第39卷第11期
2009年11月

中国科学技术大学学报
JOURNAL OF UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA

Vol. 39, No. 11
Nov. 2009

Article ID: 0253-2778(2009)11-1125-05

# DOA estimation for uncorrelated and coherent signals with centre-symmetric circular array

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**Abstract**: A new direction of arrival (DOA) estimation method for the centre-symmetric circular array is proposed to cope with the scenario where both uncorrelated signals and pairs of coherent signals are presented. By constructing a centre-symmetric array manifold and exploiting its properties, coherent signals in pairs can be decorrelated and then estimated without the interference of uncorrelated signals, while the uncorrelated signals are estimated by utilizing the uniqueness condition of array manifold. The two-stage estimation method is simple but effective. It does not need beamspace transform and is performed directly in element space. It has higher estimation precision. Simulation results demonstrate the effectiveness and efficiency of the proposed method.

**Key words**: centre-symmetric circular array; direction of arrival (DOA); coherent signals

**CLC number**: TN957　　**Document code**: A

# 基于中心对称圆阵的不相关源和相干源的 DOA 估计

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**摘要**:基于中心对称的圆阵,提出了一种针对不相关源和成对相干源的波达方向(DOA)估计方法.这是一种分两步估计的方法,首先利用阵列流形的唯一性条件估计出不相关源;然后通过构造中心对称的阵列流形来去除不相关源的干扰,同时达到去除成对相干源相干性的目的.这种方法简单有效,由于直接在阵元空间进行估计而不需要进行波束变换,因此具有较高的估计精度.仿真实验结果验证了该方法的有效性.

**关键词**:中心对称圆阵;波达方向(DOA);相干源

## 0 Introduction

Direction of arrival (DOA) estimation of multiple narrowband sources is a major research issue in array signal processing. Many high-resolution DOA estimation methods have been developed over the years. Compared with the rectilinear array, the circular array has many merits, such as 360° azimuthal coverage, almost invariant directional pattern and constant azimuthal resolution, and has received more and more attention[1]. However, the circular array manifold

**Received**: 2008-03-28; **Revised**: 2008-09-17

**Foundation item**: Supported by the Youth Science Foundation of USTC (KA2100060003).

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does not have the Vandermonde form. Some useful techniques based on the rectilinear array cannot be applied directly, which makes DOA estimation for the circular array more complex especially when the signals are highly correlated or coherent due to multi-path propagation. The dominant method for the circular array is through the phase mode excitation technique, which employs beamspace transform to rebuild a manifold with the Vandermonde form. Based on this technique, some computationally efficient methods for the circular array have been proposed. Ref. [2] proposes real beamspace MUSIC with reduced computation and enhanced performance, and furthermore a novel closed-form algorithm of ESPRIT to pair the 2-D angle automatically. To combat the multi-path propagation, Refs. [3, 4] extend the spatial smoothing to the circular array and further analyze the effects of imperfections, such as mutual coupling, transform error, and directional sensor. Ref. [5] employs a unitary transformation procedure in root-MUSIC and realizes a lower estimation variance. However, these methods generally work properly only with a large number of sensors. Because the beamspace transform error caused by a small number of sensors may degrade the estimation performance. Ref. [6] presents a qualitative and quantitative analysis of such error and then introduces an improved method, which first synthesizes the dominant term of the error and then removes it in an alternate manner. Array interpolation is a technique that can map arrays of any geometry to a particular array geometry. The circular array, as an example, can also be mapped to a virtual uniform linear array through this technique. However, it needs to split the entire azimuthal range into several sectors and processes each sector separately, which may leave unavoidable mapping error and cause significant DOA bias[7,8]. In Ref. [9], a spatial averaging method is directly performed in element space to improve DOA performance of uncorrelated signals for the centre-symmetric circular array. In this paper, we furthermore propose a simple but effective DOA estimation method for this kind of circular array to cope with the scenario in which uncorrelated signals and pairs of coherent signals coexist. It is a two-stage method. By constructing a centre-symmetric array manifold and exploiting its properties, the uncorrelated signals and coherent signals can be estimated in element space separately. Theoretical analysis and simulation results both demonstrate that the two-stage method has higher precision.

## 1 Formulation of the problem

Consider $K$ narrowband far-field uncorrelated signal sources impinging upon a centre-symmetric circular array with $M=2N$ omni-directional sensors as configured in Fig. 1. Assume that some source undergoes multi-path propagations and may impinge from two different paths. Define the number of such sources as $P$, and the total number of received signals is $K+P$. Assume all signals are coming from distinct directions. Arrange the output of each sensor as follows

$$\begin{aligned}\boldsymbol{x}(t)&=[x_1(t),\cdots,x_N(t),x_{2N}(t),\cdots,x_{N+1}(t)]^{\mathrm{T}}=\\&\sum_{k=1}^{P}\sum_{l=1}^{2}\boldsymbol{a}(\theta_{kl})\rho_{kl}s_k(t)+\sum_{k=P+1}^{K}\boldsymbol{a}(\theta_k)s_k(t)+\boldsymbol{n}(t)=\\&\boldsymbol{A}_c\boldsymbol{s}_c(t)+\boldsymbol{A}_u\boldsymbol{s}_u(t)+\boldsymbol{n}(t)\end{aligned}\qquad(1)$$

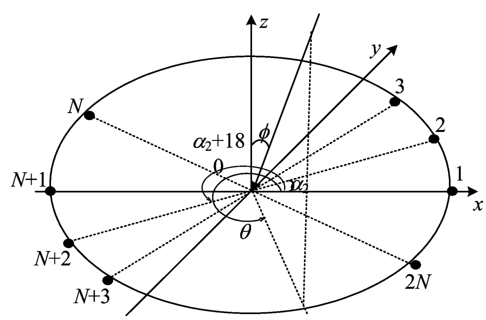


**Fig. 1 Uniform circular array geometry**

where $\boldsymbol{a}(\gamma)=[a_1(\gamma),\cdots,a_N(\gamma),a_{2N}(\gamma),\cdots,a_{N+1}(\gamma)]^{\mathrm{T}}$ is the steering vector from direction $\gamma=(\theta,\phi)$ with $a_m(\gamma)=\exp\{(\mathrm{j}2\pi r/\lambda)\cos(\theta-\alpha_m)\sin(\phi)\}$, $\theta$ is the azimuth angle, $\phi$ is the elevation angle, $\alpha_m$ is the angle between the $m$th and 1st sensor, $r$ is the array radius, $\lambda$ is the wavelength, $\rho_{kl}$ is the complex fading coefficient of the $l$th path of the $k$th source, $l=1,2$, $\boldsymbol{\rho}_k=[\rho_{k1},\rho_{k2}]^{\mathrm{T}}$, $\boldsymbol{A}_k=[\boldsymbol{a}(\gamma_{k1}),\boldsymbol{a}(\gamma_{k2})]$, $\boldsymbol{A}_c=[\boldsymbol{A}_1\boldsymbol{\rho}_1,\cdots,\boldsymbol{A}_P\boldsymbol{\rho}_P]$, $\boldsymbol{A}_u=[\boldsymbol{a}(\gamma_{p+1}),\cdots,\boldsymbol{a}(\gamma_k)]$, $\boldsymbol{S}_c(t)=[s_1(t),\cdots,s_P(t)]^{\mathrm{T}}$, and $\boldsymbol{s}_u(t)=[s_{P+1}(t),\cdots,s_K(t)]^{\mathrm{T}}$. The power of $s_k(t)$ is $\sigma_k^2$. The entries of signal vector $\boldsymbol{s}_c(t)$, $\boldsymbol{s}_u(t)$, and noise vector $\boldsymbol{n}(t)$ are all zero mean wide-sense stationary random processes and are uncorrelated to each other.

From Eq. (1), we define the array covariance matrix as

$$\boldsymbol{R}_x=E\{\boldsymbol{x}(t)\boldsymbol{x}^H(t)\}=\boldsymbol{A}_c\boldsymbol{R}_c\boldsymbol{A}_c^H+\boldsymbol{A}_u\boldsymbol{R}_u\boldsymbol{A}_u^H+\sigma_n^2\boldsymbol{I} \tag{2}$$

where $\boldsymbol{R}_c=E\{\boldsymbol{s}_c(t)\boldsymbol{s}_c^H(t)\}$, $\boldsymbol{R}_u=E\{\boldsymbol{s}_u(t)\boldsymbol{s}_u^H(t)\}$, and $\boldsymbol{I}$ is an $M\times M$ identity matrix. Due to the assumption of uncorrelatedness, we have $\boldsymbol{R}_c=\mathrm{diag}\{\sigma_1^2,\cdots,\sigma_P^2\}$ and $\boldsymbol{R}_u=\mathrm{diag}\{\sigma_{P+1}^2,\cdots,\sigma_K^2\}$.

## 2 DOA estimation

The DOA estimation proceeds in two different stages, which exploit two different properties of array manifold respectively. The first property is that any collection of steering vectors from the array manifold should be linearly independent, which is generally considered as a precondition for unique localization of narrowband sources[10]. This means that any linear combination of steering vectors can not result in a new steering vector.

Since $\boldsymbol{A}_c\boldsymbol{R}_c\boldsymbol{A}_c^H+\boldsymbol{A}_u\boldsymbol{R}_u\boldsymbol{A}_u^H$ in Eq. (2) is only of rank $K$, only $K$ eigenvectors $e_1,\cdots,e_K$ corresponding to the first $K$ maximal eigenvalues span the signal subspace when performing eigendecomposition on $\boldsymbol{R}_x$, which is also spanned by $\boldsymbol{A}_1\boldsymbol{\rho}_1,\cdots,\boldsymbol{A}_P\boldsymbol{\rho}_P$ and $\boldsymbol{a}(\gamma_{P+1}),\cdots,\boldsymbol{a}(\gamma_K)$ jointly. The rest $e_{K+1},\cdots,e_M$ span the noise subspace. Exploiting the property that the signal subspace is orthogonal to the noise subspace, we have

$$|\boldsymbol{e}_i^H\boldsymbol{A}_k\boldsymbol{\rho}_k|^2=0,\ i=K+1,\cdots,M;\ k=1,\cdots,P \tag{3}$$

$$|\boldsymbol{e}_i^H\boldsymbol{a}(\gamma_k)|^2=0,\ i=K+1,\cdots,M;\ k=P+1,\cdots,K \tag{4}$$

Then define a function as

$$p_1(\gamma)=1/\sum_{i=K+1}^{M}|\boldsymbol{e}_i^H\boldsymbol{a}(\gamma)|^2 \tag{5}$$

Since no linear combination of steering vectors can result in a new steering vector, no angles but $\gamma_{P+1},\cdots,\gamma_K$ make $p_1(\gamma)$ form peaks, from which we can obtain the DOAs of uncorrelated signals as long as $K<M$.

The second property is that the structure of $\boldsymbol{a}(\gamma)$ is centre-symmetric[9], i. e. $a_m(\gamma)=a_{N+m}^*(\gamma)$. We have $\boldsymbol{a}(\gamma)=\boldsymbol{J}\boldsymbol{a}^*(\gamma)$. Then

$$\boldsymbol{J}(\boldsymbol{A}_u\boldsymbol{R}_u\boldsymbol{A}_u^H)^*\boldsymbol{J}=\boldsymbol{J}\boldsymbol{A}_u^*\boldsymbol{R}_u(\boldsymbol{J}\boldsymbol{A}_u^*)^H=\boldsymbol{A}_u\boldsymbol{R}_u\boldsymbol{A}_u^H \tag{6}$$

where $\boldsymbol{J}$ is an $M\times M$ anti-diagonal matrix. Exploiting this property, we define a new matrix as

$$\begin{aligned}\boldsymbol{R}&=\boldsymbol{R}_x-\boldsymbol{J}\boldsymbol{R}_x^*\boldsymbol{J}=\boldsymbol{A}_c\boldsymbol{R}_c\boldsymbol{A}_c^H-\boldsymbol{J}(\boldsymbol{A}_c\boldsymbol{R}_c\boldsymbol{A}_c^H)^*\boldsymbol{J}=\\&\sum_{k=1}^{P}\boldsymbol{A}_k\sigma_k^2(\boldsymbol{\rho}_k\boldsymbol{\rho}_k^H-\boldsymbol{\rho}_k^*\boldsymbol{\rho}_k^{\mathrm{T}})\boldsymbol{A}_k^H=\\&\sum_{k=1}^{P}\boldsymbol{A}_k\boldsymbol{R}_{ck}\boldsymbol{A}_k^H=\\&[\boldsymbol{A}_1,\cdots,\boldsymbol{A}_P]\mathrm{BLKdiag}\{\boldsymbol{R}_{c1},\cdots,\boldsymbol{R}_{cP}\}[\boldsymbol{A}_1,\cdots,\boldsymbol{A}_P]^H\end{aligned} \tag{7}$$

in which only components of coherent signals remain. Since $\boldsymbol{R}_{ck}=\sigma_k^2(\boldsymbol{\rho}_k\boldsymbol{\rho}_k^H-\boldsymbol{\rho}_k^*\boldsymbol{\rho}_k^{\mathrm{T}})$ is a skew-symmetric matrix, whose eigenvalues always come in pairs $\pm\mu$, its rank is two in general. Then $\boldsymbol{R}$ has the rank of $2P$, which means that we can perform eigendecomposition directly on $\boldsymbol{R}$ and then find the peaks of $p_2(\gamma)$ to estimate the DOAs of coherent signals as long as $2P<M$

$$p_2(\gamma)=1/\sum_{k=2P+1}^{M}|\boldsymbol{u}_k^H\boldsymbol{a}(\gamma)|^2 \tag{8}$$

where $\boldsymbol{u}_1,\cdots,\boldsymbol{u}_M$ are the eigenvectors of $\boldsymbol{R}$ corresponding to the eigenvalues whose absolute values are in descending order.

Compared with the averaging method in Ref. [9], this two-stage method has more dimension of noise subspace. The dimension for the uncorrelated and coherent signals is $M-K$ and $M-2P$ respectively, while it is $M-K-P$ for the averaging method. Therefore, when computing from limited snapshots, the DOA estimates for uncorrelated signals are more accurate by this two-stage method in general. Moreover, the subtraction operation in Eq. (7) eliminates the components of uncorrelated signals and noises, reduces their effects and then improves the performance of coherent signals in low SNR. However, some powers of coherent signals are also lost during the process, which may make the improvement less obvious than that by the averaging method as SNR increases with limited snapshots.

## 3 Simulation experiments and discussion

Since the averaging method in Ref. [9] can also resolve pairs of coherent signals, here we select it and the forward-backward spatial smoothing method in Ref. [4] for comparison. A uniform circular array with $r=0.8\lambda$ is selected as the centre-symmetric circular array. For simplicity, assume that the elevation angle is $\phi=90°$ and the number of signals is known which can also be obtained by Ref. [11]. Here the signals and noises are selected to be zero mean complex Gaussian processes. The input SNR of the $k$th source is defined as $10\log_{10}(\sigma_k^2/\sigma_n^2)$. Assume that $\sigma_1^2=\cdots=\sigma_K^2$ and $\sigma_n^2=1$. The average root mean square error (RMSE) of the DOA estimates from 200 Monte Carlo runs is used as the performance index

$$\mathrm{RMSE}=\sqrt{\sum_{n=1}^{200}\sum_{k=1}^{I}(\hat{\theta}_k(n)-\theta_k)^2/(200I)} \quad (9)$$

where $\hat{\theta}_k(n)$ is the estimate of $\theta_k$ for the $n$th run, and $I$ is the number of all the uncorrelated or all the coherent signals. Note that the RMSE is calculated for uncorrelated and coherent signals respectively, because they are estimated at two different stages.

The first experiment considers four uncorrelated signals from 50°, 170°, 330°, 255° and two pairs of coherent signals when the number of array sensors is $M=12$. One pair of coherent signals come from 80°, 210° with fading coefficient $\exp\{j143.05°\}$, $0.9\exp\{j218.46°\}$ and the other from 130°, 290° with $\exp\{j83.21°\}$, $0.8\exp\{j174.95°\}$ respectively. The maximum mode order is selected to be five in Ref. [4]. The MUSIC spectra at two different stages by the new method are first shown in Fig. 2 for one run with SNR=0 dB and 500 snapshots, from which we can see that the sharp peaks have been formed at the correct DOAs. The RMSE of the DOA estimates versus input SNR is shown in Fig. 3 with 500 snapshots. These figures illustrate that the estimates of uncorrelated signals by the new method are more accurate than those by the method in Ref. [9] and the case is also for the coherent signals at low SNR, which is coincident with the analysis in Section 2. Since Ref. [4] is based on the phase mode excitation technique and needs a large number of sensors to work properly, it has the worst performance.

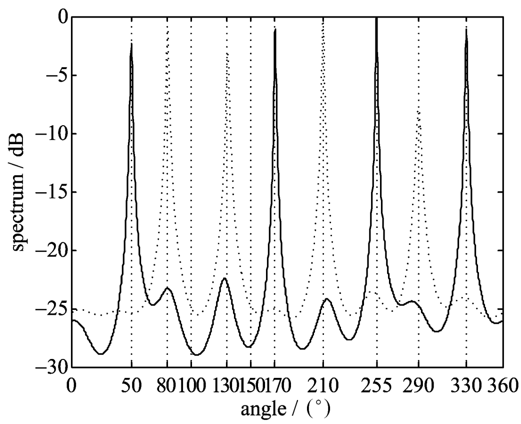


**Fig. 2 MUSIC spectra for uncorrelated signals (solid line) and coherent signals (dotted line)**

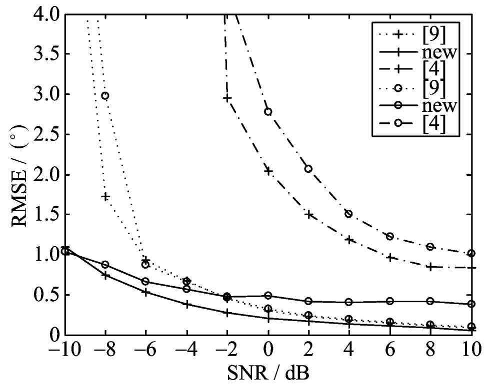


**Fig. 3 RMSE versus input SNR for uncorrelated signals (+) and coherent signals (o) with 500 snapshots**

The second experiment considers the same scenario as the first one except that there are fewer sensors with $M=10$. The RMSE of the DOA estimates versus input SNR is shown in Fig. 4 with 500 snapshots. The methods in Refs. [4, 9] have both failed to estimate the DOAs correctly in such a scenario, while the new method still has better performance. The RMSE of the DOA estimates versus the number of snapshots is shown in Fig. 5 with SNR = 0 dB, which illustrates that the estimates are more accurate as the number of snapshots increases.

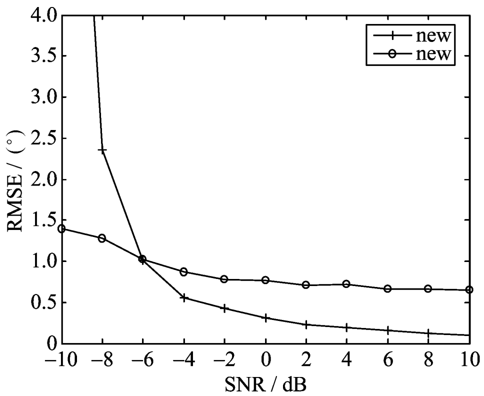


**Fig. 4　RMSE versus input SNR for uncorrelated signals (+) and coherent signals (o) with 500 snapshots**

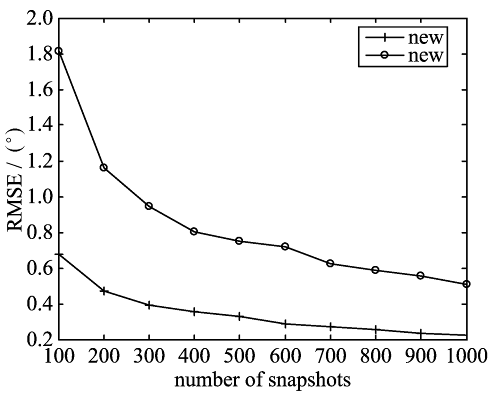


**Fig. 5　RMSE versus number of snapshots for uncorrelated signals (+) and coherent signals (o) with SNR=0 dB**

## 4　Conclusion

In this paper, we propose a two-stage DOA estimation method in element space for the centre-symmetric circular array when both uncorrelated signals and pairs of coherent signals are presented. Simulation results validate the effectiveness of this method and illustrate that the new method has higher estimation precision especially at low SNR and small number of sensors. However, a disadvantage of the new method is that it only resolves coherent signals in pairs.

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