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Study $a_0(980)$-$f_0(980)$ mixing in charmonium decays*

WU Jia-Jun(吴佳俊)$^{1;1)}$ ZHAO Qiang(赵强)$^{1,2;2)}$ ZOU Bing-Song(邹冰松)$^{1,2;3)}$

^{1} Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

^{2} Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, China

Abstract In this proceeding, we propose to directly measure the $a_0^0(980)$-$f_0(980)$ mixing in $J/\psi \to \phi\, f_0(980) \to \phi a_0^0(980)$ and $\chi_{c1} \to \pi^0 a_0^0(980) \to \pi^0 f_0(980)$ with the upgraded Beijing Electron Positron Collider(BEPCII) with BESIII detector. We show that a narrow peak of about 8 MeV will be produced by the $a_0^0(980)$-$f_0(980)$ mixing , and the predominant feature makes it standing out from the background contributions. The predicted branching ratios for these two reactions are both expected to be about $O(10^{-6})$, which is unambiguously accessible with 10^9 J/ψ and 3×10^8 χ_{c1} at BESIII.

Key words $a_0^0(980)$, $f_0(980)$, mixing, BES

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1 Introduction

More than thirty years after their discovery, today the nature of light scalar mesons $f_0(980)$ and $a_0^0(980)$ is still in controversy. They have been described in the literature as candidates for quark-antiquark, four quarks, $K\bar{K}$ molecule, hybrid, and so on. Nowadays, the study of their nature has become a central problem in the light hadron spectroscopy.

The $a_0^0(980)$-$f_0(980)$ mixing was first suggested theoretically in Ref. [1] in the late 1970s. Its mixing intensity is expected to shed important light on the nature of these two resonances, and has hence been studied extensively on its different aspects and possible manifestations in various reactions [2–14]. But unfortunately no firm experimental determination on this quantity is available yet.

There is only one experiment which gives the value of this mixing of $|\xi| = (8\pm3)\%$ [2] in the reaction $pp \to p_s(\eta\pi^0)p_f$. The result is based on the data [15] of the $a_0^0(980)$ central production in the reaction $pp \to p_s(\eta\pi^0)p_f$ and assumes that the $a_0^0(980)$ peak comes from the $a_0^0(980)$-$f_0(980)$ mixing. However, there are two problems with the assumption. Firstly, the experimental justification of such an assumption requires measuring the reaction $pp \to p_s(\eta\pi^0)p_f$ at a much higher energy to exclude a possible effect of the secondary Regge trajectories, for which the $\eta\pi^0$ production is not forbidden by G parity [2–4]. Secondly, the width of $a_0^0(980)$ peak in the $\eta\pi^0$ invariant mass spectrum is found to be 72 ± 16 MeV similar to that of the $a_0^-(980)$ peak, 61 ± 19 MeV, in the $\eta\pi^-$ invariant mass spectrum from the WA102 experiment [15]. This indicates that the $a_0^0(980)$ peak from WA102 experiment is unlikely mainly coming from the $a_0^0(980)$-$f_0(980)$ mixing mechanism.

Obviously, more solid and precise measurements on this quantity are needed, such as by polarized target experiment in $\pi^-p \to \eta\pi^0 n$ [3], and $dd \to \alpha\eta\pi^0$ reaction from WASA at COSY [12].

In all these previous proposals, the $f_0(980)$ is produced first, then transits to the $a_0^0(980)$ by the $a_0^0(980)$-$f_0(980)$ mixing , i.e., $f_0(980) \to a_0^0(980)$ transition. In this proceeding, we investigate in detail the difference between $a_0^0(980) \to f_0(980)$ and $f_0(980) \to a_0^0(980)$ transitions. We define two mixing intensities ξ_{fa} and ξ_{af} for $f_0(980) \to a_0^0(980)$ and $a_0^0(980) \to f_0(980)$, respectively. It shows that ξ_{fa} has more dependence on the parameters of $a_0^0(980)$ from Eq. (7), while the ξ_{af} has more dependence on the

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1) E-mail: wujiajun@mail.ihep.ac.cn

2) E-mail: zhaoq@mail.ihep.ac.cn

3) E-mail: zoubs@mail.ihep.ac.cn

parameters of $f_0(980)$ from Eq. (8). For this reason, we consider two reactions to measure ξ_{fa} and ξ_{af}.

The first reaction which we proposal to study ξ_{fa} is $J/\psi \to \phi\ a_0^0(980) \to \phi\eta\pi^0$. This reaction is an isospin breaking process with initial state of isospin 0 and final state of isospin 1. It can occur through the isospin breaking $f_0(980) \to a_0^0(980)$ by $J/\psi \to \phi f_0(980) \to \phi a_0^0(980)$. The decay $J/\psi \to \phi f_0(980)$ has already been clearly observed in $J/\psi \to \phi\ \pi^+\pi^-$ by BES II experiment [16]. However, it should be noted that besides the $a_0^0(980)$-$f_0(980)$ mixing mechanism, the $J/\psi \to \gamma^* \to \phi\ a_0^0(980)$ and $J/\psi \to K^*\bar{K}+c.c. \to \phi\ a_0^0(980)$ can also contribute to the $J/\psi \to \phi\ a_0^0(980)$ final state. So we need to estimate the relative strength of these mechanisms to see whether ξ_{fa} can be reliably extracted.

The second reaction is $\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi\pi$, from which we can study ξ_{af}. An experimental study of the $\chi_{c1} \to \pi^+\pi^-\eta$ reaction is reported by CLEO Collaboration [17], where the $a_0^{\pm}(980)$ resonances are clearly showing up and dominant. From isospin symmetry, the $\chi_{c1} \to \pi^0 a_0^0(980)$ should be produced with the same rate as $\chi_{c1} \to \pi^{\pm} a_0^{\mp}(980)$ at BES. This may provide an ideal place for studying the $a_0^0(980)$-$f_0(980)$ mixing in the $a_0^0(980) \to f_0(980)$ transition.

BES II did not give any information of these two reactions because of poor performance for measuring multi-photon final states and small event sample. With $10^9/J/\psi$ and 3×10^8 χ_{c1} events expected at the BEPC II with much improved BESIII detector, the measurements of the $J/\psi \to \phi\eta\pi^0$ and $\chi_{c1} \to \pi^0\pi\pi$ reactions are possible. From our estimation, more than 600 and 300 events can be reconstructed by the BESIII detector in the narrow peak around the mass of 990 MeV in the $\pi\eta$ and $\pi\pi$ invariant mass spectra, respectively.

In Sect. 2, we give a brief review of the theory for the $a_0^0(980)$-$f_0(980)$ mixing term. In the Sect. 3, we define two mixing intensities and investigate their differences. In Sects. 4 and 5 we calculate the decay rate for $J/\psi \to \phi\eta\pi^0$ and $\chi_{c1} \to \pi^0\pi\pi$, respectively. Finally we give a summary in Sect. 6.

2 The mixing amplitude

The basic theory for the $a_0^0(980)$-$f_0(980)$ mixing was proposed by Achasov and collaborators [1]. The nearly degenerate $a_0^0(980)$ (isospin 1) and $f_0(980)$ (isospin 0) both can decay into $K\bar{K}$. Due to isospin breaking effect, the charged and neutral kaon thresholds are different by about 8 MeV. Between the charged and neutral kaon thresholds the leading term to the $a_0^0(980)$-$f_0(980)$ mixing amplitude is dominated by the unitary cuts of the intermediate two-kaon system and proportional to the difference of phase spaces of the charged and neutral kaon systems.

Considering the $a_0^0(980)$-$f_0(980)$ mixing, the propagator of $a_0^0(980)/f_0(980)$ can be expressed as [4]:

$$G=\frac{1}{D_f D_a-|D_{af}|^2}\begin{pmatrix} D_a & D_{af} \\ D_{af} & D_f \end{pmatrix}, \qquad (1)$$

where D_a and D_f are the denominators for the usual propagators of $a_0^0(980)$ and $f_0(980)$, respectively:

$$D_a=m_a^2-s-i\sqrt{s}[\Gamma^a_{\eta\pi}(s)+\Gamma^a_{K\bar{K}}(s)], \qquad (2)$$

$$D_f=m_f^2-s-i\sqrt{s}[\Gamma^f_{\pi\pi}(s)+\Gamma^f_{K\bar{K}}(s)], \qquad (3)$$

$$\Gamma^a_{bc}(s)=\frac{g^2_{abc}}{16\pi\sqrt{s}}\rho_{bc}(s), \qquad (4)$$

$$\rho_{bc}(s)=\sqrt{\left(1-\frac{(m_b-m_c)^2}{s}\right)\left(1-\frac{(m_b+m_c)^2}{s}\right)}. \qquad (5)$$

The D_{af} is the mixing term. From Ref. [1, 3], the mixing due to $K\bar{K}$ loops gives

$$D_{af,\ K\overline{K}} = \frac{g_{a_0^0(980)K^+K^-}\,g_{f_0(980)K^+K^-}}{16\pi}\times \{i[\rho_{K^+K^-}(s)-\rho_{K^0\bar{K}^0}(s)]- \mathcal{O}(\rho^2_{K^+K^-}(s)-\rho^2_{K^0\bar{K}^0}(s))\}. \qquad (6)$$

Since the mixing mainly comes from the $K\bar{K}$ loops, we have $D_{af}\approx D_{af,K\bar{K}}$, and this is the amplitude of $a_0^0(980)$-$f_0(980)$ mixing .

In Eq. (6), the D_{af} becomes large only when the $\sqrt{s}$ is between the $2M_{K^+}$ and $2M_{K^0}$. Therefore, it is a narrow peak with a width of about 8 MeV.

3 Two mixing intensities

3.1 Two types of reactions

Two types of reactions can be used to study this mixing: $X \to f_0(980)\ Y \to a_0^0(980) Y \to \pi^0\eta Y$ and $X \to a_0^0(980)\ Y \to f_0(980)\ Y \to \pi\pi Y$.

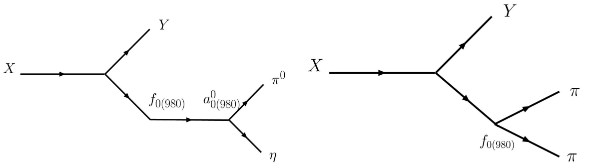

Fig. 1. The Feynman diagrams for $\mathrm{X} \to \mathrm{Yf_0(980)} \to \mathrm{Ya_0^0(980)} \to \mathrm{Y}\pi^0\eta$ (left) and $\mathrm{X} \to \mathrm{Yf_0(980)} \to \mathrm{Y}\pi\pi$ (right).

For the reaction $\mathrm{X} \to \mathrm{Yf_0(980)} \to \mathrm{Ya_0^0(980)} \to \mathrm{Y}\pi^0\eta$ as shown in the Fig. 1 (left), the influence of various X and Y on the $\mathrm{a_0^0(980)}$-$\mathrm{f_0(980)}$ mixing can be removed by its comparison with the corresponding reaction $\mathrm{X} \to \mathrm{Yf_0(980)} \to \mathrm{Y}\pi\pi$ as shown in Fig. 1 (right). We define the mixing intensity ξ_{fa} for the $\mathrm{f_0(980)} \to \mathrm{a_0^0(980)}$ transition as the following

$$\xi_{\mathrm{fa}}(s) = \frac{\mathrm{d}\Gamma_{\mathrm{X}\to\mathrm{f_0(980)Y}\to\mathrm{a_0^0(980)Y}\to\pi^0\eta\mathrm{Y}}(s)}{\mathrm{d}\Gamma_{\mathrm{X}\to\mathrm{Yf_0(980)}\to\mathrm{Y}\pi\pi(\mathrm{s})}}$$

$$= \frac{|D_{\mathrm{af}}|^2}{\Gamma^{\mathrm{f}}_{\pi\pi}\Gamma^{\mathrm{a}}_{\pi\eta}}\left(\frac{\Gamma^{\mathrm{a}}_{\pi\eta}}{|D_{\mathrm{a}}|}\right)^2. \quad (7)$$

where s is the invariant mass squared of the final state mesons. Similarly, for $\mathrm{X} \to \mathrm{Ya_0^0(980)} \to \mathrm{Yf_0(980)} \to \mathrm{Y}\pi\pi$, we can define the mixing intensity ξ_{af} for the $\mathrm{a_0^0(980)} \to \mathrm{f_0(980)}$ transition as the following

$$\xi_{\mathrm{af}}(s) = \frac{\mathrm{d}\Gamma_{\mathrm{X}\to\mathrm{Ya_0^0(980)}\to\mathrm{Yf_0(980)}\to\mathrm{Y}\pi\pi(\mathrm{s})}}{\mathrm{d}\Gamma_{\mathrm{X}\to\mathrm{Ya_0^0(980)}\to\mathrm{Y}\pi^0\eta(\mathrm{s})}}$$

$$= \frac{|D_{\mathrm{af}}|^2}{\Gamma^{\mathrm{f}}_{\pi\pi}\Gamma^{\mathrm{a}}_{\pi\eta}}\left(\frac{\Gamma^{\mathrm{f}}_{\pi\pi}}{|D_{\mathrm{f}}|}\right)^2. \quad (8)$$

With Eqs. (1-6), one can get the $\xi_{\mathrm{fa}}(\mathrm{s})$ and $\xi_{\mathrm{af}}(\mathrm{s})$ in details. The detailed calculation of the two intensities can be found in Ref. [18].

3.2 Predictions of ξ_{af} and ξ_{fa} from various models and experiment information

In the above equations, the mixing intensities $\xi_{\mathrm{af(fa)}}$ depend on $g_{\mathrm{a_0^0(980)K^+K^-}}$, $g_{\mathrm{f_0(980)K^+K^-}}$, $g_{\mathrm{a_0^0(980)}\pi^0\eta}$, $g_{\mathrm{f_0(980)}\pi^0\pi^0}$, m_{f} and m_{a}. Various models for two mesons give different predictions for the coupling constants and masses [14, 19–21] as listed in Table. 1 by No. A-D. There have also been some experimental measurements on these couplings and masses [16, 22–28] as listed by No. E-H. The dependence of

Table 1. Meson masses $m_{\mathrm{a_0^0(980)}}$ (MeV), $\mathrm{m_{f_0(980)}}$ (MeV) and coupling constants $g_{\mathrm{a_0}\pi\eta}$ (GeV), $g_{\mathrm{a_0K^+K^-}}$ (GeV), $g_{\mathrm{f_0K^+K^-}}$ (GeV) and $g_{\mathrm{f_0}\pi^0\pi^0}$ (GeV) from various models (A-D) and experimental measurements (E-H), and calculated values of ξ_{af} and ξ_{fa} at $\sqrt{s} = 991.4$ MeV by Eqs. (7,8).

No.	model or experiment	m_{a}	$g_{\mathrm{a_0}\pi\eta}$	$g_{\mathrm{a_0K^+K^-}}$	m_{f}	$g_{\mathrm{f_0}\pi^0\pi^0}$	$g_{\mathrm{f_0K^+K^-}}$	$\lvert\xi_{\mathrm{fa}}\rvert$	$\lvert\xi_{\mathrm{af}}\rvert$
A	$\mathrm{q\bar{q}}$ model [14]	983	2.03	1.27	975	0.64	1.80	0.023	0.010
B	$\mathrm{q^2\bar{q}^2}$ model [14]	983	4.57	5.37	975	1.90	5.37	0.068	0.062
C	$\mathrm{K\bar{K}}$ model [19, 20, 29]	980	1.74	2.74	980	0.65	2.74	0.21	0.15
D	$\mathrm{q\bar{q}g}$ model [21]	980	2.52	1.97	975	1.54	1.70	0.005	0.006
E	SND [22, 23]	995	3.11	4.20	969.8	1.84	5.57	0.088	0.089
F	KLOE [24, 25]	984.8	3.02	2.24	973	2.09	5.92	0.034	0.025
G	BNL [26]	1001	2.47	1.67	953.5 [27]	1.36 [27]	3.26 [27]	0.019	0.014
H	CB [28]	999	3.33	2.54	965 [16]	1.66 [16]	4.18 [16]	0.027	0.023

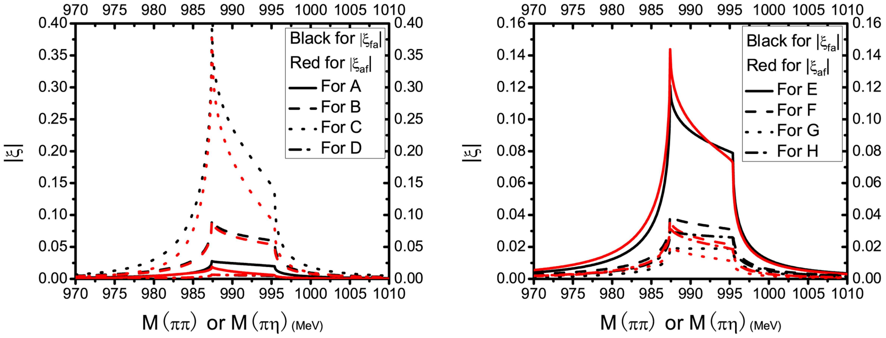

Fig. 2. Predictions for the $\mathrm{a_0^0(980)}$-$\mathrm{f_0(980)}$ mixing intensities ξ_{af} and ξ_{fa} vs two-meson invariant mass from Table 1 A-D (left) and Table 1 E-H (right).

$a_0^0(980)$-$f_0(980)$ mixing intensities ξ_{af} and ξ_{fa} vs the two-meson invariant mass is shown in Fig. (2). It turns out that the results from indirect experiments suffer from large uncertainties, hence they cannot distinguish the model differences. A direct measurement of the mixing intensities thus would be important.

3.3 Discuss the difference between $\boldsymbol{\xi_{af}}$ and $\boldsymbol{\xi_{fa}}$

It shows that ξ_{af} has more dependence on $f_0(980)$, while the ξ_{fa} has more dependence on $a_0^0(980)$. This is due to two reasons. The first one is that the values of ξ_{af} and ξ_{fa} are very different as shown in Fig. 2 when we use the same parameter sets. Secondly, the two sets of parameters give almost identical ξ_{fa} but very different ξ_{af}. For example, We give the two sets of parameters in Table. 2. Set No. 1 is close to the SND values in Table. 1. Set No. 2 changes the not-well-measured g_{aKK} and g_{fKK} in their experimental uncertainties. We plot the corresponding diagrams of ξ_{af} and ξ_{fa} vs two-meson invariant mass M_2 as shown in the Fig. 3. The two sets of parameters give almost identical ξ_{fa} but very different ξ_{af}.

Table 2. Two typical parameter sets. The units of parameters are the same as Table. 1.

No.	m_a	$g_{a_0\pi\eta}$	$g_{a_0K^+K^-}$	m_f	$g_{f_0\pi^0\pi^0}$	$g_{f_0K^+K^-}$
1	980	3.2	4.2	980	1.5	4.0
2	980	3.2	3.0	980	1.5	5.12

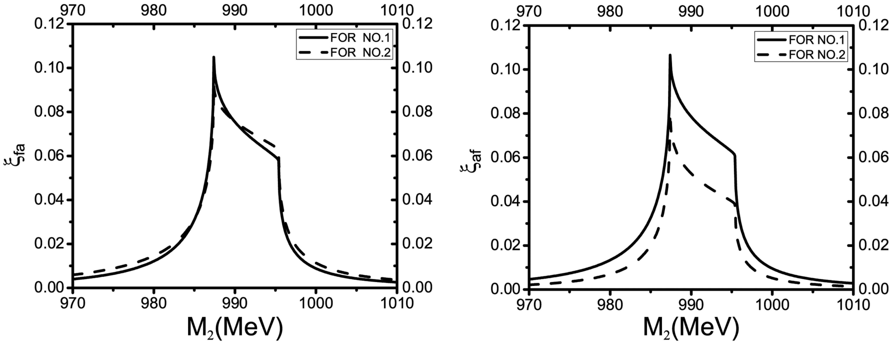

Fig. 3. Diagrams of the ξ_{af} and ξ_{fa} vs two-meson invariant mass with parameter sets No. 1 and No. 2 of Table. 2.

The above analysis suggests significant differences between ξ_{af} and ξ_{fa}. Both mixing intensities depend on six parameters: m_f, m_a, $g_{a_0^0(980)K^+K^-}$, $g_{f_0(980)K^+K^-}$, $g_{a_0^0(980)\pi^0\eta}$, $g_{f_0(980)\pi^0\pi^0}$, which are all important for understanding the nature of the $a_0^0(980)$ and $f_0(980)$ mesons, but have not been well determined yet. Therefore, to measure ξ_{af} and ξ_{fa} will be very useful for pinning down these parameters.

4 Possibility of measuring $\boldsymbol{\xi_{fa}}$ from $J/\psi \to \phi\eta\pi^0$

The reactions $J/\psi \to \omega a_0^0(980)$ and $J/\psi \to \phi a_0^0(980)$ were suggested by Close and Kirk [2] who predicted branching ratios of $O(10^{-5})$. BESII experiments [16, 30] have studied the J/ψ decays into $\phi f_0(980)$ and $\omega f_0(980)$. Although these two channels are found to have similar branching ratios, the $f_0(980)$ peak appears outstanding in the $\pi\pi$ invariant mass spectrum in $J/\psi \to \phi\ \pi^+\pi^-$ [16], but is much buried by other components in $J/\psi \to \omega\ \pi^+\pi^-$ [30]. Therefore, we choose $J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0$ to measure ξ_{fa}. Apart from the contribution from the $a_0^0(980)$-$f_0(980)$ mixing mechanism, we also examine those from two background reactions $J/\psi \to \gamma^* \to \phi\ a_0^0(980)$ and $J/\psi \to K^*\bar{K}$+c.c. $\to \phi\ a_0^0(980)$. The corresponding Feynman diagrams are shown in Fig. 4.

The invariant amplitudes for these three channels can be written as

$$\mathcal{M}_{\rm mix} = g_{J/\psi f_0(980)\phi}\varepsilon^{\mu}_{J/\psi}\varepsilon^{*}_{\phi\mu}\frac{D_{af}}{D_f D_a}g_{a_0^0(980)\pi^0\eta}, \tag{9}$$

$$\mathcal{M}_{\rm EM} = g_{J/\psi\gamma}\varepsilon^{\mu}_{J/\psi}\frac{-{\rm i}g_{\mu\nu}}{k^2+{\rm i}\varepsilon}g_{\gamma\phi a(k^2)}\varepsilon^{\nu}_{\phi(\lambda)}, \tag{10}$$

$$\begin{aligned}\mathcal{M}_{K^*\bar{K}} = {\rm i}\sum_{K}\int\frac{{\rm d}^4p_{2(K)}}{(2\pi)^4}\left(\frac{{\rm i}g_{\phi}\epsilon^{\mu\nu\alpha\beta}p_{\phi\mu}\varepsilon_{\phi\nu}p_{2(K)\alpha}}{m_{\phi}}\right)\times\\ \left(-g_{\beta\lambda}+\frac{p_{1(K)\beta}p_{1(K)\lambda}}{p^2_{1(K)}}\right)\times\\ \left(\frac{{\rm i}g_{\psi}\epsilon^{\lambda\sigma\tau\delta}p_{\psi\sigma}\varepsilon^{*}_{\psi\tau}p_{3(\bar{K})\delta}}{M_{J/\psi}}\right)\times\\ ({\rm i}g_a)F(p^2_{2(K)})G_{(K*1)}G_{(K2)}G_{(K3)},\end{aligned} \tag{11}$$

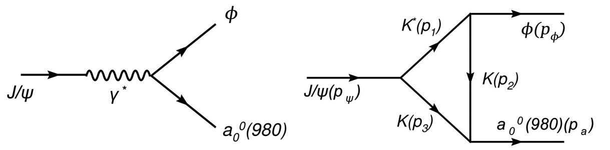

Fig. 4. Feynman diagrams for reactions J/ψ → $\gamma^* \rightarrow$ ϕ$a_0^0(980)$ (left) and J/ψ → $K^*\bar{K}$ → ϕ $a_0^0(980)$ (right).

where $G_{(K*1)}$, $G_{(K2)}$ and $G_{(K3)}$ are the propagators of K^* and two kaons in Fig. 4. In the calculation we only consider that K_2 is off-shell, while K∗ and K_3 are both on-shell [31]. A form factor is introduced for the off-shell kaon. Two different types of form factors, monopole and dipole form factor, are adopted as follows:

$$F(p_2^2) = \frac{\Lambda_K^2 - m_{2(K)}^2}{\Lambda_K^2 - p_{2(K)}^2}, \qquad (12)$$

$$F(p_2^2) = \left(\frac{\Lambda_K^2 - m_{2(K)}^2}{\Lambda_K^2 - p_{2(K)}^2}\right)^2, \qquad (13)$$

where Λ_K is the cut-off energy.

We adopt parameter set No. H in Table. 1 to calculate these channels. The details of calculation can be found in Ref. [32]. The the branching ratios for J/ψ → $K^*\bar{K}$ + c.c. → ϕ $a_0^0(980)$ with the monopole and dipole form factors are listed in Table. 3. For $\Lambda_K = \infty$, it is equivalent to without form factor, i.e., $F(p_2^2) = 1$. We also draw the πη invariant mass spectrum for this channel in Fig. 5 (right: line B and C). It shows that they are both broad peaks with $\Gamma = 70$ MeV. The predicted branching ratio for J/ψ → γ^* → ϕ $a_0^0(980)$ is $O(10^{-7})$ with the broad width of 70 MeV. This is much smaller than the contribution from $a_0^0(980)$-$f_0(980)$ mixing and $K^*\bar{K}$ loop, and almost close to zero in Fig. 5.

Table 3. Branching ratios of J/ψ → $K^*\bar{K}$ → ϕ $a_0^0(980)$ with typical Λ_K cut-off parameters for the monopole and dipole form factors.

Λ_K/GeV	monopole F.F.	dipole F.F.
1.0	1.5×10^{-6}	0.4×10^{-6}
1.5	3.8×10^{-6}	2.1×10^{-6}
2.0	5.7×10^{-6}	4.6×10^{-6}
∞	12.3×10^{-6}	12.3×10^{-6}

The predicted branching ratio due to the $a_0^0(980)$-$f_0(980)$ mixing is 2.0×10^{-6}. We also present the $\eta\pi^0$ invariant mass spectrum from $a_0^0(980)$-$f_0(980)$ mixing in J/ψ → ϕ $\eta\pi^0$ in Fig. 5 (right: line A). A narrow outstanding peak with a width about 8 MeV is highlighted, which is much narrower than the usual width (50 ∼ 100 MeV) of $a_0^0(980)$. With 10^9 J/ψ events and a detection efficiency about 30%, more than 600 events would be observed in this channel and mostly accumulated in the narrow gap of between 987.4 and 995.4 MeV. In addition, we plot the observed $f_0(980)$ contribution to J/ψ → ϕ ππ [16] in Fig. 5 (left) with an integration over $m_{\pi\pi}$ equal to the measured branching ratio $(5.4\pm0.9)\times10^{-4}$ for this channel. These will allow us to extract the ξ_{fa} easily.

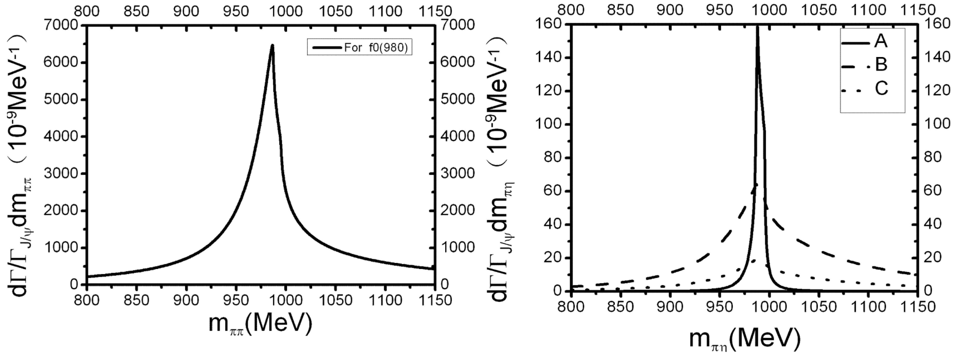

Fig. 5. ππ invariant mass spectrum for J/ψ → ϕ $f_0(980)$ → ϕ ππ [16] (left) and the πη invariant mass spectrum for J/ψ → ϕ ηπ via a_0^0-f_0 mixing (right: line A). Contributions from $K^*\bar{K}$ rescattering (line B for without form factor; C for monopole form factor with $\Lambda_K = 1.5$ GeV) are also shown.

As shown by Table. 3 and Fig. 5, the branching ratio given by the K^*K loops is 12.3×10^{-6} without form factor, which is almost 6 times as that from $a_0^0(980)$-$f_0(980)$ mixing . Although the integration of line B is about 5 times of line A, the peak of line A is still more than a factor of 2 over the peak of line B. The contribution from the K^*K loops gives a much broader distribution in the πη invariant mass spectrum than that from the $a_0^0(980)$-$f_0(980)$ mixing in Fig. 5 (right). So we conclude that the background cannot influence the observation of this predominant narrow peak due to the $a_0^0(980)$-$f_0(980)$ mixing . As a consequence, the ξ_{fa} could be unambiguously and precisely measured at BESIII.

5 Possibility of measuring ξ_{af} from $\chi_{c1}\to\pi^0\pi\pi$

The Feynman diagram for $\chi_{c1}\to\pi^0a_0^0(980)\to\pi^0f_0(980)\to\pi^0\pi\pi$ is shown in Fig. 6. The invariant amplitude for this reaction is

$$\mathcal{M}_{\chi_{c1}\to\pi\pi\pi^0} = g_{\chi_{c1}a_0^0(980)\pi^0}\varepsilon^{\mu}_{\chi_{c1}}(p_{\pi^0}-p_{f_0(980)})_\mu \times \frac{D_{af}}{D_f D_a}\sqrt{3}g_{f_0(980)\pi^0\pi^0}. \qquad (14)$$

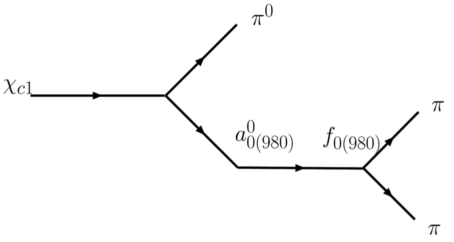

Fig. 6. Feynman diagram for $\chi_{c1}\to\pi^0a_0^0(980)\to\pi^0f_0(980)\to\pi^0\pi\pi$.

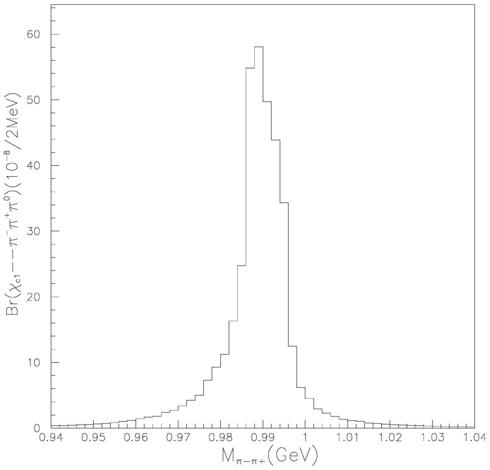

Fig. 7. $\pi^+\pi^-$ invariant mass spectrum for $\chi_{c1}\to\pi^0a_0^0(980)\to\pi^0f_0(980)\to\pi^0\pi^+\pi^-$.

We also apply the parameters in set No. H in Table. 1. The detailed calculation can be found in Ref. [18]. The branching ratio of this reaction is 4.6×10^{-6} and the invariant mass spectrum of $\pi^+\pi^-$ is shown in Fig. 7, where a narrow peak with a width of about 8 MeV is also seen.

At the BEPCII with BESIII detector, about 3.2×10^9 ψ_{2s} events and hence about 2.8×10^8 χ_{c1} events can be collected per year with the branching ratio $\mathcal{B}(\psi_{2s}\to\gamma\chi_{c1})\simeq0.088$ [33]. With $\mathcal{B}(\chi_{c1}\to\pi^0\pi\pi)=4.6\times10^{-6}$, more than 300 events can be reconstructed with the reconstruction efficiency of 30%. Since all these events should gather in a narrow region of about 8 MeV around 990 MeV in the $\pi^+\pi^-$ invariant mass spectrum, the narrow peak should be easily observed. Hence, the ξ_{af} is also accessible in this reaction.

6 Summary

In this paper, we present a brief review of the theory for the $a_0^0(980)$-$f_0(980)$ mixing term and investigate in detail the difference between $a_0^0(980)\to f_0(980)$ and $f_0(980)\to a_0^0(980)$ transitions. We defined two corresponding mixing intensities ξ_{af} and ξ_{fa}. It is found that these two mixing intensities are both important, so to measure $a_0^0(980)\to f_0(980)$ and $f_0(980)\to a_0^0(980)$ transition are both necessary. Note that different models give very different predictions for these two mixing intensities, and the results from indirect experiments suffer from large uncertainties. We hence propose to do a direct measurement of these quantities in $J/\psi\to\phi\eta\pi^0$ and $\chi_{c1}\to\pi^0\pi\pi$ at BESIII. It shows that a narrow peak about 8 MeV at about 990 MeV in the $\pi\pi(\eta)$ invariant mass spectra will be produced by the mixing mechanism. The background contributions from the EM transitions and rescatterings would not submerge the narrow signal which makes it possible to directly measure the mixing intensities in experiment. With 10^9 J/ψ and 3×10^8 χ_{c1} events and a detection efficiency about 30% for the final states expected at BESIII, about 600 events for $J/\psi\to\phi f_0(980)\to\phi a_0^0(980)\to\phi\eta\pi^0$ reaction, and 300 events for $\chi_{c1}\to\pi^0a_0^0(980)\to\pi^0f_0(980)\to\pi^0\pi\pi$ can be expected.

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