How are Income and Non-Income Factors Different in Promoting Happiness? An Answer to the Easterlin Paradox*

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Abstract

This paper develops a formal economic theory to explain the Easterlin paradox—average happiness levels do not increase as countries grow wealthier. The theory analyzes the different roles of the income and non-income factors in promoting people's happiness, and provides a foundation for studying happiness from the perspectives of social welfare maximization and individuals' self-interested rationality. In line with the existing empirical findings, our theory predicts that happiness rises with income only up to a critical point, whose magnitude is determined by the amount of the non-income factors in the economy; but once the critical income level is achieved, raising income further will lead to Pareto inefficient allocations and decrease people's happiness. A policy implication is that government should promote a balanced growth between income and non-income factors. The empirical analysis provides some preliminary evidence consistent with the theory's predicitions.

Keywords: Easterlin Paradox, Social Comparison, Pareto Optimality

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1 Introduction

The Easterlin Paradox refers to the fact that economic growth does not increase happiness or life satisfaction in a variety of economies. This phenomenon challenges the traditional views of economics that individuals' utilities depend solely on their own consumption of goods and that measures of income are sufficient indices to capture well-being.[1] The Kingdom of Bhutan has even adopted the national happiness product (GHP) rather than the gross domestic product (GDP) to measure her national progress. Over the past decades, economists and psychologists have made extensive efforts to explore the puzzling relationship between income and happiness.

Two prominent approaches to explain the Easterlin Paradox are the "omitted variables" approach and the social comparison approach. Di Tella and MacCulloch (2005) coin the term "omitted variables" to stand for non-income factors which have been shown by numerous experimental and empirical studies to exhibit strong relationship with happiness.[2] These non-income factors mainly refer to personal characteristics (health, friendship, religion, marriage), macroeconomic variables (inflation, unemployment, inequality) and social variables (environment, human rights, trust, culture, governance). According to the "omitted variables" approach, it is the depreciation of some non-income factors that accounts for the stagnation of happiness levels.

The social comparison approach, proposed first by Easterlin himself (1995, 2001), focuses on income factor only and states that happiness varies directly with one's own income but varies inversely with the incomes of others. When the positive effect of one's own higher income on her happiness is offset by the negative effect of others' higher income, economic growth will not increase everyone's happiness even though it inceases everyone's wealth. Although social comparison is a social-psychology concept, it has been well modelled as interdependent preferences in economics. For example, the "keeping up with the Joneses" models, a growing literature in macroeconomics, rely on status-seeking behaviors to study asset prices (e.g., Campbell and Cochrane, 1999), taxation and growth (e.g., Liu and Turnovsky 2005).

Significant as they are, both approaches have limitations. The studies adopting "omitted variables" approach are mostly empirical or experimental analyses conducted by psychologists. Very

[1]Tian and Yang (2005) have a detailed discussion on the background of this issue.

[2]See Diener and Seligman (2004) for a review.

few economic models have been proposed to study people's happiness,[3] and partly because of this, the economics of happiness is regarded as non-mainstream economics and has been neglected by most economists. More importantly, it also seems unlikely that the non-income factors alone can completely explain the Easterlin Paradox. On the one hand, the increase in income seems too dramatic to be counterbalanced by the decrease in non-income factors in many countries. For example, Japan's real GDP per capita in 1987 is five times higher than in 1958, yet the happiness remained stationary. On the other hand, some non-income factors such as leisure and environment have gotten better off instead of worse off, which deepens the Easterlin Paradox rather than solves it. (Di Tella and MacCulloch, 2006)

The social comparison approach takes no account of the interactions between income and non-income factors in promoting happiness. This makes it fail to explain other aspects of the relationship between income and well-being. In particular, many studies find that "happiness seems to rise with income up to a point, but not beyond it." (Graham, 2005, p. 4) For example, Layard (2005) proposes that extra money buys no happiness once personal income goes above USD20,000. In the literature, this critical point is commonly explained as an *exogenous* level of income which is necessary to maintain the basic needs of life. In contrast, in this paper, we are going to show that such a critical point can be *endogenously* generated by the interaction between income and non-income factors.

Specifically, we develop a theoretical model in which individuals derive utility from both income and non-income factors, but only income factor exhibits social comparison effect. It is shown that Pareto efficiency will require free disposal of a certain amount of income once the income reaches some critical level, whose magnitude is determined by the endowment of non-income factors. In consequence, given the non-income factors, raising the income of all beyond the critical level would not raise the happiness of all. In addition, economic growth would eventually decrease social welfare if the increased income is exhausted in equilibrium and the government policies have corrected all the market failures in the pecuniary domain. Those results lead to an important policy prescription: when the critical income level is achieved, improving non-income factors is the only way to raise well-being.

[3] Exceptions are a series of studies by Yew-Kwang Ng and his coauthors (Ng and Wang, 1993; Ng and Ng, 2001; Ng, 2003).

Therefore, integrating the "omitted variables" approach and the social comparison approach sheds new light on the understanding of the Easterlin paradox. On the one hand, income social comparison justifies the *existence* of a critical income point beyond which more income does not make people happier. On the other hand, non-income factors determine the *magnitude* of the critical income level, i.e., improving non-income factors such as health and human rights can push the critical point to a higher level.

We also provide some preliminary empirical evidence to test our theoretical predictions. Specifically, we obtain estimates for the model implied critical income levels from the *World Value Survey,* and find that for countries whose incomes are below their estimated critical levels, economic growth has positive effect on happiness; but for those whose incomes are above the critical levels, economic growth has no effect even negative effect on happiness. These findings corroborate our theoretical explanations to the different evolutions of happiness in different countries.

On the top of the results we obtained, this paper *per se* also illuminates the possibility of integrating happiness studies into mainstream economics, since we conduct our theoretical analysis within a standard economics textbook model and with standard economics concepts like Pareto efficiency and social welfare functions.

Besides the "omitted variables" approach and the social comparison approach, there exist other explanations to the Easterlin Paradox. The hedonic adaptation theory (e.g., Frederick and Loewenstein, 1999), known as habit formation in economics, says that extra income initially provides extra pleasure, but it is usually only transitory, because people are hedonically adapted to the higher income. The set point theory states that every individual goes back to a presumed happiness level over time (e.g., Easterlin, 2003). Some researchers claim that happiness data itself is misleading due to the facts that happiness scores are not comparable across people and that people redefine their happiness scores over time.[4] Our results complement this literature.

The rest of the paper is organized as follows. Section 2 presents the model, highlights the different roles of income and non-income factors in promoting happiness, and gives an answer to the Easterlin Paradox. Section 3 considers some extensions to the basic model and section 4 provides some preliminary evidences. Section 5 concludes. All the proofs are collected in an appendix.

[4] See Di Tella and MacCulloch (2006) for a discussion.

2 The Model

In this section, we first describe the model and underscore the interpretation of the goods and the motivation of the utility functions. Then, we rely on two basic economics concepts, Pareto efficiency and social welfare, to explain two interesting phenomena: (i) at an individual level, raising everyone's income need not increase everyone's happiness; and (ii) at a society level, economic growth is not necessarily accompanied by increased social happiness (i.e., the Easterlin Paradox). Both explanations highlight the importance of improving non-income factors in promoting individual and social happiness.

2.1 Economic Environment

Consider an exchange economy with $I \geq 2$ consumers who consume two types of goods. Good m indexes income factor and good n indexes non-income factors, such as health, marriage, environment, employment status, etc, that is, all the other factors considered by psychologists to explain the subjective well-being differences across countries.

We offer two explanations for our categorization of goods. First, good m could be understood as *material good* and good n as *non-material good*. In reality, good m roughly corresponds to those goods and services that are currently included in GDP, and good n corresponds to those not included. The first interpretation is consistent with the empirical happiness studies where GDP is used as a measure of the *material* well-being of a society. Second, good m can also be interpreted as *positional good* and good n as *non-positional good*. This interpretation is based on the fact that social comparison does not operate equally across different goods. In economics, positional goods refers to "those things whose value depends relatively strongly on how they compare with things owed by others. Goods that depend relatively less strongly on such comparisons will be called non-positional goods." (Frank, 1985, p. 101)

In fact, the above two explanations are consistent. The positional goods literature has proposed the following empirically supported hypotheses: "(1) Income is more positional than leisure...(3) Private goods are more positional (competitive) than public goods (cf. Ng, 1987), (4) Consumption goods such as clothing and housing are more positional than health and safety." (Solnick and Hemenway, 2006, p. 147) Basically, these hypotheses say that material goods are more positional than non-material goods. Easterlin (2003) also argues that the social comparison in the "pecuniary domain" is less than that in the "nonpecuniary domain". This is true, because, with regard to the material goods domain, comparison is easily done, but, health, family life etc., "are less accessible to public scrutiny than material possessions" (Easterlin, 2003, p. 11181), or they are "inconspicuous" consumption. (Frank, 2004)

In the subsequent discussions, we will refer to good m as *income good*, and good n as *non-income good*. Consumer i's consumption of the two goods is denoted by a vector (m_i, n_i), $i = 1, ..., I$. Assume that the consumption of good m exhibits a negative externality such that the utility of consumer i is adversely affected by other consumers' income good consumption, $m_{-i} = (m_1, ..., m_{i-1}, m_{i+1}, ..., m_I)$. Since good n refers to non-positional good, we assume the consumption of good n has no social comparison effect. Consumer i's utility function is then denoted as $u_i(m_i, n_i; m_{-i})$, which is continuously differentiable, $\frac{\partial u_i}{\partial m_i} > 0$, $\frac{\partial u_i}{\partial n_i} > 0$, $\frac{\partial u_i}{\partial m_j} < 0$, $\frac{\partial^2 u_i}{\partial m_i^2} < 0$, and $\frac{\partial^2 u_i}{\partial m_j^2} \leq 0$, for $i, j = 1, ..., I$ and $j \neq i$. Initially, there are $\bar{m}$ units of income good and $\bar{n}$ units of non-income good available.

Recently, Vostroknutov (2007) provides an axiomatic foundation for interdependent preferences that can be characterized by the following utility representation:[5]

$$u_i(m_i, n_i; m_{-i}) = f(m_i, n_i) + \sum_{j \neq i} \pi_j g(m_i, m_j), \tag{1}$$

where the the first term, $f(m_i, n_i)$, is the utility from consuming income and non-income goods, and the second term, $\sum_{j \neq i} \pi_j g(m_i, m_j)$, is the utility from income social comparison. In particular, the function $g(m_i, m_j)$ describes the specific way that consumer i cares about her income consumption relative to one other consumer j, and $\{\pi_j\}_{j \neq i}$ are the weights that represent the importance of each other consumer to agent i.

For computational simplicity, in our basic model, we adopt the following specific function form to illustrate the essential idea:

$$u_i(m_i, n_i; m_{-i}) = m_i^{\alpha} n_i^{1-\alpha} - \beta \frac{\sum_{j \neq i} m_j}{I - 1}, \text{ with } \alpha \in (0, 1), \beta > 0, i = 1, ..., I. \tag{2}$$

[5] To be specific, Vostroknutov uses Anscombe and Aumann (1963) framework to show that a preference defined over a horse lottery space can be represented by utility function given by equation (1) if and only if six or seven axioms are satisfied depending on the adopted frameworks.

Here, a standard textbook Cobb-Douglas function is used to represent the utility from consumption of one's own goods, and a simple linear function is used to the capture the income social comparison effect. In terms of equation (1), we have $f(m_i, n_i) = m_i^{\alpha} n_i^{1-\alpha}$, $\pi_j = \frac{\beta}{I-1}$ and $g(m_i, m_j) = -m_j$. Our specification of social comparison is also consistent with Easterlin (1995, 2001), who uses average income to capture the negative consumption externality of m_{-i}. In section 3.3, we demonstrate that our results also hold for more general utility functions given by equation (1), provided that the income social comparison effect is sufficiently large.

More comments about the function form are in order. First, the current utility function assumes that all the consumers are in the same reference group. One will see that this assumption can be relaxed and an extension of the basic model to multiple reference groups yields similar results in section 3.1. Second, we assume that there is a negative externality in the consumption of the income goods, but there is *no* externality in the consumption of non-income good. So, our assumption is an extreme case in which there is no social comparison in non-income goods. We would see it does not affect our main results by relaxing this assumption in section 3.2. Third, some of the non-income goods are public rather than private goods, such as democracy and inflation. But the main qualitative result of this paper still holds if we assume that good n is a public good.

2.2 Pareto Efficiency and Social Happiness Maximization

2.2.1 Pareto Efficiency: When Will Raising the Incomes of All Increase the Happiness of All?

When evaluating the performance of an economic system, economists usually adopt the criterion of Pareto efficiency. Implicit in every Pareto efficient outcome is the condition that all possible improvements to a society have been exhausted. If an allocation is Pareto inefficient, some alternative allocation can be supported by consensus. In particular, Pareto efficiency is a very suitable concept for answering Easterlin's question: "will raising the incomes of all increase the happiness of all?" (Easterlin, 1995)

Definition 1 *An allocation of income and non-income goods* $\{m_i, n_i\}_{i=1}^{I} \in \mathbb{R}_{++}^{2I}$[6] *is* feasible *if*

[6]Here, we implicitly assume the consumption sets of all consumers are open sets $\mathbb{R}_{++}^{2}$, in order to apply the Kuhn-Tucker theorem easily.

$\sum_{i=1}^I m_i \leq \bar{m}$ *and* $\sum_{i=1}^I n_i \leq \bar{n}$.[7] *An allocation of income and non-income goods* $\{m_i, n_i\}_{i=1}^I$ *is* Pareto optimal (efficient) *if it is feasible, and there does not exist another feasible allocation,* $\{m_i', n_i'\}_{i=1}^I$*, such that* $u_i(m_i', n_i'; m_{-i}') \geq u_i(m_i, n_i; m_{-i})$ *for all* $i = 1, ..., I$ *and* $u_i(m_i', n_i'; m_{-i}') > u_i(m_i, n_i; m_{-i})$ *for some* i.

For our model, Pareto efficient outcomes $\{m_i^*, n_i^*\}_{i=1}^I$ are completely characterized by the following problem:

$$(PE)\begin{cases} \max\limits_{\{m_i,n_i\}_{i=1}^I \in \mathbb{R}_{++}^{2I}} m_I^\alpha n_I^{1-\alpha} - \beta\frac{m_1+...+m_{I-1}}{I-1} \\ s.t. \ \sum_{i=1}^I m_i \leq \bar{m}, \\ \sum_{i=1}^I n_i \leq \bar{n}, \\ m_i^\alpha n_i^{1-\alpha} - \beta\frac{\sum_{j\neq i} m_j}{I-1} \geq u_i^*, \forall i = 1, ..., I-1, \end{cases}$$

where $u_i^* = m_i^{*\alpha} n_i^{*1-\alpha} - \beta\frac{\sum_{j\neq i} m_j^*}{I-1}$.

By solving the above problem in appendix A, we have the following technical result on Pareto efficiency.

Lemma 1 *For a pure exchange economy with the above specific utility functions, it is necessary to destroy some income good in order to achieve Pareto efficient outcomes if and only if* $\bar{m} > \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n}$*. Specifically,*

(1) When $\bar{m} > \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n}$*, income destruction is necessary to achieve Pareto efficient allocations and the set of Pareto optimal allocations is characterized by*

$$\left\{\begin{array}{c} \{m_i, n_i\}_{i=1}^I \in \mathbb{R}_{++}^{2I} : m_i = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} n_i, \forall i = 1, ..., I, \\ \text{and } \sum_{i=1}^I n_i = \bar{n}, \ \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n} = \sum_{i=1}^I m_i < \bar{m}. \end{array}\right\}$$

(2) When $\bar{m} \leq \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n}$*, all income good should be exhausted to achieve Pareto efficient allocations and the set of Pareto optimal allocations is characterized by*

$$\left\{\begin{array}{c} \{m_i, n_i\}_{i=1}^I \in \mathbb{R}_{++}^{2I} : m_i = \frac{\bar{m}}{\bar{n}} n_i, \forall i = 1, ..., I, \\ \text{and } \sum_{i=1}^I n_i = \bar{n}, \ \sum_{i=1}^I m_i = \bar{m}. \end{array}\right\}.$$

Lemma 1 shows that once income level $\bar{m}$ achieves the critical point $\left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n}$, one has to freely dispose of some income good to achieve Pareto efficiency; otherwise the resulting allocations will

[7] If both inequalities hold with equality, then the allocation is called ballanced.

be Pareto inefficient. This result provides an answer to when raising the incomes of all will increase the happiness of all (Easterlin, 1995). Figure 1 illustrates how.

FIGURE1 GOES HERE

In Figure 1, any point corresponds to a particular economy, with the vertical (horizontal) axis coordinate representing the aggregate consumption of income (non-income) good. Since we do not specify the individual consumption bundles at a point, many allocations are compatible with an economy. But we assume that any underlying allocation in the non-shaded area is Pareto efficient, which could be implemented by a market mechanism with corrective government policies. In contrast, lemma 1 indicates that the allocations in the shaded area will be Pareto inefficient, because the aggregate consumptions of income good are greater than the critical values.

Suppose the economy is initially at point A, which is relatively poor in terms of income good. Then, increasing everyone's income while keeping the non-income constant such that the economy moves to another point B. The economic growth of this kind could potentially increase everyone's happiness, because in a richer society (point B), any initial allocation at point A is still feasible but not Pareto efficient by lemma 1, i.e., there exists a way to improve everyone's well-being when the economy moves from A to B.

However, if we keep increasing everyone's income without changing non-income from point B to point C, then this change would definitely hurt some individuals. To see why, recalling lemma 1, once the income endowment exceeds the critical point, $(\alpha/\beta)^{\frac{1}{1-\alpha}}\,\bar{n}$, income destruction is necessary to achieve Pareto efficient outcomes. Therefore, the original allocation in point B, which is assumed to be Pareto efficient in economy B, is still Pareto efficient in the more affluent economy C. As a result, some individuals are worse off as the economy moves from B to C with different allocations, which is a direct implication of the definition of Pareto efficiency. Thus, raising income alone may not benefit everyone in the economy.

However, if we simultaneously increase income and non-income goods, like from B to D, then everyone could be better off, following a similar discussion as in a change from A to B. The result has important policy implications, suggesting that improving income and non-income factors could potentially support a growth path along which everyone is sustainablely getting happier.

2.2.2 Social Welfare Maximization: When Does Economic Growth Produce Social Happiness?

How to evaluate people's happiness as a whole? What is the corresponding economics concept of the social happiness in the Easterlin Paradox? These questions involve comparing utilities across different individuals. In economics, the concept of social welfare function has been developed to achieve this.

A *social welfare function* (SWF), $W(u_1, ..., u_I)$, takes the individual utilities as arguments and generates a real number to represent the judgement of the whole society over different allocations. Usually, a SWF is assumed to be strictly monotone in individual utilities. A commonly used SWF is the utilitarian SWF:

$$W(u_1, ..., u_I) = \sum_{i=1}^{I} a_i u_i, \text{ with } a_i \geq 0,$$

which says that the social happiness is a linear sum of weighted utilities of individuals. In the happiness studies, psychologists typically use mean life satisfaction to represent a society's happiness (Diener and Seligman, 2004), which is essentially equivalent to adopting a utilitarian SWF with equal weights.

An ideal society should operate at allocations that maximize some SWF subject to the resource constraints. Clearly, the optimal allocations have to be Pareto efficient given the monotonicity of a SWF. The optimal allocations could be implemented by a market mechanism with corrective government policies. For this reason, we will refer to the *social happiness* in the Easterlin Paradox literature as the maximum social welfare that could be achieved with the feasible allocations.[8] In general the values of the social happiness depend on the choices of the SWF. Following lemma 1, we have the following proposition which characterizes the behavior of social happiness in our model.

Proposition 1 *In a pure exchange economy with the above specific utility functions,*
(1) if $\bar{m} \leq \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n}$*, i.e., if the economy is relatively poor, then for any choice of SWF, raising income alone will increase social happiness;*
(2) if $\bar{m} > \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n}$*, i.e., if the economy is relatively rich, then for any choice of SWF, raising*

[8] If one prefers to interpret social happiness as the social welfare evaluated at a competitive market equilibrium, then the following proposition 1 would change to a slightly different version: the effect of raising income on social happiness has an upper bound, and increasing non-income good will raise this upper bound.

income alone will not change social happiness; if in addition, no free disposal of income is allowed, then raising income alone will decrease social happiness; and

(3) if $\bar{m} > \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n}$*, the only way to increase social happiness is to increase the amount of non-income good.*

Proposition 1 not only provides an explanation to the Easterlin Paradox, but also gives policy prescriptions to solve the paradox, i.e., promoting income and non-income goods in a balanced way. To better understand our result, let us choose a simple utilitarian social welfare function, $W(u_1, ..., u_I) = u_1 + u_2 + ... + u_I$, which comports with using mean life satisfaction to represent social happiness in the literature.

Plugging the Pareto efficient allocations given by lemma 1 into the social welfare function $W(u_1, ..., u_I) = u_1 + u_2 + ... + u_I$, the social happiness is:

$$W = \begin{cases} \bar{m}^{\alpha}\bar{n}^{1-\alpha} - \beta\bar{m} & \text{if } \bar{m} \leq \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n} \\ \left(\frac{\beta}{\alpha} - \beta\right)\left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n} & \text{if } \bar{m} > \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n} \end{cases}.$$

If free disposal is not allowed, i.e., if all resources constraints have to be balanced, which is more likely the case in reality, then the social happiness is given by

$$W = \bar{m}^{\alpha}\bar{n}^{1-\alpha} - \beta\bar{m},$$

for all $\bar{m} > 0$ and $\bar{n} > 0$. How the social happiness varies with income $\bar{m}$ for a fixed $\bar{n}$ is graphically shown in Figure 2.

Figure 2 illustrates that increasing income alone can bring happiness only up to a point. This result helps us understand the different evolutions of happiness in countries with similar growth performances, for example, there exists no trend of happiness in the U.S., a decline in Britain, Italy and Germany, and an increase in France. (c.f. Cooper *et al.*, 2001) Specifically, if the non-income factors have not changed significantly, for those countries whose income levels are lower than their critical points, economic growth produces happiness; for those countries whose income levels exceed their critical points, economic growth has no impact on happiness if free disposal is allowed, or negative impact on happiness if free disposal is impossible. In section 4, we obtain estimates of the critical points from *World Value Survey*, and verify the above explanation for some countries like USA, Ireland, Netherlands, etc.

FIGURE2 GOES HERE

Our model suggests that the government policies should be tilted towards boosting non-income good when the income level is close to the critical point. Actually, a government can play an important role in many non-material domains, for example, fighting inflation, improving democracy and freedom, preventing crime. Diener and Seligman (2004) argue that government can also find its way to improve social relations, relieve mental disorder, etc. Also, they suggest the government should build a system of well-being indicators and focus on improving well-being directly. So, all of these suggestions by psychologists can be supported by our theoretical model.

3 Extensions

In our basic model, there is only one reference group, there is no social comparison for non-income good, and a specific utility function is used. All these assumptions will be relaxed in this section and our main result (proposition 1) qualitatively hold.

3.1 Multiple Reference Groups

When people make social comparison, they usually compare themselves to relevant others in the same reference group, say, people in the same city, of the same profession, etc. In this subsection, assume there are K groups, group k has I_k consumers, and consumers compare with the other agents in the same group. Specifically, a typical consumer i in group k has the following utility function

$$u_{ik}\left(m_{ik}, n_{ik}; m_{-ik}\right) = m_{ik}^{\alpha_k} n_{ik}^{1-\alpha_k} - \beta_k \frac{\sum_{j \neq i} m_{jk}}{I_k - 1},$$

where $0 < \alpha_k < 1$, $\beta_k > 0$, and m_{-ik} denotes the vector $(m_{1k}, ..., m_{i-1,k}, m_{i+1,k}, ...m_{I_k k})$. Our basic model corresponds to $K = 1$ and $I_1 = I$.

Two layers of allocation problems are involved in finding the Pareto efficient outcomes: (i) Allocate the society's aggregate resources among groups; and (ii) Allocate the group's aggregate resources among consumers within the group. We are going to start with the second problem.

Suppose group k has a total of $(\bar{m}_k, \bar{n}_k)$ unites of income and non-income goods available. By proposition 1, at Pareto efficiency allocations, the critical income level for group k is $\bar{m}_k^C =$

$\left(\frac{\alpha_k}{\beta_k}\right)^{\frac{1}{1-\alpha_k}} \bar{n}_k$. That is, if $\bar{m}_k > \bar{m}_k^C$, then Pareto efficiency requires free disposal of income good within group k. Therefore, for any given endowment vector $(\bar{m}, \bar{n})$ of the whole economy, Pareto efficient allocation would end up with either $\bar{m}_k \geq \bar{m}_k^C$ for all k, or $\bar{m}_k \leq \bar{m}_k^C$ for all k. Otherwise, i.e., if $\bar{m}_k > \bar{m}_k^C$ for some k and $\bar{m}_{k'} < \bar{m}_{k'}^C$ for some $k' \neq k$ at the same time, then transferring income from group k to group k' would lead to a Pareto improvement.

Given the above discussion, if the society's aggregate income is relatively high such that $\bar{m} > \sum_{k=1}^{K} \left(\frac{\alpha_k}{\beta_k}\right)^{\frac{1}{1-\alpha_k}} \bar{n}_k$, then there will be destruction of income good within some group at Pareto efficient allocations. At this time, increasing income goods only would result in the same set of Pareto efficient allocations as before, and consequently has no effect on increasing social happiness indexed by any social welfare function. We formalize this result in the following proposition.

Proposition 2 *In the economy with multiple reference groups,*
(1) if the economy is poor (i.e. $\bar{m} \leq \sum_{k=1}^{K} \left(\frac{\alpha_k}{\beta_k}\right)^{\frac{1}{1-\alpha_k}} \bar{n}_k$*), then increase in income alone will increase social happiness; and*
(2) if the economy is rich (i.e. $\bar{m} > \sum_{k=1}^{K} \left(\frac{\alpha_k}{\beta_k}\right)^{\frac{1}{1-\alpha_k}} \bar{n}_k$*), then increase in income alone has no effect on social happiness, and the only way to produce social happiness is to improve non-income good.*

3.2 Social Comparison Effect of Non-Income Good

Although non-income good is less subject to social comparison than income good, it might be too restrictive by assuming non-income good does not have any negative externality. This subsection relaxes this assumption.

To ease exposition, consider an economy with only 2 consumers. Of course, there is only one reference group in this case. Let the utility function be

$$u_i(m_i, n_i; m_j) = m_i^{\alpha} n_i^{1-\alpha} - \beta m_j - \gamma n_j,$$

where $\alpha \in (0,1), \beta > 0, \gamma > 0$, $i \in \{1,2\}, j \in \{1,2\}, j \neq i$. The parameter$\beta$ and γ captures the social comparison effect of income and non-income goods, respectively. In addition, assume that the economy adopts a utilitarian social welfare function. That is, we have the following maximization problem:

$$(SCN)\begin{cases} \max\limits_{(m_1,n_1,m_2,n_2)\in\mathbb{R}^4_{++}} m_1^{\alpha}n_1^{1-\alpha}-\beta m_2-\gamma n_2+m_2^{\alpha}n_2^{1-\alpha}-\beta m_1-\gamma n_1 \\ s.t.\ m_1+m_2\le\bar{m},\ n_1+n_2\le\bar{n}. \end{cases}$$

Let $\beta^{\frac{1}{1-\alpha}}\gamma^{\frac{1}{\alpha}}$ be a measure of the *joint social comparison effect* of income and non-income goods. It can be shown that the joint social comparison effect has to be smaller than an upper bound, $\alpha^{\frac{1}{1-\alpha}}(1-\alpha)^{\frac{1}{\alpha}}$, in order for everyone to consume both goods in an allocation which maximizes the social welfare. This condition will hold even when the income social comparison effect β is very large, as long as the non-income social comparison effect γ is sufficiently small. For example, when $\alpha=1/2$, if $\gamma=1/16$, then β can take values up to 4. The relative magnitudes of β and γ might correspond to the reality as we argued before.

In addition, if income is large enough relatively to the non-income good,

$$\bar{m}\ge\left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}}\bar{n},$$

then social welfare maximization would require free disposal of income good. The social happiness is given by

$$W=\left[\left(\frac{\alpha}{\beta}\right)^{\frac{\alpha}{1-\alpha}}-\beta\left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}}-\gamma\right]\bar{n},$$

where the coefficient $\left(\frac{\alpha}{\beta}\right)^{\frac{\alpha}{1-\alpha}}-\beta\left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}}-\gamma$ can be shown to be positive by $\beta^{\frac{1}{1-\alpha}}\gamma^{\frac{1}{\alpha}}<\alpha^{\frac{1}{1-\alpha}}(1-\alpha)^{\frac{1}{\alpha}}$. We state this result formally in the following proposition which is proved in appendix B.

Proposition 3 *Suppose that both goods have social comparison effect in the economy and that the joint social comparison is small, i.e.,* $\beta^{\frac{1}{1-\alpha}}\gamma^{\frac{1}{\alpha}}<\alpha^{\frac{1}{1-\alpha}}(1-\alpha)^{\frac{1}{\alpha}}$. *Then,*
(1) in a poor society (i.e. $\bar{m}\le\left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}}\bar{n}$*), raising income alone will increase social happiness indexed by the utilitarian SWF with equal weights; and*
(2) in a rich society (i.e. $\bar{m}>\left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}}\bar{n}$*), raising income alone has no effect on social happiness indexed by the utilitarian SWF with equal weights, and the only way to produce social happiness is to improve non-income good.*

3.3 General Utility Functions

The results obtained in the section 2 can be extended to the economies with general utility functions given by (1). For simplicity, consider a symmetric two-consumer economy and use a simple utilitarian SWF to measure social happiness. In Tian and Yang (forthcoming), the Pareto efficiency problem is considered with more general utility functions.

As indicated before, consumer i's utility function, equation (1), has its axiomatic foundation provided by Vostroknutov (2007). In a symmetric two-consumer economy, equation (1) changes to:

$$u_i(m_i, n_i; m_{-i}) = f(m_i, n_i) + g(m_i, m_j), \tag{3}$$

where the functions $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$ are twice continuously differentiable. We will solve the following maximization problem to find the allocations that maximize social welfare:

$$(GUF)\begin{cases} \max_{(m_1,n_1,m_2,n_2)\in\mathbb{R}^4_{++}} & f(m_1,n_1) + g(m_1,m_2) + f(m_2,n_2) + g(m_2,m_1) \\ & s.t.\ m_1 + m_2 \le \bar{m},\ n_1 + n_2 \le \bar{n}. \end{cases}$$

The first order conditions of the problem (GUF) are given in appendix C.

It can be shown that how social happiness varies with income depends on the value of the following function:

$$H(m,\bar{n}) = f_1(m,\bar{n}/2) + g_1(m,m) + g_2(m,m),$$

where $f_1(\cdot,\cdot)$ is the partial derivative of $f(\cdot,\cdot)$ with respect to its first argument, and a similar explanation applies to $g_1(\cdot,\cdot)$ and $g_2(\cdot,\cdot)$. In our basic model with $I = 2$, we have $f_1(m,\bar{n}/2) = \alpha m^{\alpha-1}(\bar{n}/2)^{1-\alpha}$, $g_1(m,m) = 0$ and $g_2(m,m) = -\beta$. The value $H(m,\bar{n})$ measures the marginal effect of income on happiness given $\bar{n}$ units of non-income good. The first term, $f_1(m,\bar{n}/2)$, is the marginal utility from the consumption of an extra unit of income good; the second term, $g_1(m,m)$, is the marginal utility from the increase of some consumer's social rank; and the third term, $g_2(m,m)$, is the marginal disutility from the decrease of the other consumer's social rank.

Given that $f_1(m,\bar{n}/2)$ and $g_1(m,m)$ are marginal benefits and that $g_2(m,m)$ is marginal cost, the following assumptions sound reasonable:

(A1) $f_1(m,\bar{n}/2)$, $g_1(m,m)$ and $g_2(m,m)$ are weakly decreasing in m, and at one of them is strictly decreasing in m;

(A2) $\lim_{m\to 0}[f_1(m,\bar{n}/2) + g_1(m,m)] > \lim_{m\to 0} g_2(m,m)$; and

(A3) $\lim_{m\to\infty}[f_1(m,\bar{n}/2) + g_1(m,m)] < \lim_{m\to\infty} g_2(m,m)$.

Assumption (A1) states that the marginal benefits are diminishing in income good but the marginal cost is increasing in income good. Assumption (A2) and (A3) say that the marginal benefits dominate the marginal cost as income is low and that the reverse is true as income is high. Clearly, the utility function of our basic model satisfies these two assumptions.

Assumptions (A1)-(A3) lead to the following proposition:

Proposition 4 *Suppose the quasiconcave utility functions given by (3) satisfy assumptions (A1)-(A3). Then, there exists a critical point,* $\bar{m}^C$*, which is implicitly determined by* $H\left(\bar{m}^C/2, \bar{n}\right) = 0$ *such that*

(1) in a poor society (i.e., $\bar{m} \leq \bar{m}^C$*), raising income alone will increase social happiness indexed by the utilitarian SWF with equal weights; and*

(2) in a rich society (i.e., $\bar{m} > \bar{m}^C$*), raising income alone has no effect on social happiness indexed by the utilitarian SWF with equal weights, and increase in social happiness can be achieved only by raising non-income good.*

4 Empirical Evidence

In this section, we fit the data to our theoretical model to estimate the critical values, and provide some preliminary evidence to support our theoretical results. Specifically, we demonstrate that economic growth does increase happiness for those countries whose income levels are lower than their estimated critical values, but does not for those whose income levels are larger than the estimated critical values.

4.1 Data

Our data sets are the *World Values Survey* (WVS) and the *ERS International Macroeconomic Data Set*. The *World Values Survey* has four successive waves, in 1981-1982, 1989-1993, 1995-1998, and 1999-2003, respectively. Different waves cover different but overlapping countries. The most recent survey covers more than 70 countries. We do a cross nations analysis, in which each country from each wave constitutes one observation.[9] Our main purpose is to get estimates of α and β in the utility function (2), and calculate the model implied critical values.

The WVS provides a life satisfaction variable, scaled from 1 (Dissatisfied) to 10 (Satisfied). In line with the previous empirical happiness studies, we use the mean satisfaction to index happiness

[9]This is aggregate information. The *World Values Survey* contains data at the individual level.

u. In addition, the real per capita income (in 2000 USD) in the *ERS International Macroeconomic Data Set* is used to represent the income explanatory variable m.

The non-income good n in our model should be understood as a composite good made up of a large number of factors which have significant influence on happiness. According to the previous empirical studies,[10] we focus on the following non-income factors available from the WVS data set: state of health, marital status, human rights and time with friends. We have tried other non-income factors, for example, age, and got similar results but not reported here. Other variables in the WVS, such as corruption, could also serve as candidates for non-income factors, which we did not explore in the analysis. The main reason is that in many cases the data are missing for a large number of countries in some waves, even for the U.S. and Britain.

Because we have a small sample size in the cross nations analysis, we are not going to use many non-income variables in one regression, but instead, we will try different ways to combine two of them in a Cobb-Douglas form to index the composite non-income good. That is, we assume

$$n = n_1^{\phi_1} n_2^{\phi_2}, \tag{4}$$

where $\phi_1 > 0$, $\phi_2 > 0$, and n_1, n_2 denote two non-income factors.

All non-income factors are ordered data in the WVS. For example, the variable A009 asks "(a)ll in all, how would you describe your state of health these days?" The correspondents can choose answer from "very good" to "very poor." We use percentage to measure n_1 and n_2 so that the explanatory data to be invariant of the order scale. To be specific, "state of health" is the percentage of respondents who report good health condition; "marital status" is the percentage of respondents who are "married" or "live together as married" (X007 in the WVS); "human rights" is the percentage of respondents who report "there is a lot of respect for individual human rights" (E124 in the WVS); and "time with friends" is the percentage of respondents who visit friends frequently (A058 in the WVS).

In addition, in order to control the effect of the dissolution of the Former Soviet Union, a dummy variable is introduced. For Belarus, Estonia, Latvia, Lithuania, Russia, Ukraine, this dummy variable takes value 1 and for the other countries, it takes value 0. Table 1 reports the data summary.

TABLE1 GOES HERE

[10]For a review, see Diener and Seligman (2004).

4.2 Results

We estimate the following utility function,

$$u = m^{\alpha} \left(n_1^{\phi_1} n_2^{\phi_2} \right)^{1-\alpha} - \beta m - \kappa D, \tag{5}$$

where D denotes the dummy variable to indicate whether the country belongs to the Former Soviet Union. Equation (5) implicitly assumes that individuals are identical within one country, and compare themselves only with other people in the same country.

We do non-linear least squared estimation with Eviews4, and the results for various combinations of non-income factors are reported in Table 2.[11] For example, regression I chooses n_1 and n_2 as "state of health" and "marital status" and gives the following estimated values: $\hat{\alpha} = 0.09$, $\hat{\beta} = 3.22$e-5, $\hat{\phi}_1 = 0.23$, $\hat{\phi}_2 = 0.08$, and $\hat{\kappa} = 0.52$. There are 147 observations included in this regression and the adjusted R^2 is 0.59. The t-statistics reported in parentheses indicate that $\hat{\alpha}$ and $\hat{\phi}_1$ are significant at 1% level and the other parameters are significant at 5% level. Similarly, regression II gives the result based on taking n_1 and n_2 as "state of health" and "human rights", and so on and so forth. The signs of the estimated coefficients are consistent with the previous works. For example, due to the instability effect of the dissolution in the Soviet Union, belonging to the Former Soviet Union has negative effect on happiness.

TABLE2 GOES HERE

The patterns of coefficients are very similar across all regressions. We focus on those regressions whose parameters are all significant: regression I ($n_1 =$"state of health", $n_2 =$"marital status"), regression III ($n_1 =$"marital status", $n_2 =$"human rights"), and regression V ($n_1 =$"human rights", $n_2 =$"time with friends"). According to equation (4), we can estimate the composite non-income factor by

$$\hat{n} = n_1^{\hat{\phi}_1} n_2^{\hat{\phi}_2},$$

which gives the critical income level of one country in a specific year:

$$\hat{m} = \left(\frac{\hat{\alpha}}{\hat{\beta}} \right)^{\frac{1}{1-\hat{\alpha}}} \hat{n}. \tag{6}$$

[11] Graham (2005) pointed out that the result of OLS method is almost the same as that of the ordered probit or logit model.

Table 3 and 4 report the estimated ciritical income levels for the U.S., Japan, Ireland, Nethelands, and Puerto Rico.

TABLE3 GOES HERE

Table 3 shows that in 1990s, both the U.S. and Japan are operating on the inefficient area, because their real income levels exceeded the estimated critical values. Moreover, the estimated critical income levels did not change much over time (regression I), which suggests that the non-income good did not improve much in the last decades. Therefore, according to proposition 1, we are not surprised to observe the flat trace of both countries' happiness in the last 10 years. Also note that the critical levels are very similar across regressions. For example, the critical income level of the U.S. in 1999 is, 24729.09 as "state of health" and "marital status" are selected as non-income factors (regression I), 25816.65 as "marital status" and "human rights" are non-income factors (regression III), and 24763.60 as "human rights" and "time with friends" are non-income factors (regression V). Thus, the results are quite robust.

TABLE4 GOES HERE

The estimated model can also predict increase in happiness for less developed countries such as Albania, Ireland, Mexico, Netherlands, Puerto Rico, Slovenia, etc. Table 4 reports the result for Ireland, Netherlands, and Puerto Rico. We could see that, in these three countries, the real income do not exceed the estimated critical levels (which are again almost constant over time), and the increase in income does add to happiness.

In addition, we fix the non-income good at the mean of its estimates, $\tilde{n}$, and get an explicit relationship between happiness and income:

$$u = \tilde{n}^{1-\hat{\alpha}} m^{\hat{\alpha}} - \hat{\beta} m.$$

Then, we calculate the response of happiness to an increase in income, $\frac{\partial u}{\partial m}\frac{m}{u} = \frac{\hat{\alpha}\tilde{n}^{1-\hat{\alpha}} m^{\hat{\alpha}} - \hat{\beta} m}{\tilde{n}^{1-\hat{\alpha}} m^{\hat{\alpha}} - \hat{\beta} m}$, and the result based on regression V is reported in Table 5. According to regression V, the mean of estimated non-income good is $\tilde{n} = 3.10$, and the estimated preference parameters are $\hat{\alpha} = 0.11$ and $\hat{\beta} = 3.85\text{e-}5$. Plugging those estimates into equation (6), we could find an estimated critical income level at 23,405 USD.

TABLE5 GOES HERE

Table 5 illustrates that the elasticity is decreasing in income for a given amount of non-income good. In particular, the elasticity does not vary much once income level exceeds 10,000 dollars, and will become negative once income is beyond the estimated critical level, 23,405 USD.[12] This observation is consistent with the previous cross nations studies, which state that below USD 10,000 per capita, the effect of income is significant in increasing happiness, and above that level, the effect is pretty small or no effect (Frey and Stutzer, 2002; Helliwell, 2003; Schyns, 2003).

5 Conclusion

In this paper, we develop an economic theory to study happiness. Our model highlights the idea that social comparison affects utility less in nonpecuniary than in pecuniary domains. We show that there exists a critical point beyond which raising income alone has no effect on social happiness. More importantly, the critical income level is determined by the amount of non-income factors in the society, and improving non-income factors could raise the critical income point. We further provide empirical evidence for our theoretical predictions.

These results have important policy implications. In particular, government should promote a balanced growth between income and non-income factors. In many countries, the reality might be that policy makers have overemphasized economic growth and the economies have produced so many income good, which has led to their happiness stagnation problem. A simple but effective solution to this problem is to convert income good into non-income good.

[12] If we allow free disposal of income, then the elasticities vanish for those income larger than the critical level.

Appendix

Appendix A. Proof of proposition 1.

In the problem (PE), the Pareto efficient points are completely characterized by the first order conditions (FOCs), because the objective function and constraints are continuously differentiable and concave on $\mathbb{R}^{2I}_{++}$.

Define the Lagrangian function as:

$$
\begin{aligned}
L \;=\;& m_I^{\alpha} n_I^{1-\alpha} - \beta\left(\frac{m_1 + ... + m_{I-1}}{I-1}\right) + \lambda_m\left(\bar{m} - \sum_{i=1}^{I} m_i\right) + \lambda_n\left(\bar{n} - \sum_{i=1}^{I} n_i\right) \\
&+\mu_1\left[m_1^{\alpha} n_1^{1-\alpha} - \beta\left(\frac{m_2 + m_3 + ... + m_{I-1} + m_I}{I-1}\right) - u_1^*\right] \\
&+\mu_2\left[m_2^{\alpha} n_2^{1-\alpha} - \beta\left(\frac{m_1 + m_3 + ... + m_{I-1} + m_I}{I-1}\right) - u_2^*\right] \\
&+... \\
&+\mu_{I-1}\left[m_{I-1}^{\alpha} n_{I-1}^{1-\alpha} - \beta\left(\frac{m_1 + m_2 + ... + m_{I-2} + m_I}{I-1}\right) - u_{I-1}^*\right].
\end{aligned}
$$

We can obtain the following FOCs:

$$
m_i \;:\; -\frac{\beta}{I-1} - \lambda_m + \mu_i \alpha m_i^{\alpha-1} n_i^{1-\alpha} - \beta\left(\frac{\mu_1 + ... + \mu_{i-1} + \mu_{i+1} + ... + \mu_{I-1}}{I-1}\right) = 0, \tag{7}
$$

$$
n_i \;:\; -\lambda_n + \mu_i (1-\alpha) m_i^{\alpha} n_i^{-\alpha} = 0, \tag{8}
$$

$$
m_I \;:\; \alpha m_I^{\alpha-1} n_I^{1-\alpha} - \lambda_m - \beta\left(\frac{\mu_1 + ... + \mu_{i-1} + \mu_i + \mu_{i+1} + ... + \mu_{I-1}}{I-1}\right) = 0, \tag{9}
$$

$$
n_I \;:\; (1-\alpha) m_I^{\alpha} n_I^{-\alpha} - \lambda_n = 0, \tag{10}
$$

$$
\lambda_m \;:\; \sum_{i=1}^{I} m_i \leq \bar{m},\; \lambda_m \geq 0,\; \lambda_m\left(\bar{m} - \sum_{i=1}^{I} m_i\right) = 0, \tag{11}
$$

$$
\lambda_n \;:\; \sum_{i=1}^{I} n_i \leq \bar{n},\; \lambda_n \geq 0,\; \lambda_n\left(\bar{n} - \sum_{i=1}^{I} n_i\right) = 0, \tag{12}
$$

$$
\mu_i \;:\; m_i^{\alpha} n_i^{1-\alpha} - \beta\frac{\sum_{j\neq i} m_j}{I-1} \geq u_i^*,\; \mu_i \geq 0,\; \mu_i\left(m_i^{\alpha} n_i^{1-\alpha} - \beta\frac{\sum_{j\neq i} m_j}{I-1} - u_i^*\right) = 0, \tag{13}
$$

where (7), (8) and (13) hold for any $i = 1, ..., I-1$.

By (10), we have $\lambda_n > 0$. Thus, by (12), we have

$$
\sum_{i=1}^{I} n_i = \bar{n}. \tag{14}
$$

Using (7)-(10), we have

$$\mu_i\left(\mu_i^{\frac{1-\alpha}{\alpha}}\alpha m_I^{\alpha-1}n_I^{1-\alpha}+\frac{\beta}{I-1}\right)=\alpha m_I^{\alpha-1}n_I^{1-\alpha}+\frac{\beta}{I-1},$$

which implies that $\mu_i = 1$ for any i, since the left hand side is an increasing function in μ_i.

By $\mu_i = 1$, $\lambda_n > 0$, equations (7) and (8), we have

$$\lambda_m = \lambda_n^{-\frac{1-\alpha}{\alpha}}\alpha\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}} - \beta. \tag{15}$$

In addition, (8), (10), $\mu_i = 1$ and $\lambda_n > 0$ yield

$$n_i = (1-\alpha)^{\frac{1}{\alpha}}\lambda_n^{-\frac{1}{\alpha}}m_i, \tag{16}$$

for $i = 1, 2, ..., I$.

Summing up (16) over i and using (14), we have

$$\lambda_n = (1-\alpha)\left(\frac{\sum_{i=1}^{I}m_i}{\bar{n}}\right)^{\alpha}, \tag{17}$$

which implies that,

$$n_i = \frac{m_i\bar{n}}{\sum_{i=1}^{I}m_i}, \tag{18}$$

for $i = 1, 2, ..., I$.

Substituting (17) into (15) yields

$$\lambda_m = \alpha\left(\frac{\bar{n}}{\sum_{i=1}^{I}m_i}\right)^{1-\alpha} - \beta, \tag{19}$$

which will be used to determine the critical income level for Pareto efficiency.

Since $\lambda_m \geq 0$ at equilibrium, there are two cases to consider:

Case 1. $\lambda_m > 0$. In this case, we must have $\sum_{i=1}^{I}m_i < \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}}\bar{n}$ by (19), and thus by (11),

$$\sum_{i=1}^{I}m_i = \bar{m}. \tag{20}$$

Therefore, if $\bar{m} < \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}}\bar{n}$, the income good should be exhausted in order to achieve Pareto efficiency.

In addition, by (18)

$$n_i = \frac{\bar{n}}{\bar{m}}m_i, \tag{21}$$

for $i = 1, 2, ..., I$.

Case 2. $\lambda_m = 0$. Then, by (19), we must have $\sum_{i=1}^{I} m_i = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n}$, which is true for any $\bar{m} \geq \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n}$.

By (18) and $\sum_{i=1}^{I} m_i = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}} \bar{n}$, we have

$$n_i = \left(\frac{\beta}{\alpha}\right)^{\frac{1}{1-\alpha}} m_i, \tag{22}$$

for $i = 1, 2, ..., I$.

Summarizing the two cases gives rise to proposition 1.

Appendix B. Proof of proposition 3.

Set up the Lagrangian for problem (SCN) as

$$\begin{aligned} L &= m_1^{\alpha} n_1^{1-\alpha} - \beta m_2 - \gamma n_2 + m_2^{\alpha} n_2^{1-\alpha} - \beta m_1 - \gamma n_1 \\ &\quad + \lambda_m (\bar{m} - m_1 - m_2) + \lambda_n (\bar{n} - n_1 - n_2) . \end{aligned}$$

The FOCs related to the choices of m and n are

$$\begin{aligned} m_1 &: \alpha m_1^{\alpha-1} n_1^{1-\alpha} - \beta - \lambda_m = 0, \\ n_1 &: (1-\alpha) m_1^{\alpha} n_1^{-\alpha} - \gamma - \lambda_n = 0, \\ m_2 &: \alpha m_2^{\alpha-1} n_2^{1-\alpha} - \beta - \lambda_m = 0, \\ n_2 &: (1-\alpha) m_2^{\alpha} n_2^{-\alpha} - \gamma - \lambda_n = 0, \end{aligned}$$

which imply

$$\left(\frac{n_1}{m_1}\right)^{1-\alpha} = \left(\frac{n_2}{m_2}\right)^{1-\alpha} = \frac{\beta + \lambda_m}{\alpha}, \tag{23}$$

$$\left(\frac{n_1}{m_1}\right)^{-\alpha} = \left(\frac{n_2}{m_2}\right)^{-\alpha} = \frac{\gamma + \lambda_n}{1-\alpha}. \tag{24}$$

Equations (23) or (24) implies $\frac{n_1}{m_1} = \frac{n_2}{m_2}$. Equations (11), (12), (23) and (24) consist of a system to characterize the solutions.

There are four cases to consider:

Case 1. $\lambda_m > 0, \lambda_n > 0$. *In this case, we must have*

$$m_1 + m_2 = \bar{m}, n_1 + n_2 = \bar{n},$$

which imply that

$$\frac{n_1}{m_1} = \frac{n_2}{m_2} = \frac{\bar{n}}{\bar{m}}. \tag{25}$$

(25), (23) and (24) give us

$$\lambda_m = \alpha \left(\frac{\bar{n}}{\bar{m}}\right)^{1-\alpha} - \beta > 0, \tag{26}$$

$$\lambda_n = (1-\alpha)\left(\frac{\bar{n}}{\bar{m}}\right)^{-\alpha} - \gamma > 0, \tag{27}$$

which are true when $\left(\frac{\gamma}{1-\alpha}\right)^{\frac{1}{\alpha}} < \frac{\bar{m}}{\bar{n}} < \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}}$. This would imply

$$\beta^{\frac{1}{1-\alpha}}\gamma^{\frac{1}{\alpha}} \leq \alpha^{\frac{1}{1-\alpha}}(1-\alpha)^{\frac{1}{\alpha}}. \tag{28}$$

Case 2. $\lambda_m = 0, \lambda_n > 0$. *By* (12) and $\lambda_n > 0$, we have (14).

By (23) and $\lambda_m = 0$,

$$\frac{n_1}{m_1} = \frac{n_2}{m_2} = \left(\frac{\beta}{\alpha}\right)^{\frac{1}{1-\alpha}}. \tag{29}$$

Equations (29), (14) and (11) require

$$\bar{m} \geq \left(\frac{\beta}{\alpha}\right)^{-\frac{1}{1-\alpha}} \bar{n}.$$

By (29) and (24),

$$\lambda_n = (1-\alpha)\left(\frac{\beta}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}} - \gamma > 0,$$

which implies the weak inequality (28) holds.

Case 3. $\lambda_m > 0, \lambda_n = 0$. By (11) and $\lambda_m > 0$, we have

$$m_1 + m_2 = \bar{m}.$$

By (24) and $\lambda_n = 0$,

$$\frac{n_1}{m_1} = \frac{n_2}{m_2} = \left(\frac{\gamma}{1-\alpha}\right)^{-\frac{1}{\alpha}}. \tag{30}$$

Equations (30) and (12) imply

$$\bar{m} < \left(\frac{\gamma}{1-\alpha}\right)^{\frac{1}{\alpha}} \bar{n}.$$

Equations (23) and (30) require

$$\lambda_m = \alpha\left(\frac{\gamma}{1-\alpha}\right)^{-\frac{1-\alpha}{\alpha}} - \beta > 0,$$

which is equivalent to $\beta^{\frac{1}{1-\alpha}}\gamma^{\frac{1}{\alpha}} < \alpha^{\frac{1}{1-\alpha}} (1-\alpha)^{\frac{1}{\alpha}}$.

Case 4 $\lambda_m = 0, \lambda_n = 0$. By (23) and (24), this is true only when

$$\frac{m_1}{n_1} = \frac{m_2}{n_2} = \left(\frac{\gamma}{1-\alpha}\right)^{\frac{1}{\alpha}} = \left(\frac{\alpha}{\beta}\right)^{\frac{1}{1-\alpha}}, \tag{31}$$

which implies

$$\beta^{\frac{1}{1-\alpha}}\gamma^{\frac{1}{\alpha}} = \alpha^{\frac{1}{1-\alpha}} (1-\alpha)^{\frac{1}{\alpha}}.$$

The conditions in proposition 3 ensure that case 2 holds. The results follow directly.

Appendix C. Proof of proposition 4.

Set up the Lagrangian for problem (GUF) as

$$\begin{aligned} L &= f(m_1, n_1) + g(m_1, m_2) + f(m_2, n_2) + g(m_2, m_1) \\ &\quad + \lambda_m (\bar{m} - m_1 - m_2) + \lambda_n (\bar{n} - n_1 - n_2). \end{aligned}$$

The FOCs related to the choices of m and n are

$$m_1 : \frac{\partial f(m_1, n_1)}{\partial m_1} + \frac{\partial g(m_1, m_2)}{\partial m_1} + \frac{\partial g(m_2, m_1)}{\partial m_1} - \lambda_m = 0, \tag{32}$$

$$n_1 : \frac{\partial f(m_1, n_1)}{\partial n_1} - \lambda_n = 0, \tag{33}$$

$$m_2 : \frac{\partial g(m_1, m_2)}{\partial m_2} + \frac{\partial f(m_2, n_2)}{\partial m_2} + \frac{\partial g(m_2, m_1)}{\partial m_2} - \lambda_m = 0, \tag{34}$$

$$n_2 : \frac{\partial f(m_2, n_2)}{\partial n_2} - \lambda_n = 0. \tag{35}$$

Equations (32)-(35), (23) and (24) consist of a system to characterize the solutions.

In particular, $m_1 = m_2 \equiv m$, $n_1 = n_2 = \bar{n}/2$, $\lambda_n = \frac{\partial f(m, \bar{n}/2)}{\partial n_1}$, and $\lambda_m = f_1(m, \bar{n}/2) + g_1(m, m) + g_2(m, m)$ satisfy this system. Given the quasiconcavity of the objective function, FOCs are both necessary and sufficient, and the solution is unique. Therefore, we have

$$\lambda_m = f_1(m, \bar{n}/2) + g_1(m, m) + g_2(m, m) \equiv H(m, \bar{n}),$$

which is the marginal effect of increasing income on social happiness, by Envelope theorem. In addition, assumptions (A1)-(A3) imply that for any fixed $\bar{n}$, $H(m, \bar{n})$ varies inversely from positive to negative as m increases from 0 to $+\infty$. Then, the results follow directly.

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Figure 1 Does Raising the Incomes of All Increase the Happiness of All?

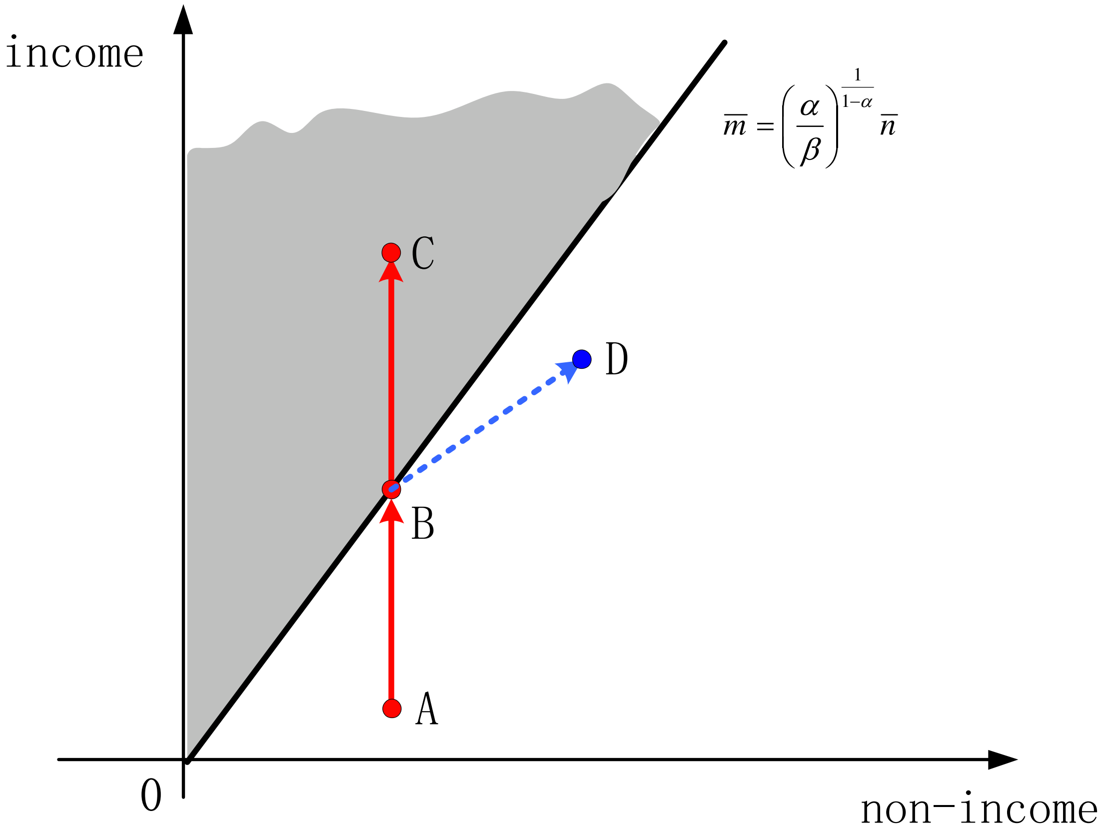

Figure 2 Income VS Happiness

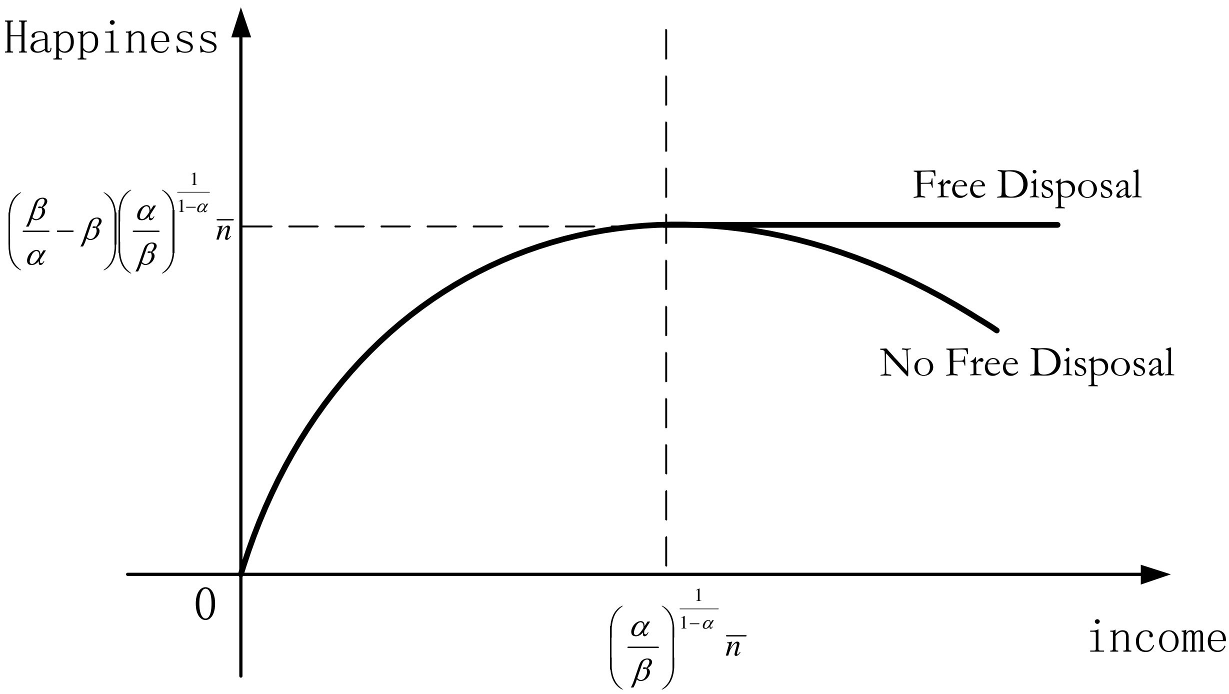

Table 1 Data Summary

	Min	Max	Mean	S.D.	# Obs.
Mean life satisfaction	3.73	8.49	6.63	1.09	187
GDP per capita (2000 US$)	261.00	37459.00	9210.81	9408.34	187
State of health	25.93	89.46	60.64	15.29	148
Marital status	39.81	87.46	64.46	8.20	185
Human rights	0.41	61.90	13.18	11.81	79
Time with friends	58.47	97.78	81.12	10.19	69
Former Soviet Union	0.00	1.00	0.09	0.29	187

Table 2 Estimation Result (Nonlinear Least Squared)

	I	II	III	IV	V
α	0.09***	0.09***	0.13***	0.10***	0.11***
	(7.17)	(3.46)	(7.83)	(5.00)	(5.85)
β	3.22e-5**	2.84e-5	4.21e-5**	1.42e-5	3.85e-5*
	(2.20)	(0.93)	(2.02)	(0.64)	(1.78)
State of health	0.23***	0.27***			
	(6.65)	(6.15)			
Marital status	0.08**		0.20***	0.11	
	(2.14)		(7.28)	(1.63)	
Human rights		0.04	0.07***		0.05***
		(1.59)	(4.60)		(3.07)
Time with friends				0.16**	0.23***
				(2.35)	(7.33)
Former Soviet Union	0.52**	0.22	0.56**	0.93***	0.55*
	(2.13)	(0.54)	(2.33)	(3.05)	(1.74)
# observations	147	46	79	69	68
Adjusted R^2	0.59	0.58	0.73	0.61	0.64

Note: The t-statistics are shown in parentheses. The superscripts *, **, and *** indicate the coefficients are significant at 10%, 5%, and 1% significance level, respectively.

Table 3 Economic Growth Does Not Bring Happiness

Year	Mean Satisfaction	Real Income	Critical Level		
			I	III	V
		Panel A:	the U.S.		
1982	7.67	22518.19	24284.04	*NA*	*NA*
1990	7.75	28467.86	24621.31	*NA*	*NA*
1995	7.68	29910.29	24688.02	*NA*	*NA*
1999	7.65	33717.43	24729.09	25816.65	24763.60
		Panel B: Japan			
1981	6.59	24176.56	21652.87	*NA*	*NA*
1990	6.53	33438.54	21865.49	*NA*	*NA*
1995	6.72	35332.73	22958.70	*NA*	*NA*
2000	6.48	37459.16	22886.39	24790.58	21261.54

Table 4　Economic Growth Does Bring Happiness

Year	Mean Satisfaction	Real Income	Critical Level		
			I	III	V
		Panel A:　Ireland			
1981	7.82	9915.67	24239.53	*NA*	*NA*
1990	7.88	13444.14	24778.52	*NA*	*NA*
1999	8.17	22952.64	*NA*	26773.26	25402.69
		Panel B:　Netherlands			
1981	7.70	15564.10	24332.80	*NA*	*NA*
1990	7.76	18498.68	23940.34	*NA*	*NA*
1999	7.88	22669.39	*NA*	26525.55	25676.55
		Panel C:　Puerto Rico			
1995	7.70	11502.26	23646.51	*NA*	*NA*
2001	7.88	13394.87	24013.54	25563.74	22895.78

Table 5 Happiness-Income Elasticity

Income(2000 USD)	1,000	2,000	3,000	5,000	10,000
Happiness-income Elasticity	0.1035	0.0983	0.0935	0.0840	0.0612
Income(2000 USD)	15,000	23,405	25,000	30,000	40,000
Happiness-income Elasticity	0.0386	0.0000	−0.0075	−0.0313	−0.0812

Note: Here the report is based on regression V with $\tilde{n} = 3.10$, $\hat{\alpha} = 0.11$, and $\hat{\beta} = 3.85e-5$.