Turk J Phys
30 (2006) , 115 – 126.

Cherenkov Radiation in Homogeneous Isotropic Chiral Media

Pierre HILLION
Institut Henri Poincaré, 86 Bis Route de Croissy, 78110 La Vésinet-FRANCE
e-mail: pierre.hillion@wanadoo.fr

Received 25.07.2005

Abstract

We present a Fourier transform analysis of Cherenkov radiation for a point charge uniformly moving inside a 2D-homogeneous, isotropic, unbounded, chiral medium endowed with the Drude-Born-Fedorov constitutive relations when the chirality parameter β and the wave number k make negligible the $\beta^2 k^2$ terms. The electromagnetic field is not described in terms of circularly right and left polarized waves but in terms of TM and TE components. The Cherenkov radiation arises when the velocity v of the point charge is greater than the phase velocity c/n, where n is the refractive index of the chiral medium and the electromagnetic field stands inside the Mach cone with opening angle c/nv.

Key Words: Cherenkov, chiral, charge, Fourier, TM, TE waves.

1. Introduction

The Fourier transform of Cherenkov radiation generated by a point charge (electron) uniformly moving inside a homogeneous, isotropic, unbounded chiral medium has been analyzed many years ago for two types of constitutive relations with two different techniques [1, 2]. Because time-harmonic fields in these media are circularly birefringent, they are expressed in terms of two components: left handed and right handed, a different wave number being associated with each handedness. So, the Fourier transform of the Cherenkov radiation uses these two components.

We present a different description starting with the remark that, for harmonic plane waves in homogeneous, isotropic, chiral media, the wave equations satisfied by the electric and magnetic fields transform in absence of currents into a linear homogeneous system of equations with a solution only if its determinant is null; a condition fulfilled, as just stated, by two circularly right and left eigenmodes. But, in the presence of a current, and for the same situation, the wave equations give a nonhomogeneous linear system of equations with a solution free of constraints on its determinant, a result used to revive the Cherenkov radiation analysis.

For a point charge uniformly moving in a homogeneous, isotropic, unbounded, chiral medium endowed with the Drude-Born-Fedorov constitutive relations [2] the electric displacement and magnetic field are described by

$$\mathbf{D} = \varepsilon(\mathbf{E} + \beta\nabla \wedge \mathbf{E}) , \qquad \mathbf{B} = \mu(\mathbf{H} + \beta\nabla \wedge \mathbf{H}) \tag{1}$$

in which permittivity ε, permeability μ and chirality $ß$ are function of the frequency ω. The Maxwell equations are

$$\nabla \wedge \mathbf{E} + i\omega\mu/c \ (\mathbf{H} + \beta \nabla \wedge \mathbf{H}) = 0, \qquad \nabla \wedge \mathbf{H} - i\omega\varepsilon/c \ (\mathbf{E} + \beta \nabla \wedge \mathbf{E}) = \mathbf{J}. \tag{2}$$

The time factor exp(iωt) is implicit, c is the velocity of light and $\mathbf{J}$ is the vector current density.

A simple calculation gives the wave equations satisfied by $\mathbf{E}$ and $\mathbf{H}$

$$\Delta\mathbf{E} + 2\beta\gamma^2\nabla \wedge \mathbf{E} + \gamma^2\mathbf{E} = im^2\left(\mathbf{J} + \beta \nabla \wedge \mathbf{J}\right); \tag{3}$$

$$\Delta\mathbf{H} + 2\beta\gamma^2\nabla \wedge \mathbf{H} + \gamma^2\mathbf{H} = -\nabla \wedge \mathbf{J}, \tag{3a}$$

in which

$$\gamma^2 = k^2(1 - ß^2k^2)^{-1}, k^2 = \omega^2n^2/c^2, n^2 = \varepsilon\mu, and m^2 = \omega\mu\gamma^2/ck^2. \tag{3b}$$

Then, to simplify the Cherenkov radiation analysis, without prejudicing its conclusions, calculations are made in a 2D-space: fields and current do not depend on the y coordinate. We suppose, in addition, $|ßk|$ is small enough to make negligible the $ß^sk^s$ terms for $s \geq = 2$ and we work to approximation order O($ß^2k^2$), O denoting the Landau symbol.

The components of the current density $\mathbf{J}$ for a point charge uniformly moving along the z-axis with the velocity v are [1]

$$J_x = J_y = 0, J_z = q\delta(x)\exp(i\omega z/v) \tag{4}$$

where $\delta(x)$ is the Dirac distribution and q is a constant depending on the charge.

Taking into account (4), the Maxwell equations (2) in a 2D-space reduce to

$$\partial_z H_y = -i\omega\varepsilon/c(E_x - ß\partial_z E_y) \tag{5a}$$

$$\partial_z H_x - \partial_x H_z = i\omega\varepsilon/c[E_y + ß(\partial_z E_x - \partial_x E_z)] \tag{5b}$$

$$\partial_x H_y = i\omega\varepsilon/c(E_z + ß\partial_x E_y) + J_z \tag{5c}$$

$$\partial_z E_y = i\omega\mu/c(H_x - -ß\partial_z H_y) \tag{6a}$$

$$\partial_z E_x - \partial_x E_z \ = \ - i\omega\mu/c[H_y + ß(\partial_z H_x - \partial_x H_z)] \tag{6b}$$

$$\partial_x E_y = -i\omega\mu/c(H_z + ß\partial_x H_y). \tag{6c}$$

For harmonic plane waves,

$$\mathbf{E} = \mathbf{e}\exp[i(\chi_x x + \chi_z z)], \qquad \mathbf{H} = \mathbf{h}\exp[i(\chi_x x + \chi_z z)], \chi_x^2 + \chi_z^2 = \chi^2, \tag{7}$$

and wave equations (3) transform for $J = 0$ into an homogeneous system of equations with determinant

$$Q = \begin{vmatrix} \gamma^2 - \chi^2 & -2i\beta\gamma\chi_z & 0 \\ 2i\beta\gamma\chi_z & \gamma^2 - \chi^2 & 2i\beta\gamma\chi_x \\ 0 & 2i\beta\gamma\chi_x & \gamma^2 - \chi^2 \end{vmatrix}, \tag{8}$$

and $Q = 0$ gives the equation χ^4–-2 $\gamma^2\chi^2$ (1+2 $\beta^2\gamma^2$) + γ^4 = 0 with the solutions

$$\begin{aligned} \chi_\pm &= \left[1 + 2\beta^2\chi^2 \pm 2\beta\gamma\left(1 + \beta^2\gamma^2\right)^{1/2}\right]^{1/2} \\ &= \gamma\left(1 \pm \beta\gamma\right) + O\left(\beta^2 k^2\right), \end{aligned} \tag{8a}$$

where

$$\begin{aligned} \gamma &= k + O\left(\beta^2 k^2\right), \\ k &= n\omega/c, \end{aligned} \tag{8b}$$

thus supplying the phase velocity:

$$v_\pm = \omega/\chi_\pm = cn^{-1}(1 \pm ß\gamma) + O(ß^2 k^2). \tag{9}$$

We first look for the solutions of equations (5), (6) in a homogeneous, isotropic, achiral medium when *ß* = 0.

2. Cherenkov Radiation in 2D-homogeeous, Isotropic Media

With ß = 0, Maxwell equations (5), (6) reduce to

$$\partial x_z H_y = -i\omega\varepsilon/cE_x \tag{10a}$$

$$\partial_z H_x - \partial_x H_z = i\omega\varepsilon/cE_y \tag{10b}$$

$$\partial_x H_y = i\omega\varepsilon/cE_z + J_z \tag{10c}$$

$$\partial_z E_y = i\omega\mu/cH_x \tag{11a}$$

$$\partial_z E_x - \partial_x E_z = -i\omega\mu/cH_y \tag{11b}$$

$$\partial_x E_y = -i\omega\mu/cH_z. \tag{11c}$$

The symmetry of these equations implies $H_x = H_z = E_y = 0$, reducing the Cherenkov radiation to a TM electromagnetic wave and we are left with equations (10a), (10c) and (11b), from which we get the nonhomogeneous wave equation satisfied by the component H_y:

$$(\partial_x^2 + \partial_z^2 + k^2)H_y = \partial_x J_z, k^2 = \omega^2 n^2/c^2. \tag{12}$$

We look for the solution of equation (12) in the form

$$H_y(x,\ z) = h(x)\exp(i\omega z/v). \tag{13}$$

Substituting (13) into (12) and using (4) gives the differential equation

$$(\partial_x^2 + \lambda^2)h(x) = j(x), j(x) = q\delta d'(x) \tag{14}$$

$$\lambda^2 = k^2 - \omega^2/v^2 = \omega^2 n^2/c^2(1 - -c^2/n^2v^2), \tag{14a}$$

where $\delta d'(x)$ is the derivative of the Dirac distribution and $\lambda^2 > 0$ for $v > c/n$.

Then, the solution of equation (11) is [3]

$$\begin{aligned} h(x) &= \textstyle\int_{-\infty}^{\infty} j(x')g(x,x')\ dx'; \\ g(x,x') &= \tfrac{i}{2\lambda}\exp\left(i\lambda|x-x'|\right), \end{aligned} \tag{15}$$

in which g$(x,\ x')$ is Green's function of the 1D-Helmholtz equation. Then

$$\begin{aligned} h(x) &= \tfrac{iq}{2\lambda}\textstyle\int_{-\infty}^{x}\delta'(x')\exp\left[-i\lambda(x'-x)\right]dx' + \tfrac{iq}{2\lambda}\int_{x}^{\infty}\delta'(x')\exp\left[i\lambda(x-x')\right]dx' \\ &= \tfrac{iq}{2\lambda}\left[-\tfrac{i}{2}\partial_x\exp(-i\lambda x) - \tfrac{i}{2}\partial_x\exp(i\lambda x)\right] \\ &= \tfrac{iq}{2}\sin\left(\lambda x\right). \end{aligned} \tag{16}$$

Taking into account (13), we get from (16)

$$H_y = (iq/2)\sin\left(\lambda x\right)\exp(i\omega z/v). \tag{17}$$

Substituting (17) into (10a) and (11b) gives the components E_x, E_z as

$$E_x = -iqc/2\varepsilon v \sin(\lambda x) \exp(i\omega z/v)$$
$$E_z = \lambda qc/2\omega\varepsilon \cos(\lambda x) \exp(i\omega z/v) \qquad (17a)$$

so that $E_x/E_z = icn/v(1 - c^2n^2/v^2)\ -1/2 \tan(\lambda x) = i\ \cos\theta(1 - \cos^2\theta)^{-1/2}\ \tan(\lambda x)$, implying that the radiation is contained within the 2D Mach cone of opening angle θ. (A more complete discussion may be found in [4], [5].)

Remark. These mathematical results are also valid with minor changes for a point charge moving in a homogeneous, isotropic, Veselago, chiral medium [6] in which ε, μ, n are negative. According to (17), λ becomes $-\lambda$, and the phase velocity of plane waves is in the opposite direction being antiparallel to the Poynting vector. And according to (17a), the sign of E_x is different, implying that the θ angle of the Mach cone is changed to $\pi - \theta$, with nontrivial physical consequences.

3. Cherenkov Radiation in 2D-homogeneous, Isotropic, Chiral Media

We now come back to Maxwell's equations (5), (6). The results of Section 2 suggest that the components H_x, H_z, E_y, of the Cherenkov radiation, null in achiral media, become ß-linear with the form

$$H_x = (i\omega\varepsilon/c)ßM_x, H_z = (i\omega\varepsilon/c)ßM_z, E_y = (i\omega\mu/c)ßN_y. \qquad (18)$$

Substituting (18) into (5), (6) shows that equations (5a) (5c) and (6b) reduce to (10a), (10c) and (11b) so that the TM components H_y, E_x, E_z have still the expressions (17), (17a) as for ß = 0. This statement is easily justified, for instance, with respect to equation (5a) which becomes

$$\partial_z H_y = -i\omega\varepsilon/cE_x - -ß^2k^2\partial_z N_y = -i\omega\varepsilon/cE_x + O(ß^2k^2). \qquad (18a)$$

Now, taking into account (18), equations (5b), (6a) and (6c) become

$$\partial_z M_x - \partial_x M_z = i\omega\mu/cN_y + A_1; A_1 = \partial_z E_x - \partial_x E_z \qquad (19a)$$

$$\partial_z N_y = i\omega\varepsilon/cM_x - A_2; A_2 = \partial_z H_y \qquad (19b)$$

$$\partial_x N_y = -i\omega\varepsilon/cM_z - -A_3; A_3 = \partial_x H_y, \qquad (19c)$$

from which we get the nonhomogeneous wave equation satisfied by N_y:

$$(\partial_x^2 + \partial_z^2 + k^2)N_y = i\omega\varepsilon/cA_1 - \partial_z A_2 - \partial_x A_3; \qquad (20)$$

and using (11b), (12) to explicit A_1, A_2, A_3, gives

$$(\partial_x^2 + \partial_z^2 + k^2)N_y = 2k^2 H_y - \partial_x J_z. \tag{20a}$$

We look for the solution to (20a) in the form

$$N_y(x, z) = N(x)\exp(i\omega z/v), \tag{21}$$

and taking into account (4), (13) and (14) wave equation (20a) becomes

$$(\partial_x^2 + \lambda^2)N(x) = 2k^2 h(x) - j(x). \tag{22}$$

According to (15) and (16) the solution to (22) is

$$N(x) = N_1(x) + N_2(x), \tag{23}$$

where

$$N_1(x) = -h(x) \tag{23a}$$

$$N_2(x) = (ik^2/\lambda)\int_{-\infty 8}^{\infty 8} h(x')\exp(i\lambda|x - x'|)dx'. \tag{23b}$$

Using additional results from Appendix A for x in the interval $\{(m-1)\pi p/\lambda,\ (m+1)\pi p/\lambda\}$, where m is an arbitrary integer, we get

$$N_2(x) = iqk^2/2\lambda^2[(\lambda x - m\pi p)\cos(\lambda x) - (1 - i\pi p)\sin(\lambda x], (m \pm 1) are even integers \tag{24a}$$

$$N_2(x) = iqk^2/2\lambda^2[(\lambda x - m\pi p)\cos(\lambda x) + i\{\cos(\lambda x) + \pi p\sin(\lambda x)\}], (m \pm 1) are odd integers.(24b) \tag{24b}$$

Substituting (23) into (21) gives

$$N_y = -H_y + N_2(x)\exp(i\omega z/v); \tag{25}$$

and according to (19b) and (19c),

$$M_x = c/\varepsilon v N_2(x)\exp(i\omega z/v) \tag{25a}$$

$$M_z = ic/\varepsilon\omega\partial_x N_2(x)\exp(i\omega z/v). \tag{25b}$$

Finally, substituting (25) and (25a) into (18) gives the TE components of the Cherenkov radiation inside the Mach cone as

$$E_y(x,z) = -(i\omega\mu ß/c)[H_y - N_2(x)\exp(i\omega z/v)], \tag{26}$$

$$H_x(x,z) = (i\omega ß/v)N_2(x)\exp(i\omega z/v)], \tag{26a}$$

$$H_z(x,z) = -ß\partial_x N_2(x)\exp(i\omega z/v)]. \tag{26b}$$

So,

$$\begin{aligned} H_x/H_z &= \tfrac{-i\omega}{v}\tfrac{N_2}{\partial_x N_2} \\ &= -i\,\cos\theta\,\left(1-\cos^2\theta\right)^{-1/2}\tfrac{N_2}{\partial_x N_2}, \end{aligned} \tag{27}$$

where θ is the opening angle of the Mach cone. The presence of H_y in N_y makes the decoupling of TE and TM components only partial.

4. Conclusions

To O($ß^2 k^2$) order, the electric field has the same expression as in achiral media with an identical Mach cone, and the Cherenkov radiation takes place for superluminal velocities $v > c/n$ and not superluminal with respect to the chiral phase velocities (9).

The analysis of the Cherenkov radiation has been simplified with the help of two particular features: the approximation of O($ß^2 k^2$), and the 2D dimensional space in which the point charge propagates.

To illustrate how we may get higher order approximations, we consider the approximation of O($ß^3 k^3$) for TM components E_x, E_z, H_y with the TE components H_x, H_z, E_y given by (18), in which M_x, M_z, N_y are expressions (25) and (25a). Noting E_x^0, E_z^0, H_y^0 are the solutions obtained in Section 2 for $ß = 0$, we write

$$E_x = E_x^0 + ß^2k^2E_x^1, E_z = E_z^0 + ß^2k^2E_z^1, H_y = H_y^0 + ß^2k^2H_y^1, \tag{28}$$

in which E_x^0, E_z^0, H_y^0 are solutions of equations (10a), (10c) and (11b) and M_x, M_z, N_y are solutions of equations (19).

Then, the Maxwell equations (5a), (5c) and (6b) give the following equations for E_x^1, E_z^1, H_y^1:

$$\partial_z H_y^1 = -i\omega\varepsilon/cE_x^1 - \partial_z N_y \tag{29a}$$

$$\partial_x H_y^1 = i\omega\varepsilon/cE_z^1 + \partial_x N_y \tag{29b}$$

$$\partial_z E_x^1 - \partial_x E_z^1 = -i\omega\mu/cH_y^1 + \partial_z M_x - \partial_x M_z, \tag{29c}$$

from which it is easy to get a nonhomogeneous wave equation for H_y^1 with the Helmholtz operator $(\partial_x^2 + \partial_z^2 + k^2)$; and giving the solutions in the form $H_y^1 = h^1(x)\exp(i\omega z/v)$, a differential equation with the differential operator $(\partial_x^2 + \lambda^2)$ acting on $h^1(x)$ whose solutions are obtained with the Green's function in (15).

In the same way, we may use (18) to get the O(ß^{4}k^4) of the TE components, just by writing

$$H_x = H_x^0 + ß^3k^3H_x^1, H_z = H_z^0 + ß^3k^3H_z^1, E_y = E_y^0 + ß^3k^3E_y^1, \tag{30}$$

in which H_x^0 , H_z^0 , E_y^0 are expressions (26). Substituting (30) into equations (5b), (6a) and (6c) would supply the equations satisfied by H_x^1, H_z^1, E_y^1.

This process may be iterated to achieve higher order approximations. At any order, the approximations $E_y^{(m)}$, $H_y^{(m)}$ of the E_y, H_y components may be written

$$E_y^{(m)}(x,z) = e^{(m)}(x)\exp(i\omega z/v), H_y^{(m)}(x,z) = h^{(m)}(x)\exp(i\omega z/v), \tag{31}$$

$e^{(m)}$(x), $h^{(m)}$(x) being solutions of a differential equation with the differential operator (∂_x^2+ λ^2) to order m. So, to any order, the Cherenkov radiation takes place for the superluminal velocities v ¿ c/n.

We now discuss the second point mentioned earlier. Taking into account the divergence equations $\nabla \cdot \mathbf{E} = \nabla \cdot \mathbf{H} = 0$, we get for E_y, H_x, H_z, when ß = 0,

$$E_y = 0, H_x = ic/\mu\omega\partial_y E_z, H_z = -ic/\omega\mu\omega\partial_y E_x; \tag{32}$$

and while E_x, E_z are solutions of the differential equations,

$$\partial_y^2 E_x + n^2\omega^2c^{-2}E_x = i\omega\mu/c\partial_z H_y, \partial_y^2 E_z + n^2\omega^2c^{-2}E_z = -i\omega m\mu c\partial_x H_y - J_z, \tag{33}$$

with the nonhomogeneous wave equation for H_y being

$$(\partial_x^2 + \partial_y^2 + \partial_z^2 + k^2)H_y = \partial_x J_z, J_z = q_1\delta(x)\delta(y)\exp(i\omega z/v). \tag{34}$$

In the chiral medium with ß $\neq$ 0, and to order O(ß$_2$k^2), E_x, E_z, H_y are solutions of equations (33) and (34), while components E_y, H_x, H_z become

$$E_y = i\omega\mu/cßN_y, H_x = H_x^0 + i\omega\varepsilon/cßM_x, H_z = H_z^0 + i\omega\varepsilon/cßM_z, \tag{35}$$

with H_x^0 and H_z^0 given by (32). Calculations are only a bit more intricate than in a 2D-space to reach the same conclusions.

Finally, it is interesting to investigate the role of the constitutive relations. Suppose [1]

$$D = \varepsilon E + i\xi B, B = \mu H - i\mu\xi E. \tag{36}$$

Then the 2D-Maxwell equations are

$$\partial_z H_y = -i\omega\varepsilon/cE_x + \omega\mu\xi/c(H_x - i\xi E_x), \partial_z E_y = i\omega\mu/c(H_x - i\xi E_x) \tag{37a}$$

$$\partial_z H_x - \partial_x H_z = iwe/cE_y - \omega\mu\xi/c(H_y - i\xi E_y), \partial_z E_x - \partial_x E_z = -i\omega\mu/c(H_y - i\xi E_y) \tag{37b}$$

$$\partial_x H_y = iwe/cE_z - \omega\mu\xi/c(H_z - i\xi E_z) + J_z, \partial_x E_y = -i\omega\mu/c(H_z - i\xi E_z). \tag{37c}$$

The O(ξ^2) approximation of the Cherenkov radiation is discussed in Appendix B and we get, for instance, for its TM components in which H_x^0 , E_x^0 , E_z^0 are the expressions (17), (17a),

$$H_x(x,z) = H_x^0(x,z) + \xi \exp[i\lambda x + \omega z/v)] \tag{38a}$$

$$E_x(x,z) = E_{x0}(x,z) - c\xi/\varepsilon v \exp[i(\lambda x + \omega z/v)] \tag{38b}$$

$$E_z(x,z) = E_z^0(x,z) + c\xi\lambda/\omega\varepsilon \exp[i(\lambda x + \omega z/v)]. \tag{38c}$$

These results complete those obtained with constitutive relations (1), because the O(ξ^2) approximation is less stringent than the $O(ß^2 k^2)$ approximation, since $O(\xi^2)$ does not call out the frequency ω through k.

Cherenkov radiation devices are extensively used in high energy physics as instruments for velocity measurements, as mass analyzers when combined with momentum analysis and as discriminators against slow particles [5]. According to the present analysis of Cherenkov radiation, the use of Cherenkov counters in chiral media would not suffer any ambiguity since only one Mach cone intervenes. With the previous analysis [1, 2], some ambiguity may arise because of the relative value of particle's velocity with respect to the two right handed and left handed phase velocities. We ignore whether an experiment has been performed to check the properties of the Cherenkov radiation in a chiral medium.

Appendix A

Using the relation (37) for h(x), integral (23a) (rewritten here)

$$N_2(x) = (ikr^2/\lambda) \int_{-\infty}^{\infty} h(x') \exp(i\lambda|x - x'|)dx' \tag{23a}$$

becomes for x in the interval $\{t = (m-1)\pi/\lambda, u = (m+1)\pi/\lambda\}$, where m is an arbitrary integer,

$$N_2(x) = (-qk^2/2\lambda) \int_t^u dx' \sin(\lambda x') \exp(i\lambda|x - x'|) \tag{A.1}$$
$$= -(qk^2/2\lambda) \int_t^x dx' \sin(\lambda x') \exp[i\lambda(x - x')]$$

$$-(qk^2/2\lambda) \int_x^u dx' \sin(\lambda x') \exp[i\lambda(x' - x)]. \tag{A.1a}$$

Now when $m \pm 1$ are even integers,

$$\int_t^x dx' \sin(\lambda x') \exp[i\lambda(x - x')] = \exp(i\lambda x)/2i \int_t^x dx'[1 - \exp(-2\lambda x')] \tag{A.2}$$

$$= 1/2i\lambda\ [\lambda x - (m-1)\pi]\ \exp(i\lambda x) - 1/2i\lambda\ \sin(\lambda x) \tag{A.2a}$$

and

$$\int_x^u dx' \sin(\lambda x') \exp[i\lambda(x'-x)] = \exp(-i\lambda x)/2i \int_t^x dx'[\exp(2\lambda x') - 1] \tag{A.3}$$

$$= 1/2i\lambda[\lambda x - (m+1)\pi] \exp(-i\lambda x) - 1/2i\lambda \sin(\lambda x). \tag{A.3a}$$

Substituting (A.2a) and (A.3a) into (A.1a) gives

$$N_2(x) = (iqk^2/2\lambda^2)[(\lambda x - m\pi] \cos(\lambda x) - (1 - i\pi) \sin(\lambda x)]. \tag{A.4}$$

For the case when $m \pm 1$ are odd integers, we have only to change $-1/2i\lambda \sin(\lambda x)$ into $1/2\lambda \cos(\lambda x)$ in (A.2) and (A.3) so that

$$N_2(x) = (iqk^2/\lambda^2)[(\lambda x - m\pi] \cos(\lambda x) + i\{\cos(\lambda x) + \pi \sin(\lambda x)\}]. \tag{A.5}$$

Appendix B

Neglecting the ξ^2 terms in (37) gives this system of equations:

$$\partial_z H_y = -i\omega\varepsilon/cE_x + \omega\xi^2/cH_x(B.1a), \partial_z E_y = i\omega\mu/c(H_x - i\xi E_x) \tag{B.2a}$$

$$\partial_z H_x - \partial_x H_z = i\omega\varepsilon/cE_y - \omega\mu\xi/cH_y(B.1b), \partial_z E_x - \partial_x E_z = -i\omega\mu/c(H_y - i\xi E_y) \tag{B.2b}$$

$$\partial_x H_y = i\omega\varepsilon/cE_x - \omega\mu\xi H_z + J_z(B.1c), \partial_x E_y = -i\omega\mu/c(H_z - -i\xi E_z). \tag{B.2c}$$

For $\xi = 0$ these equations reduce to systems (10), (11); the TE components are null and the TM components, which are E_x^0, E_z^0, H_y^0, have the expressions (17), (17a) and we write

$$E_x = E_x^0 + \xi E_x^1, E_z = E_z^0 + \xi E_z^1, H_y = H_y^0 + \xi H_y^1 \tag{B.3a}$$

$$H_x = \xi H_x^1, H = \xi H_z^1, E_y = \xi E_y^1. \tag{B.3b}$$

To $O(\xi^2)$ order, and taking into account (B.3b), equations (B.2a), (B.2c) and (B.1b) become

$$\partial_z E_y^1 = i\omega\mu/c(H_x^1 - iE_x^0) \tag{B.4a}$$

$$\partial_x E_y^1 = -i\omega\mu/c(H_z^1 - iE_z^0) \tag{B.4b}$$

$$\partial_z H_x^1 - \partial_x H_z^1 = i\omega\varepsilon/cE_y^1 - \omega\mu/cH_y^0. \tag{B.4c}$$

Note that the component E_y^1 satisfies the wave equation

$$(\partial_x^2 + \partial_z^2 + k^2)E_y^1 = -2i\omega^2\mu^2/c^2 H_y^0. \tag{B.5}$$

Looking for $E_y^1(x,\ z)$ in the form $E_y^1(x,\ z) = E(x)\exp(i\omega z/v)$ gives the differential equation

$$(\partial_x^2 + \lambda^2)E(x) = 2i\omega^2\mu^2/c^2 h(x), \tag{B.6}$$

where h(x) is the function (16); and using (15)

$$E(x) = \omega^2\mu^2/\lambda c^2 \int_{-\infty}^{\infty} dx' h(x')\exp(i\lambda|x - x'|); \tag{B.7}$$

and taking into account (23a),

$$E(x) = -i\omega^2\mu^2/k^2c^2 N_2(x), \tag{B.8}$$

with $N_2(x)$ given by (A.4) and (A.5) so that

$$E_y^1 = -i\omega^2\mu^2/k^2c^2 N_2(x)\exp(i\omega z/v). \tag{B.8a}$$

Substituting (B.8a) into the first two equations (B.4) gives H_x^1 and H_z^1.

Similarly, taking into account (B.3a) and (B.3b), we get from (B.1a), (B.1c) and (B.2b) to $O(\xi^2)$ order, since E_x^0, E_z^0, H_y^0 satisfy (10a), (10c) and (11b),

$$\partial_z H_y^1 = -i\omega\varepsilon/cE_x^1, \partial_x H_y^1 = i\omega\varepsilon/cE_z^1(\tag{B.9}$$

$$\partial_z E_x^1 - \partial_x E_z^1 = i\omega\mu/cH_y^1. \tag{B.9a}$$

So, H_y^1 is a solution of the wave equation $(\partial_x^2 + \partial_z^2 + k^2)H_y^1 = 0$ with the solution

$$H_y^1(x,\ z) = \exp[i(\lambda x + \omega z/v); \tag{B.10}$$

and substituting (B.10) into the firsf two relations (B.9) gives

$$E_x^1(x,z) = -c/\varepsilon v \exp[i(\lambda x + \omega z/v), E_z^1(x,z) = c\lambda/\omega\varepsilon \exp[i(\lambda x + \omega z/v) \tag{B.10a}$$

which allows, according to (B.3a), one to determine E_x, E_z and H_y.

References

[1] N. Engheta and S. Bassiri, *J. Appl. Phys.*, **68**, (1990), 4393.

[2] A. Lakhtakia, *Optik*, **3**, (1991), 133.

[3] P. M. Morse and H. Feschbach, *Methods of Theoretical Physics,* (Mac Graw Hill, New York, 1953) p. 981.

[4] J. Van Bladel, *Relativity and Engineering,* (Springer Verlag, Berlin, 1979) p. 116.

[5] J. D. Jackson, *Classical Electrodynamics.*, (Wiley, New York, 1975) p. 638.

[6] V. E. Veselago, *Sov. Phys. Usp.*, **10**, (1968), 509.