal reservoir, i.e $\mathcal{M}_1 = \mathcal{M}_2 = 0$ and we assume that the two reservoirs have the same number of photons, $\mathcal{N}_1 = \mathcal{N}_2 = n$. In Fig.(1a), we consider the case where the source supplies the partners Alice and Bob with maximum entangled state. It is clear that for small values of $n$, we can see that the entanglement decays asymptotically and the time of the sudden death is delayed. For large values of $n$ the decay of entanglement is hastened and the time of entanglement sudden death becomes shorter.

In Fig.(1b), we quantify the amount of survival amount of entanglement contained in a density operator initially prepared in a partially entangled state. It is clear that, the entanglement decays faster and the time of the entanglement sudden death is shorter. From Figs.(1a) and Fig.(1b), we see that the entanglement decay and the sudden death of entanglement are sensitive to the initial entangled state. So, by controlling $n$ and $\Gamma$ one can prolonge the time of lived entanglement and delayed the time of the sudden death .

In Fig.(2a), we investigate the dynamics of entanglement at some specific values of the mean photon number, $n$, where we assume that the qubits interact with thermal reservoir. For small values of $n = 0.00001$, the phenomena of long-lived entanglement is seen, where the entanglement decreases asymptotically. On the other hand, as $n$ increases, the time at which the entanglement vanishes decreases. The entanglement sudden death phenomena appears for much larger values of the mean photon number, $n = 6$. The behavior of the
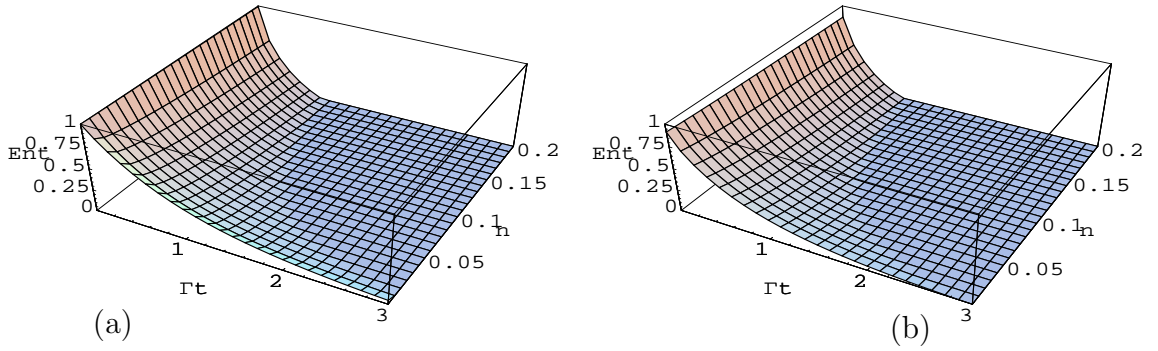


Figure 1: The effect of the mean photon number $\mathcal{N}_1 = \mathcal{N}_2 = n$ on the degree of entanglement(a) The initial state is maximum entangled state $|\phi^+\rangle\langle\phi^+|$ and (b) For a partial entangled state with $c_1 = c_2 = c_3 = 0.85$.
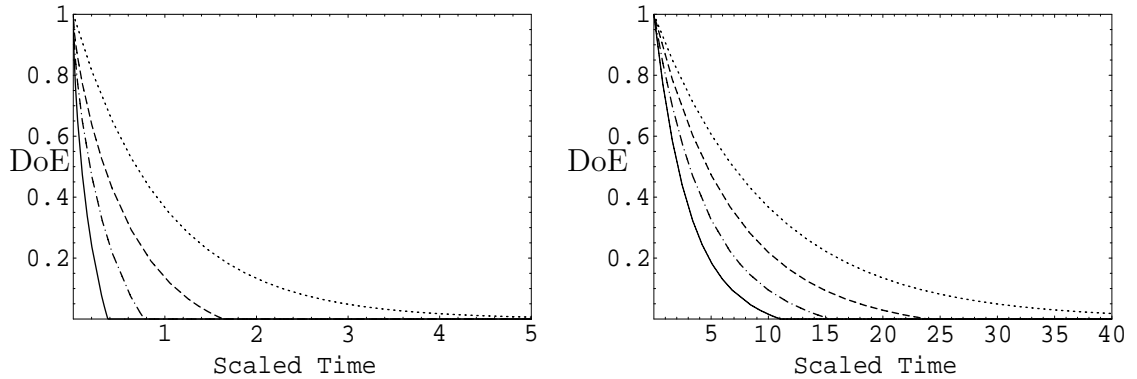


Figure 2: The dot, dash-dot, dash and slid curves represent the degree of entanglement(a) inside a thermal reservoir $\mathcal{N}_1 = \mathcal{N}_2 = n = 0.00001, 0.05, 0.2, 0.6$ and (b)inside a squeezed reservoir with $\mathcal{M}_1 = \mathcal{M}_2 = 0.05, 0.2, 0.4, 0.6$ and

entanglement when the qubits interact with squeezed reservoir is depicted in Fig.(2a). In this case the entanglement decays smoothly and for small values of the squeezed parameter, $\mathcal{M}$, one observes the long lived entanglement. Also, the vanishing time of entanglement is much larger than that has been shown in Fig.(2a).

# 4    Information dynamics

Quantum information science has emerged as one of the most exciting scientific developments in the past decade. Let us assume that Alice and Bob have coded information, say a quantum secret key, in their shared entangled state. But due to the reality there is no isolated systems, the entangled state which carries the information interacts with its surroundings. These undesirable interactions cause a loss of information. Therefore quantum information, in fact, cannot be perfectly copied, neither locally [20] nor at distance[21].

Our aim in this section is to investigate the dynamics of information which is carried by the shared entangled state. In our calculations, we consider the effect of the thermal and the squeezed reservoirs. In this treatments, we investigate two phenomena, the disturbance of information and the exchange information between the shared state and its local environments

One says that a system is *disturbed* when its initial and final states do not coincide. Since the information is coded on the input states, then it may be quantified in terms of fidelities [22]. The closeness of the output quantum state $\rho_f$ to the input one $\rho_i$ is expressed by the quantum fidelity $\mathcal{F}$, where $0 \leq \mathcal{F} \leq 1$ [23]. Now, we can define the disturbance, $\mathcal{D}$ as

$$\mathcal{D} = 1 - \mathcal{F}, \quad \mathcal{F} = Tr\{\rho_f \rho_i\}. \tag{13}$$

Fig(3a), shows, the behavior of the disturbance of information in the case of the thermal
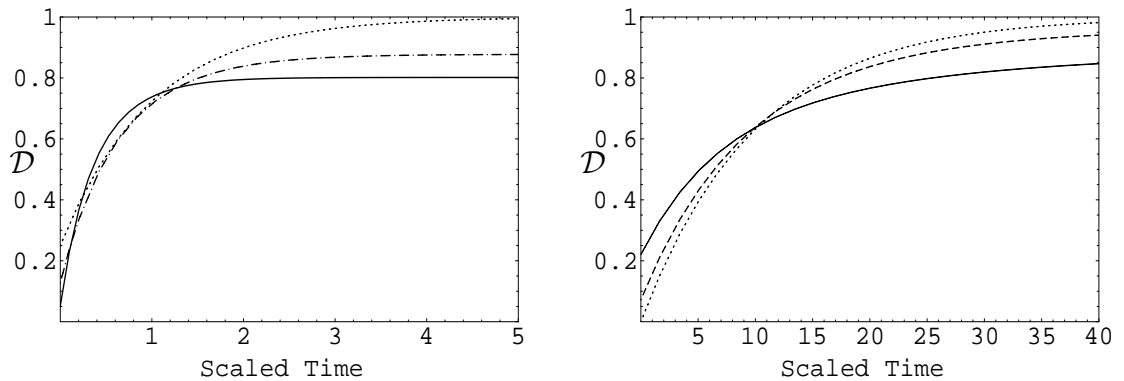
Figure 3: The dot, dash-dot and the solid curves represent the Disturbance $\mathcal{D}$, (a) For the thermal reservoir with $\mathcal{N}_1 = \mathcal{N}_2 = 0.00001, 0.2, 0.6$ (b) For the squeezed reservoir with $\mathcal{M}_1 = \mathcal{M}_2 = 0.001, 0.2, 0.4$

reservoir. It is clear that the disturbance increases as the scaled time, $\Gamma t$ increases. For small values of the thermal photon the disturbance increases gradually at the expense of the fidelity of the teleported state. For scaled time $\Gamma t \geq 4$, the disturbance $\mathcal{D}$ reaches its maximum value. This means that the input and the output states are completely different with the entangled state converted to separable state see(Fig.(2a)). As one increases the thermal photon number, the disturbance increases with time and reaches to a constant value as soon as the entanglement disappears. So, the entangled state is completely separable and there is no more information to be disturbed. For the values which causes a sudden-death of entanglement as depicted in Fig.(1a), the disturbance sudden be constant. This means at this values there as a sudden death of information.

Fig.(3b), describes the behavior of the disturbance of information in the presence of the local reservoir. In general, the same behavior is seen as that depicted for the thermal reservoir, but the disturbance $\mathcal{D}$ increases slowly and consequently the time loss of information is large. This behavior is due to the long-lived entanglement as seen in Fig.(2b). So, for small values of the squeezed reservoir parameters, the time of disturbed information can be infinite.

To quantify the amount of information exchange between the state and the environment during the evaluation, we use the entropy exchange [24],

$$S_e = -Tr\{\rho \log \rho\}. \tag{14}$$

If there is no interaction between the system and its surroundings, then the entropy exchange is zero and consequently there is no information loss from the system. One can look at the environment as an Eavesdropper, who wants to gain more information from the entangled system, which carries this information. In Fig.(4), we investigate the dynamics of this phenomenon also for the thermal and squeezed environment. In both cases, the exchange information increases as one increases the reservoir parameters, but as soon as the state turns into a separable state, the exchange information decreases. Then there is no more interaction with the local environment, therefore the exchange information becomes a constant. The effect of the thermal reservoir is seen in Fig.(4a), where for small values of the mean photon number $\mathcal{N}_1 = \mathcal{N}_2 = 10^{(-4)}$, the exchange entropy increases to reach its maximum values and then decreases with gradually time until it reaches to a constant value at $\Gamma t \geq 5$. We notice that, at this time the state turns into a separable state (see Fig.(2a)) so there is no more exchange between the environment and the state. As one increases the mean photon number, the exchange information becomes constant at $\Gamma t \cong 1.3$. For larger value of the mean photon number, say $\mathcal{N}_1 = \mathcal{N}_2 = 0.2$, the exchange information becomes constant at much earlier time .

The behavior of the exchange information in the presence of the local squeezed reservoir is the same as that shown for the thermal reservoir. But the time in which the exchange
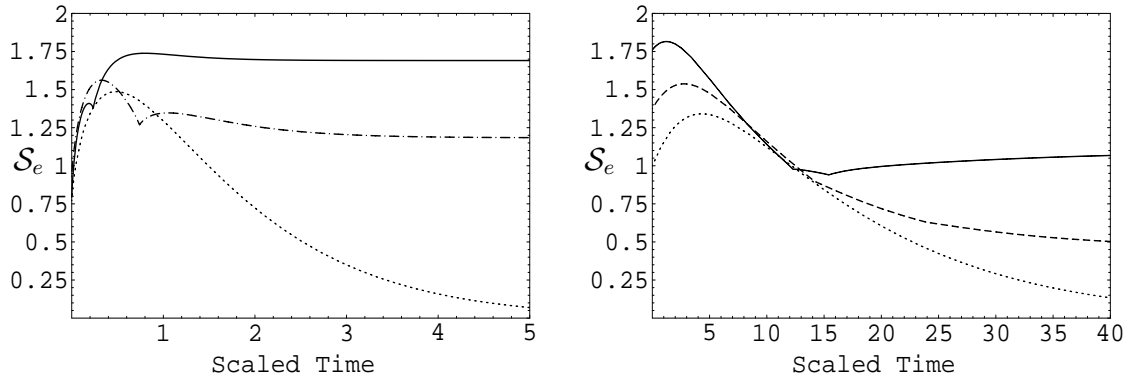
Figure 4: The exchange information, (a) For the thermal reservoir with same values in as Fig.(2a). (b) For the squeezed reservoir with same values as in Fig.(2b).

information becomes constant is much larger than that depicted for the thermal case. Science in the squeezed reservoir case the entanglement is long-lived.

## 5    Conclusion

In this contribution, we use the Kraus operators to investigate the dynamics of entangled state passes through a thermal or squeezed reservoir. The phenomenon of the entanglement decay and the sudden death of entanglement are shown for both reservoirs. we show that the entanglement lived longer for the squeezed reservoir. Also, the disturbance of information is discussed for both environment, where it is very sensitive to the thermal reservoir parameter much more than the squeezed vacuum reservoir parameters. For large values of the thermal photon reservoir, the information is suddenly disturbed, but it is disturbed gradually for the squeezed reservoir. The loss of information is quantified by the means of the entropy exchange between the environment and the shared entangled state. For both environments, the exchange information increases and leads to a constant when the system turns into separable state. For thermal reservoir, the exchange information becomes constant faster than that for the squeezed reservoir.

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