Probabilistic teleportation via rotation operating and computation basis measuring*

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A scheme for teleporting an unknown quantum state via a partially entangled particle pair is proposed. We show that teleportation can be successful realized with unit fidelity but less than unit probability if the sender performs a rotational operation on the teleported qubit and then performs computation basis measurements on the teleported particle and her half of the entangled particle pair respectively. The probability of successful teleportation is determined by both Schmidt coefficients of the entangled pair. This method can be generalized to the teleportation of an arbitrary and unknown multi-qubit state.

Keywords: teleportation, entangled state, rotational operation, computation basis measurement

OCIS: 270.0270

Quantum teleportation, first proposed by Bennett *et al.* [1] and experimentally realized by Bouwmeester *et al.* [2], is a technique for moving quantum state around, even in the absence of a quantum communications channel linking the sender (Alice) of the quantum state to the recipient (Bob). Here's how quantum teleportation works: Alice interacts the teleported qubit $|y\rangle$ with her half of the EPR pair, and then measures the two qubits in her possession in Bell basis, obtaining one of four uniformly distributed random two-bit classical results. She sends this information to Bob. Depending on Alice's classical message, Bob performs one of four operations on his half of the EPR pair. Amazingly, by doing this he can deterministically recover the original state $|y\rangle$! Considering that an entangled state may be partially entangled (due to some imperfection at the source), Li *et al.*[3] have investigated a partially entangled quantum channel. In their report, Alice performs a Bell measurement on her side while Bob performs a corresponding general evolution to reestablish the initial state with certain probability. Based on the probabilistic teleportation protocol in Ref. $[3]$, Lu *et al.*[4,5], Gu *et al.*[6], Ye *et al.*[7] and Chen[8] have presented some schemes for the probabilistic teleportation of unknown multi-particle states. We should mention that there have been another proposals to teleport an unknown state using a partially entangled state but using generalized measurement such as POVMs[9], non-maximally entangled measurements[10]. Also, there has been a qubit assisted conclusive teleportation process[11]. However, those schemes are different than ours as we will see below.

As an interface between the quantum and classical words, measurement is generally considered to be an irreversible operation, destroying quantum information and replacing it with classical information. But teleportation demonstrates that in certain carefully designed case this need not be true, as teleportation uses measurement to transfer states from one place to another. Though Bell measurement is an essential task of teleportation, it is very difficult to fully access it. It has been shown that Bell measurement cannot be performed perfectly with linear element[12,13], and this difficulty can be seen in some teleportation experiments[2]. The projective measurements in computational basis may be more efficient. Computation basis measurements have many nice properties. In particular, it is easy to experimentally realize. On the other hand, in Refs. $[3-8]$, the receiver must introduce an auxiliary qubit and operate a collective unitary transformation on his qubit and the auxiliary qubit in order to realize the probabilistic teleportation, which is also more complicated to experimentally realize. In this letter, we show, the same objective, teleportation an unknown qautum state via a partially entangled quantum channel with unit fidelity but less than unit probability, can be accomplished in a simpler manner if Alice performs a rotation about the *Y* axis on her teleported qubit and then measures the two qubits in her possession in computational basis respectively. We discuss various special cases from probabilistic to deterministic teleportation of unknown states. Further, we generalize the scheme to the teleportation of an arbitrary and unknown *N*-qubit state.

Consider that the sender Alice has an unknown quantum state labeled *A*. She wants to teleport the unknown state $|\psi\rangle_A$ to a receiver Bob who is at a remote station. The state $|\psi\rangle_A$ may be expressed as

$$|\psi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A , \tag{1}$$

where α and β are unknown amplitudes. Alice and Bob are supposed to shared a partially entangled state

$$|\varphi\rangle_{ab} = \cos\theta|00\rangle_{ab} + \sin\theta|11\rangle_{ab} , \tag{2}$$

the particles *a* and *b* belong to Alice and Bob, respectively. In the circuit, the input state $|\psi\rangle_0$ is

$$\begin{aligned} |\psi\rangle_0 &= |\psi\rangle_A|\varphi\rangle_{ab} \\ &= (\alpha\cos\theta|000\rangle + \alpha\sin\theta|011\rangle + \beta\cos\theta|100\rangle + \beta\sin\theta|111\rangle)_{Aab} . \end{aligned} \tag{3}$$

Alice sends her qubits *A* and *a* through a local CNOT gate $U_{CNOT}(A;a)$ (with the control being qubit *A*), and then sends the first qubit *A* through the $R_Y(\theta')$ gate

$$R_Y(\theta') = \begin{pmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{pmatrix} , \tag{4}$$

where $R_Y(\theta')$ corresponds to the rotation about the y axes. After this, the state (3) will be

$$\begin{aligned} |\psi\rangle_1 &= R_Y(\theta')\, U_{CNOT}(A;a)\, |\psi\rangle_0 \\ &= |0\rangle_a \big[\alpha\cos\theta(\cos\theta'|0\rangle + \sin\theta'|1\rangle)_A|0\rangle_b + \beta\sin\theta(-\sin\theta'|0\rangle + \cos\theta'|1\rangle)_A|1\rangle_b \big] \end{aligned}$$

$$+|1\rangle_a \left[a \sin q (\cos q'|0\rangle + \sin q'|1\rangle)_A |1\rangle_b + b \cos q (-\sin q'|0\rangle + \cos q'|1\rangle)_A |0\rangle_b \right]. \qquad (5)$$

This state may be re-written in the following way, simply by regrouping terms:

$$|y\rangle_1 = |00\rangle_{aA} Z_b (a \cos q \cos q'|0\rangle + b \sin q \sin q'|1\rangle)_b$$

$$+|10\rangle_{aA} (a \cos q \sin q'|0\rangle + b \sin q \cos q'|1\rangle)_b$$

$$-|01\rangle_{aA} iY_b (a \sin q \cos q'|0\rangle + b \cos q \sin q'|1\rangle)_b$$

$$+|11\rangle_{aA} X_b (a \sin q \sin q'|0\rangle + b \cos q \cos q'|1\rangle)_b . \qquad (6)$$

Then Alice performs the computation basis measurements on her qubits a and A, respectively, and the qubits a and A are subsequently discarded. From the previous expression we can read off Bob's post-measurement state, given the results of Alice's measurement:

$$|00\rangle_{aA} \mapsto Z_b (a \cos q \cos q'|0\rangle + b \sin q \sin q'|1\rangle)_b = Z_b |y(00)\rangle_b , \qquad (7)$$

$$|10\rangle_{aA} \mapsto (a \sin q \cos q'|0\rangle + b \sin q \cos q'|1\rangle)_b = |y(10)\rangle_b , \qquad (8)$$

$$|01\rangle_{aA} \mapsto -iY_b (a \sin q \cos q'|1\rangle + b \sin q \cos q'|0\rangle)_b = -iY_b |y(01)\rangle_b , \qquad (9)$$

$$|11\rangle_{aA} \mapsto X_b (a \sin q \sin q'|0\rangle + b \cos q \cos q'|1\rangle)_b = X_b |y(11)\rangle_b . \qquad (10)$$

Here, we write X ,Y, and Z instead of s_x, s_y, and s_z; X_b stands for the operator X acting on the qubit b, and so on. Depending on Alice's measurement outcomes, Bob's qubit will end up in one of these four possible states. Of course, to know which state it is in, Bob must be told the result of Alice's measurement--which is two classical bits (two c-bits) of information. Once Bob has learned the measurement outcome, Bob can "fix up" his state by applying the appropriate quantum gate. We now wish to have faithful transportation with nonzero probability. Let us consider several scenarios involving various choices of the parameter q', given the value of q. Choice is at the disposal of Alice.

(1) Complete teleportation: Let us choose $q' = q = p/4$, then $R_Y(\frac{p}{4}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$=$HZ$, and faithful teleportation is possible with unit fidelity and unit probability.

(2) Probabilistic teleportation-- this is one of the main results of our letter: If we make the choice $q' = q \neq p/4$, then for any of these choices, reliable teleportation is possible for only two out of four possible results of the measurement. For example, in the case where the measurement yields $|00\rangle_{Aa}$, the outcome state of particle *b* is the state (7), which cannot be rotated back to the desired state without having any knowledge of the state parameters a and b. The teleportation fails. If the measurement is $|01\rangle_{Aa}$, the state of the particle *b* collapses to state (9), a Pauli operator Y is performed on qubit *b*, then teleportation can be achieved. Synthesizing all conditions (two kinds), we obtain the probability of successful teleportation is $2\sin^2 q \cdot \cos^2 q$. This scheme consumes a two-particle partially entangled state $|j\rangle_{ab}$ and two c-bits from Alice to Bob. The amount of entanglement of the state $|j\rangle_{ab}$ is given by its entropy of entanglement

$$E(|j\rangle_{ab}) = -\cos^2 q \log_2(\cos^2 q) - \sin^2 q \log_2(\sin^2 q) . \qquad (11)$$

That is, the amount of entanglement required to implement the probabilistic teleportation is given by Eq.(11). In this scheme, single qubit state needs only two particles to teleport compared to Li's [3] three particles. The price to be paid, however, is that our scheme succeeds with $2\sin^2 q \cdot \cos^2 q$ probability. In Li's [3] scheme, the probability of successful teleportation is $2\sin^2 q$ $(|\sin q| < |\cos q|)$.

(3) No teleportation: If the value of q' is not related with that of q, then teleportation is not possible with unit fidelity.

This method can be generalized to the teleportation of a multi-particle system. Consider a *N*-particle system ($A_1, A_2, \cdots, A_N$) at Alice's side with the state

$$\begin{aligned} |y\rangle_A &= a_1|00\cdots00\rangle_{A_1\cdots A_N} + a_2|00\cdots01\rangle_{A_1\cdots A_N} + \cdots + a_{2^N}|11\cdots11\rangle_{A_1\cdots A_N} \\ &= \sum_{i=1}^{2^N} a_i \prod_{j=1}^{N} |u_{ij}\rangle_{A_j} , \end{aligned} \qquad (11)$$

where $\prod_{j=1}^{N}|u_{ij}\rangle_{A_j} = |u_{i1}\rangle_{A_1}|u_{i2}\rangle_{A_2}\cdots|u_{iN}\rangle_{A_N}$ is the *i*th basis vector in the 2^N dimensional space; $u_{ij} \in \{0,1\}$; $|0\rangle_j$ and $|1\rangle_j$ stand for two orthogonal states of the *j*th particle. Without loss of generality, the quantum channels between Alice and Bob are N independent entangled pairs with the state $\prod_{j=1}^{N}(\sin q_j|00\rangle+\cos q_j|11\rangle)_{a_j b_j}$ (any other pure quantum channel can be transformed to this by local operations). The particles $a_1, a_2, \cdots, a_N$ belong to Alice and particles $b_1, b_2, \cdots, b_N$ belong to Bob. The total initial state can be written as

$$|y\rangle=|y\rangle_A \otimes \prod_{j=1}^{N}(\sin q_j|00\rangle+\cos q_j|11\rangle)_{a_j b_j} . \tag{13}$$

Their goal is that Bob ends up with the state

$$\begin{aligned}|y\rangle_{b_1,b_2,\cdots,b_N} &= a_1|00\cdots00\rangle_{b_1\cdots b_N}+a_2|00\cdots01\rangle_{b_1\cdots b_N}+\cdots+a_{2^N}|11\cdots11\rangle_{b_1\cdots b_N}\\ &=\sum_{i=1}^{2^N} a_i \prod_{j=1}^{N}|u_{ij}\rangle_{b_j} .\end{aligned} \tag{14}$$

To do this, it is necessary for Alice and Bob to perform N manipulation in the same manner as in the case of mono-qubit teleportation. In the ith step, Alice applies a local quantum CNOT gate on qubits A_i and a_i with A_i as control, then sends the qubit A_i through a $R_{YA_i}(q_i)$ gate. This is then followed by computation basis measurements on qubit A_i and a_i respectively. Qubit A_i and a_i are subsequently discarded. According to Alice's measurement results, Bob performs corresponding unitary transformation on the qubit b_i. Then the ith manipulation on the particles (A_i, a_i), as well as particle b_i, ends. After the N-time manipulation, the final state of the N particles ($b_1, b_2, \cdots, b_N$) belonging Bob is given by

$$\begin{aligned}|y\rangle_B &= R_{a_N A_N}\cdots R_{a_2 A_2} R_{a_1 A_1}\langle u_{a_N}|\langle u_{A_N}|\cdots\langle u_{a_2}|\langle u_{A_2}|\langle u_{a_1}|\langle u_{A_1}| R_{YA_N}(q_N)\cdots R_{YA_2}(q_2)R_{YA_1}(q_1)\\ &\quad U_{CNOT}(A_N;a_N)\cdots U_{CNOT}(A_2;a_2)U_{CNOT}(A_1;a_1)|y\rangle\\ &=\prod_{i=1}^{N} R_{a_i A_i}\cdot\prod_{i=1}^{N}\langle u_{a_i}|\langle u_{A_i}|\cdot\prod_{i=1}^{N} R_{YA_i}(q_i)\cdot\prod_{I=1}^{N} U_{CNOT}(A_i;a_i)|y\rangle ,\end{aligned} \tag{15}$$

where $U_{CNOT}(A_I;a_i)$ stands for the CNOT gate acting on the qubits A_i and a_i (with A_i as control and a_i as target); $R_{YA_i}(q_i)$ is the rotation (about Y axes) operation on qubit A_i; $|u_{A_i}\rangle$ ($|u_{a_i}\rangle$)=$|0\rangle_{A_i}, |1\rangle_{A_i}$ ($|0\rangle_{a_i}, |1\rangle_{a_i}$) is the computation basis in which the particle A_i (a_i) is measured; $R_{a_i A_i}$ is either the identity if $\left|u_{a_i}\right\rangle \left|u_{A_i}\right\rangle = |1\rangle_{a_i} \; |0\rangle_{A_i}$ or the single qubit Pauli operator $R_{01} = -iY_{b_i}$, $R_{00} = Z_{b_i}$ and $R_{11} = X_{b_i}$.

By calculation, Eq.(15) can reduce to the following expression

$$\prod_{i=1}^{N} \sin q_i \cos q_i \; (a_1|00\cdots00\rangle_{b_1\cdots b_N} + a_2|00\cdots01\rangle_{b_1\cdots b_N} + \cdots + a_{2^N}|11\cdots11\rangle_{b_1\cdots b_N})$$

$$= \prod_{i=1}^{N} \sin q_i \cos q_i \; (\sum_{i=1}^{2^N} a_i \prod_{j=1}^{N} \left|u_{ij}\right\rangle_{b_j}) \; . \qquad (16)$$

Comparing Eq.(16) with Eq.(14), we find that the probability of successful teleportation is $\prod_{i=1}^{N} \sin^2 q_i \cos^2 q_i$. Synthesizing all conditions, if $q_i \neq p/4$, the total successful probability is $2^N \prod_{i=1}^{N} \sin^2 q_i \cos^2 q_i$ (2^N kinds); if $q_i = p/4$, the successful probability is $P=1$ (2^{2N} kinds), faithful teleportation is possible with unit fidelity and unit probability.

In addition, in the same way Alice can teleport the *M*-particle information of *N* message particles to Bob by means of *M* independent entangled pairs $\prod_{j=1}^{M} (\sin q_j |00\rangle + \cos q_j |11\rangle)_{a_j b_j}$ ($M < N$).

In conclusion, we have proposed a scheme for teleporting an arbitrary quantum state via a partially entangled particle pair. We show that the teleportation can be realized with a unit fidelity and albeit with reduced successful probability if the sender performs a rotational operation on the teleported qubit and then performs computation basis measurements on the teleported particle and half of the entangled particle pair in her possession respectively. The manipulation is much simpler than the one shown in Refs. [3,9–11], and all of which are within the reach of current technology. The probability of successful teleportation is determined by both Schmidt coefficients of the entangled pair. We have generalized this method to teleport a arbitrary and unknown multi-particle state from Alice to Bob.

Finally, we should indicate that our scheme may also be generalized to teleported a arbitrary and unknown N-particle state to N distant users belonging to N different space domains by means of N independent entangled pairs between the sender and the N receivers. For an entangled N-particle system, the teleportation of the N-particle entangled state to N distant users presents a method to establish the multi-particle entangled in quantum network communication [14] and distributed quantum computer [15,16].

This project has been supported by the Natural Science Foundation of Guangdong Province, China (Grant No.020127) and the Natural Science Research Foundation of Education Department of Guangdong Province (Grant No.Z02069). L.Chen's e-mail address is chlibing@foshan.net.

References

[1] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W. K.Wootters, Phys. Rev. Lett. **70**, 1895 (1993)

[2] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H.Weinfuter, A. Zeilinger, Nature. **390**, 575 (1997)

[3] W.-L. Li, C.-F. Li, G.-C. Cuo, Phys. Rev. A **61**, 034301 (2000)

[4] H. Lu, Chin. Phys. Lett. **18**, 1004 (2001)

[5] H. Lu, G.-C. Guo, Phys.Lett. A **276**, 209 (2000)

[6] Y.-J. Gu, Y.-Z. Zheng, G.-C. Guo, Chin. Phys. Lett. **18**, 1543 (2001)

[7] L. Ye, C.-M. Yao, G.-C. Guo, Chin. Phys. **10**, 1001 (2001)

[8] L.-B. Chen, Chin. Phys. **11**, 999 (2002)

[9] T. Mor, quant-ph/9608005; T. Mor, P. Horodecki, quant-ph/9906039

[10] P. Agrawal, A.K. Pati, quant-ph/0210004

[11] S. Bandyopadhyay, Phys. Rev. A **59**, 012308 (2000)

[12] L. Vaidman, N. Yoran, Phys. Rev. A **59**, 116 (1999)

[13] N. Lǚkenhaus, J. Calsamiglia, K. A. Suominen, Phys. Rev. A **59**, 3295 (1999);

J. Calsamiglia, N. Lǚkenhaus, quant-ph/0007058

[14] A. Chefles, C.-R. Gilson, S.-M. Barnett, Phys. Rev. A **63**, 2314,(2001)

[15] A.Barenco, Contemp. Phys. **37**, 375,(1996)

[16] J. I. Cirac, A.Ekert, S. F. Huelga, C. Macchiavello, Phys. Rev. A **59**, 064294,(1999)

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