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Joint Optimization of Capacity Expansion, Production and Maintenance in Stochastic Manufacturing Systems

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Abstract

This paper presents an analytical model for the joint determination of optimal production, corrective maintenance and capacity expansion policies for a repairable production system subject to random failures. The production system have to make decisions regarding production and maintenance, as well as investment in capacity expansion in order to minimize costs of investment, production, maintenance, inventories and backlogs in an uncertain environment. The introduction of corrective maintenance strategy in the proposed model improves the availability of the machines and hence reduces the total incurred cost compared to available models. The control variables are the random stopping times at which to purchase a new capacity, the production and corrective maintenance rates before and after capacity purchase. The objective is

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to minimize production, capacity investment, inventories, backlogs and maintenance costs over an infinite planning horizon. Optimality conditions are given and numerical methods are used to solve them and to determine the control policy. A Numerical example and sensitive analyses are presented to illustrate the usefulness of the proposed approach.

Keywords: Optimal Control, Numerical Methods, Production Planning, Capacity Management, Maintenance Control, Manufacturing Systems

1 Introduction

This paper models and illustrates an optimal policy for a manufacturing system with a simultaneous control of capacity expansion, production and corrective maintenance strategies. As demand increases, new machines must be purchased at given epoch (or stopping time) while production and preventive maintenance are well planned before and after the stopping time at which to purchase a new capacity. The system under study, described in [16], consists of a firm that must satisfy a given demand rate for its product over time to minimize its discounted cost of investment and inventory/shortage. The firm has an existing machine that is failure-prone with given rates of breakdowns and repairs. At a given time, due to demand fluctuations, the demand for the firm's product is higher than the average production capacity of the existing machine. However, the firm has some initial inventory of its products to absorb the excess demand for a few initial periods. Such a firm may have to increase its production capacity at some future time. For this purpose, the firm has an option to purchase a new machine, identical to the existing machine, at a given fixed cost, in order to double its average production capacity. This assumes that the firm has sufficient repair capacity to handle two machines event when they are both broken down during some time interval. The purpose of this paper is to extend the model presented in [16] by controlling both production with capacity expansion and corrective maintenance in order to reduce the overall incurred cost.

The objective of this work is, for the aforementioned purpose, to find the optimal policy which integrates simultaneously capacity management strategy, production and maintenance control for a flexible manufacturing system with a wafer demand trend approximated in this paper by a multiple steps staircase structure of the demand rate. The demand rate is assumed constant in the proposed model for a given planning horizon. For the considered manufacturing system, this assumption is motivated by the fact that the rate of change in the machines states is much larger than the rate at which the demand rate change significantly. In a more general context, capacity expansion, results of capacity purchase, can be done by machines, manpower and technology acquisitions or by using subcontracting to hedge against sudden demand jumps.

In this work, we focus only on capacity expansion by machine purchase. There have been some attempts to deal with the problem of capacity expansion by machines purchase under uncertainty. An extensive survey of early work of stochastic capacity expansion problems is provided in [13]. Models that appeared after [13] are presented in [5] where it is noted that when the time is a continuous variable rather than a discrete one, the capacity expansion problem may be formulated as an optimal control problem. For a single product with a stochastic demand process, [6] used an investment rate function as the control variable to regulate the expansion rate. For multiple products with deterministic demand rates, [16] and [15] studied machine capacity expansion when machines can break down randomly. They proposed a capacity planning using the mean available production capacity for a multiple identical machines system. When breakdowns and repairs happen sufficiently fast, they showed that the cost of their expansion plan asymptotically converges to the optimal cost. Unfortunately, as breakdowns increase (failures occurrence become fast), the system availability decreases. Although, there is a great literature based on the using of maintenance to increase production system availability. We refer the reader to [2] and [9] for preventive maintenance and to [4, 9, 10] and references therein, for corrective maintenance. None of those studies, based on production and maintenance planning, examined the case in which capacity expansion is considered. They assumed that the average machines capacity is sufficient to satisfy demand of produced parts in the considered horizon. In stochastic environment, while the system average capacity remains the same with possible demand jumps, the backlogs can increase, possibly without bounds, and hence increasing the overall incurred cost. In this paper, we propose to improve the capacity of the manufacturing system by capacity expansion and by controlling the machines repair rates.

An important question that arises is to know if the contribution of the approach proposed herein in terms of total cost reduction is significant compared to a fixed repair rate situation as in [16]. The theory presented in this paper answers this question in the affirmative under reasonable assumptions (demand rates of various products are constants, the machines are completely flexible, etc). This theory is based on the fact that the structure of the control policy (production, capacity expansion and machine repair rates) can be obtained by using the fact that the value function is the unique solution to the associated Hamilton-Jacobi-Bellaman (HJB) equations. We first used a numerical approach to determine an approximate value function, instead of the true value function, to construct the control policy. Under certain appropriate conditions, the control policy constructed is asymptotically optimal as the difference between the true value function goes to zero (see [10] for details). Finally, we presented a numerical example and a sensitive analysis that illustrate the usefulness of the proposed approach.

The paper is organized as follows: In the next section, we state the model of the problem under consideration. In section 3, we present the HJB equations and show in section 4 that a numerical scheme can provide an approximation of the value function. Then in section 5, we present a numerical example and a sensitive analysis to illustrate the contribution of the paper. The paper is finally concluded in section 6.

2 Problem statement

The system under study consists of n machines producing one part type. The machines capacities are assumed herein to be described by a finite state Markov chain. After the investment in new capacity, the enhanced capacity process is represented by another finite state Markov process having a larger average capacity. The stochastic nature of the system is due to the machines that are subject to random breakdowns and repairs. At any given time, the system is in state $\boldsymbol{k}_1(t) \in \{1, 2, ..., m_1\}$ before capacity purchase and $\boldsymbol{k}_2(t) \in \{1, 2, ..., m_1 + m_2\}$ if there is additional new capacity purchase m_2 at time $t = 0$ with m_1 describing the existing maximum capacity. Each of the process $k_1(t)$, before capacity expansion, and $k_2(t)$, after the capacity expansion, is a Markov chain with state at time t describing the number of operational machines, called here capacity process of the system at time t.

Let $\{\mathcal{F}_1(t)\}$ and $\{\mathcal{F}_2(t)\}$ denoted the filtration generated by $k_1(s)$ and , $k_2(s)$,$0 \leq s < t$ respectively, i.e. $\{\mathcal{F}_1(t)\} = \sigma\{k_1(s), s \leq t)\}$ and $\{\mathcal{F}_2(t)\} = \sigma\{k_2(s), s \leq t)\}$, $0 \leq s \leq t$. We can describe the dynamics of the system by jump processes corresponding to the discrete states of the machines generated by a continuous time and discrete states Markov process $k_1(t)$ or $k_2(t)$ with values in $M_1 = \{1, 2, ..., m_1\}$ or $M_2 = \{1, 2, ..., m_2\}$. For any $\{\mathcal{F}_1(t)\}$-Markov $\tau \geq 0$, the state of the system can then be described by a new process $k(t)$ as follows:

$$k(t) = \begin{cases} k_1(t) & \text{if } t < \tau \\ & \\ k_2(t) & \text{if } t \geq \tau \end{cases} \quad and \quad k(\tau) = k_2(0) := k_1(\tau) + m_2 \tag{1}$$

where τ is the purchase time of additional capacity at a cost K. Note that $0 \leq \tau \leq \infty$ and $\tau = 0$ means not to purchase additional capacity.

Each machine in the system is either up or down and the system is either down $k(t) = 0$ or up $k(t) \neq 0$) having a maximum of $k(t) \neq 0$ units of capacity available at time t. The total production rate at each instant is limited by the capacity of the operational machines. Hence, at time t, the production rate depends on the machines states and thus is subject to sudden changes due to the dynamics of the stochastic process $k(t)$. The production constraint is then defined by:

$$p.u(t) \leq k(t) \tag{2}$$

where $p = (p_1, p_2, ..., p_n)$ is the vector of processing times of the part type on the machine M_i, with $i = 1, ..., m_1$) (before capacity expansion) or $i = 1, ..., m_1 + m_2$ (after capacity expansion); and $u = (u_1, ..., u_n)$ is the vector of the corresponding production rates ($0 \leq u_k \leq U_k^{max}$).

To increase the system availability, we considered that the transition rate from a failure mode to operation mode is a control variable, called here $u_r(t)$. Thus, by controlling $u_r(t)$, one acts on the mean time to repair. The system capacity is then described by a finite state Markov chain that depends on the corrective maintenance policy. The transition rates matrix in such a situation is given by:

$$\boldsymbol{Q}_m(u_r) = \{q^m_{\alpha\beta}\} \tag{3}$$

with $q^m_{\alpha\beta}(u_r) \geq 0$ if $\alpha \neq \beta$ and $q^m_{\alpha\alpha}(u_r) = -\sum_{\beta\neq\alpha} q^m_{\alpha\beta}(u_r)$. where $q^m_{\alpha\beta}(u_r)$ describes the transition rate of the system from mode α to mode β, with $m = 1$ before capacity expansion and $m = 2$ after capacity expansion.

For a production rate $\boldsymbol{u}(t) \in \mathbb{R}^n$, the surplus ($\boldsymbol{x}(t) \in \mathbb{R}^n$), of the manufacturing system under consideration (corresponding to inventory if positive or to backlog if negative) is described by the following equation:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{u}(t) - \boldsymbol{z} \qquad \boldsymbol{x}(0) = \boldsymbol{x} \tag{4}$$

where $\boldsymbol{z} \in \mathbb{R}^n$ denotes the constant demand rate and $\boldsymbol{x}$ the initial surplus level.

For any capacity $k(t) \in M = M_1$ or M_2, let

$$\boldsymbol{U}_{rk(t)} = \{u_{rk}, \boldsymbol{U}^{min}_{rk} \leq u_{rk} \leq \boldsymbol{U}^{max}_{rk}\}$$

$$\boldsymbol{U}_{k(t)} = \{u_{k(t)} = (u_1, u_2, ..., u_n) \geq 0, p_1u_1 + p_2u_2 + ... + p_3u_3 \leq k(t)\}$$

with $u_k \leq \boldsymbol{U}^{max}_k$

The set of admissible decisions at state $k(t)$ is defined by:

$$\Gamma_r(k) = \left\{ \begin{array}{l} (\tau, u_{k(t)}, u_{rk(t)}) = ((\tau, u_1, u_{rk}), (\tau, u_2, u_{rk(t)}), ..., (\tau, u_n, u_{rk(t)})) \\ u_{k(t)} \in bmU_{k(t)}, \boldsymbol{U}^{min}_{rk} \leq u_{rk} \leq \boldsymbol{U}^{max}_{rk}, \quad 0 \leq u_{k(t)} \leq \boldsymbol{U}^{max}_k \end{array} \right\} \tag{5}$$

The control policy at state $k(t)$ is $(\tau, u(.), u_r(.))$ and $\boldsymbol{U}^{min}_{rk}, \boldsymbol{U}^{max}_{rk}, \boldsymbol{U}^{max}_k$, are minimum repair, maximum repair and maximum production rates respectively. Such a policy states that the decisions variables are production rate $\boldsymbol{u}(.)$, repair rate $\boldsymbol{u}_r(.)$ and time of additional capacity purchase $\tau \geq 0$.

The objective of the control problem is to minimize the following discounted function:

$$J(\boldsymbol{x}, \boldsymbol{k}, \tau, \boldsymbol{u}(.), \boldsymbol{u}_r(.)) = E\left\{\int_0^{\infty} e^{-\rho t} G(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{u}_r(t))dt + K e^{-\rho\tau}\right\} \quad (6)$$

where $x(0) = x$; $k(0) = k$;

$G(x, u, u_r) = c_1^+ x_1^+ + c_1^- x_1^- + c_r u_r$ is the instantaneous cost, $c_1^+ \geq 0$, $c_1^- \geq 0$, $c_r \geq 0$ are incurred costs per unit produced parts for inventory, backlog and corrective maintenance respectively. In addition $x^+ = \max\{0, x\}$, $x^- = \max\{-x, 0\}$.

The considered stochastic optimal control problem is to find an admissible control $(\tau, \boldsymbol{u}(.), \boldsymbol{u}_r(.))$ that minimizes $J(.)$ given by equation (6) and subject to equations (1)-(5). This problem is formulated in the next section as a dynamic stochastic optimization problem with a stopping time at which to purchase new capacity, corrective maintenance and production rates over time before and after the acquisition of the new capacity as decision variables.

3 Optimality Conditions

Let $v(\boldsymbol{x}, \boldsymbol{k})$ denote the value function or minimum expected discounted cost if there is no capacity purchase ($\tau = +\infty$):

$$v(\boldsymbol{x}, \boldsymbol{k}) = \inf_{(\infty, \boldsymbol{u}(.), \boldsymbol{u}_r(.) \in \Gamma_r(k))} J(\boldsymbol{x}, \boldsymbol{k}, \tau, \boldsymbol{u}(.), \boldsymbol{u}_r(.)), \qquad \forall k \in M_1 \quad (7)$$

Let us define

$$\boldsymbol{h}(\boldsymbol{x}) = c_1^+ x_1^+ + c_1^- x_1^-$$

$$c(\boldsymbol{u}_r) = c_r u_r$$

and

$$J_1(\boldsymbol{x}, \boldsymbol{k}, \tau, \boldsymbol{u}(.), \boldsymbol{u}_r(.)) = J(\boldsymbol{x}, \boldsymbol{k}, \infty, \boldsymbol{u}(.), \boldsymbol{u}_r(.))$$

$$J_1(\boldsymbol{x}, \boldsymbol{k}, \tau, \boldsymbol{u}(.), \boldsymbol{u}_r(.)) = E\left\{\int_0^{\infty} e^{-\rho t} G(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{u}_r(t))dt\right\}$$

Using the previous notation,$J_1(.)$ can be rewritten as follows:

$$J_1(\boldsymbol{x}, \boldsymbol{k}, \tau, \boldsymbol{u}(.), \boldsymbol{u}_r(.)) = E\left\{\int_0^{\infty} e^{-\rho t}\{\boldsymbol{h}(\boldsymbol{x}(t)) + c(\boldsymbol{u}_r(t))\}dt, \boldsymbol{x}(0) = \boldsymbol{x}; \boldsymbol{k}(0) = \boldsymbol{k}\right\}$$

The value function $v(\boldsymbol{x}, \boldsymbol{k})$ in the case where there is no need to purchase a new capacity is given by the following equation:

$$v(\boldsymbol{x}, \boldsymbol{k}) = \min_{u \in \boldsymbol{U}_k, u_r \in \boldsymbol{U}_{rk}} J_1(\boldsymbol{x}, \boldsymbol{k}, \tau, \boldsymbol{u}(.), \boldsymbol{u}_r(.)) = \inf_{\boldsymbol{u}(.)} \inf_{\boldsymbol{u}_r(.)} J_1(\boldsymbol{x}, \boldsymbol{k}, \tau, \boldsymbol{u}(.), \boldsymbol{u}_r(.)) \quad (8)$$

It is shown in [4] and in [10] that such a value function satisfies the set of Hamilton-Jacobi-Bellman equations given by the following expression:

$$\rho v(\boldsymbol{x},\boldsymbol{k}) = \min_{u\in\boldsymbol{U}_k, u_r\in\boldsymbol{U}_{rk}} \{(\boldsymbol{u}-\boldsymbol{z})v_x(\boldsymbol{x},\boldsymbol{k}) + \boldsymbol{Q}_1(\boldsymbol{u}_r)v(\boldsymbol{x},.)(k) + \boldsymbol{c}(\boldsymbol{u}_r)\} + \boldsymbol{h}(x)$$

$\forall \boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{k} \in M_1$
That is :

$$\rho v(\boldsymbol{x},\boldsymbol{k}) = \min_{u\in\boldsymbol{U}_k, u_r\in\boldsymbol{U}_{rk}} \{(\boldsymbol{u}-\boldsymbol{z})v_x(\boldsymbol{x},\boldsymbol{k}) + \boldsymbol{Q}_1(\boldsymbol{u}_r)v(\boldsymbol{x},.)(k) + \boldsymbol{G}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{u}_r)\}$$

$\forall \boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{k} \in M_1$
where $v_x(\boldsymbol{x},\boldsymbol{k})$ is the gradient $\frac{\partial}{\partial \boldsymbol{x}}v(\boldsymbol{x},\boldsymbol{k})$

Let $v_a(\boldsymbol{x},\boldsymbol{k})$ denote the value function or minimum expected discounted cost if there is a capacity purchase at initial time ($\tau = 0$).

$$v_a(\boldsymbol{x},\boldsymbol{k}+\boldsymbol{m}_2) = \inf_{(0,\boldsymbol{u}(.),\boldsymbol{u}_r(.)\in\Gamma_r(k)} J_1(\boldsymbol{x},\boldsymbol{k},\tau,\boldsymbol{u}(.),\boldsymbol{u}_r(.)), \boldsymbol{k} \in M_1 \qquad (9)$$

From the definition of the process $\boldsymbol{k}(t)$, see equation (1), it follows that:

$$v_a(\boldsymbol{x},\boldsymbol{k}+\boldsymbol{m}_2) \geq v(\boldsymbol{x},\boldsymbol{k}), \boldsymbol{k} \in M_1$$

Let $J_{a0}(.)$ be the cost function when there is capacity purchase at cost $\boldsymbol{K} = 0$ and define the corresponding value function as follows:

$$v_{a0}(\boldsymbol{x},\boldsymbol{k}) = \min_{(\tau,\boldsymbol{u}(.),\boldsymbol{u}_r(.))\in\Gamma_r(k)} J_{a0}(\boldsymbol{x},\boldsymbol{k},\tau,\boldsymbol{u}(.),\boldsymbol{u}_r(.))$$

with

$$J_{a0}(\boldsymbol{x},\boldsymbol{k},\tau,\boldsymbol{u}(.),\boldsymbol{u}_r(.)) = E\left\{\int_0^\infty e^{-\rho t}\{\boldsymbol{h}(\boldsymbol{x}(t)) + c(\boldsymbol{u}_r(t))\}dt\right\}$$

This value function corresponds to the one of the control of production and corrective maintenance rates presented in [10] in the context of multiple identical machines manufacturing systems. The value function $v_{a0}(\boldsymbol{x},\boldsymbol{k})$ satisfies the set of Hamilton-Jacobi-Bellman equations given by the following expression:

$$\rho v_{a0}(\boldsymbol{x},\boldsymbol{k}) = \inf_{u\in\boldsymbol{U}_k, u_r\in\boldsymbol{U}_{rk}} \{(\boldsymbol{u}-\boldsymbol{z})(v_{a0})_x(\boldsymbol{x},\boldsymbol{k}) + \boldsymbol{Q}_1(\boldsymbol{u}_r)v_{a0}(\boldsymbol{x},.)(k) + \boldsymbol{G}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{u}_r)\}$$

$\forall \boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{k} \in M_1$
with $(v_{a0})_x(\boldsymbol{x},\boldsymbol{k})$, the gradient $\frac{\partial}{\partial \boldsymbol{x}}v_{a0}(\boldsymbol{x},\boldsymbol{k})$

From equation 9, we have

$$v_a(\boldsymbol{x},\boldsymbol{k}+\boldsymbol{m}_2)=\inf_{(0,\boldsymbol{u}(.),\boldsymbol{u}_r(.)\in\Gamma_r(k)}J_1(\boldsymbol{x},\boldsymbol{k},0,\boldsymbol{u}(.),\boldsymbol{u}_r(.)),\boldsymbol{k}\in M_1$$

and

$$\begin{aligned}v_a(\boldsymbol{x},\boldsymbol{k})&=\inf_{(0,\boldsymbol{u}(.),\boldsymbol{u}_r(.)\in\Gamma_r(k)}J_1(\boldsymbol{x},\boldsymbol{k},0,\boldsymbol{u}(.),\boldsymbol{u}_r(.)),\boldsymbol{k}\in M_2\\&=\inf_{(0,\boldsymbol{u}(.),\boldsymbol{u}_r(.)\in\Gamma_r(k)}E\left\{\int_0^\infty e^{-\rho t}G(\boldsymbol{x}(t),\boldsymbol{u}(t),\boldsymbol{u}_r(t))dt+Ke^{-\rho\tau}\right\}\end{aligned}$$

Thus,

$$\begin{aligned}v_a(\boldsymbol{x},\boldsymbol{k})&=\inf_{(0,\boldsymbol{u}(.),\boldsymbol{u}_r(.)\in\Gamma_r(k)}E\left\{\int_0^\infty e^{-\rho t}G(\boldsymbol{x}(t),\boldsymbol{u}(t),\boldsymbol{u}_r(t))dt\right\}+K\\&=\inf_{(0,\boldsymbol{u}(.),\boldsymbol{u}_r(.)\in\Gamma_r(k)}J_{a0}(\boldsymbol{x}(t),\boldsymbol{k}(t),0,\boldsymbol{u}(.),\boldsymbol{u}_r(t))dt+K\end{aligned}$$

Hence,

$$v_a(\boldsymbol{x},\boldsymbol{k})=(v_a)_{\tau=0}(\boldsymbol{x},\boldsymbol{k})+\boldsymbol{K}$$

We then have :

$$\rho v_a(\boldsymbol{x},\boldsymbol{k})=\inf_{u\in\boldsymbol{U}_k,u_r\in\boldsymbol{U}_{rk}}\{(\boldsymbol{u}-\boldsymbol{z})(v_a)_x(\boldsymbol{x},\boldsymbol{k})+\boldsymbol{Q}_1(\boldsymbol{u}_r)v_a(\boldsymbol{x},.)(k)+\boldsymbol{c}(\boldsymbol{u}_r)+\boldsymbol{h}(x)\}+\boldsymbol{K}$$

$\forall\boldsymbol{x}\in\mathbb{R}^n,\boldsymbol{k}\in M_2$

Since we are interested in optimal purchase time, optimal production and corrective maintenance rates, we write the HJB equations as follows:

$$\begin{cases}\min\{\min_{u\in\boldsymbol{U}_k,u_r\in\boldsymbol{U}_{rk}}\{(\boldsymbol{u}-\boldsymbol{z})v_x(\boldsymbol{x},\boldsymbol{k})+\boldsymbol{G}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{u}_r)\}+\boldsymbol{Q}_1(\boldsymbol{u}_r)v(\boldsymbol{x},\boldsymbol{k})\\-\rho v(\boldsymbol{x},.)(k),v_a(\boldsymbol{x},\boldsymbol{k}+m_2)-v(\boldsymbol{x},\boldsymbol{k})\}\}=0\qquad\boldsymbol{k}\in M_1\\\min_{u\in\boldsymbol{U}_k,u_r\in\boldsymbol{U}_{rk}}\{(\boldsymbol{u}-\boldsymbol{z})(v_a)_x(\boldsymbol{x},\boldsymbol{k})+\boldsymbol{G}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{u}_r)\}+\boldsymbol{Q}_2(\boldsymbol{u}_r)v_a(\boldsymbol{x},.)(k)\\-\rho(v_a(\boldsymbol{x},\boldsymbol{k})-\boldsymbol{K})=0,\qquad\boldsymbol{k}\in M_2\end{cases}\tag{10}$$

The optimal control policy $(\tau^*,\boldsymbol{u}^*,\boldsymbol{u}_r^*)$ denotes a minimizer over of the right hand side of equation 10. This policy corresponds to the value function described by equations (7) and (9). Then, when the value functions are available, an optimal control policy can be obtained as in (10). However, an analytical solution of (10) is almost impossible to obtain. In the next section, we construct a near optimal control policy through numerical methods. It is by now well known that an approximation of the corresponding control policy or near optimal control policy can be obtained by a small perturbation of the true value function. This can be done by using numerical techniques which provide a close form of the value function under reasonable assumptions.

4 Numerical Approach

In this section, we develop the numerical method for solving the optimality conditions presented in the previous section. This method is based on the Kushner approach (see [12, 2, 10] for details). The main idea behind this approach consists of using an approximation scheme for the gradient of the value functions $v(\boldsymbol{x},\boldsymbol{k})$ and $v_a(\boldsymbol{x},\boldsymbol{k})$. Let h denotes the length of the finite difference interval of the variable x. Using h, $v(\boldsymbol{x},\boldsymbol{k})$ is approximated by $v^h(\boldsymbol{x},\boldsymbol{k})$ and $v_x(\boldsymbol{x},\boldsymbol{k})$ is approximated as in equation (11).

$$v_x(\boldsymbol{x},\boldsymbol{k})(\boldsymbol{u}-\boldsymbol{z}) = \begin{cases} \frac{1}{h}\Big(v^h(\boldsymbol{x}+\boldsymbol{h},\boldsymbol{k}) - v^h(\boldsymbol{x},\boldsymbol{k})(\boldsymbol{u}-\boldsymbol{z})\Big) \text{ if } \boldsymbol{u}-\boldsymbol{z}>0 \\ \frac{1}{h}\Big(v^h(\boldsymbol{x},\boldsymbol{k}) - v^h(\boldsymbol{x}-\boldsymbol{h},\boldsymbol{k})(\boldsymbol{u}-\boldsymbol{z})\Big) \text{ otherwise} \end{cases} \tag{11}$$

Using h, $(v_a)_x(\boldsymbol{x},\boldsymbol{k})$ is approximated as in equation (12).

$$(v_a)_x(\boldsymbol{x},\boldsymbol{k})(\boldsymbol{u}-\boldsymbol{z}) = \begin{cases} \frac{1}{h}\Big(v_a^h(\boldsymbol{x}+\boldsymbol{h},\boldsymbol{k}) - v_a^h(\boldsymbol{x},\boldsymbol{k})(\boldsymbol{u}-\boldsymbol{z})\Big) \text{ if } \boldsymbol{u}-\boldsymbol{z}>0 \\ \frac{1}{h}\Big(v_a^h(\boldsymbol{x},\boldsymbol{k}) - v_a^h(\boldsymbol{x}-\boldsymbol{h},\boldsymbol{k})(\boldsymbol{u}-\boldsymbol{z})\Big) \text{ otherwise} \end{cases} \tag{12}$$

With approximations given by equations (11) and (12) and after a couple of manipulations, the HJB equations (10) can be rewritten as follows:

$$\begin{aligned} v^h(\boldsymbol{x},\boldsymbol{k}) \;=\; & \min_{\boldsymbol{u}\in\boldsymbol{U}_k^h}\min_{\boldsymbol{u}_r\in\boldsymbol{U}_{rk}^h}\frac{1}{\rho+\boldsymbol{Q}_h^{k_1}}(\boldsymbol{u}_r)\Big\{\Big((v^h(\boldsymbol{x}+\boldsymbol{h},\boldsymbol{k})\boldsymbol{P}_x^k(1) \\ & +(v^h(\boldsymbol{x}-\boldsymbol{h},\boldsymbol{k})\boldsymbol{P}_x^k(2)+\boldsymbol{G}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{u}_r) \\ & +\sum_{\boldsymbol{k}'\neq\boldsymbol{k}}\boldsymbol{q}^1_{\boldsymbol{k}\boldsymbol{k}'}(\boldsymbol{u}_r)v^h(\boldsymbol{x},\boldsymbol{k}')\Big\} \qquad \boldsymbol{k}_1\in M_1 \end{aligned} \tag{13}$$

$$\begin{aligned} v_a^h(\boldsymbol{x},\boldsymbol{k}) \;=\; & \min_{\boldsymbol{u}\in\boldsymbol{U}_k^h}\min_{\boldsymbol{u}_r\in\boldsymbol{U}_{rk}^h}\frac{1}{\rho+\boldsymbol{Q}_h^{k_2}}(\boldsymbol{u}_r)\Big\{\Big((v_a^h(\boldsymbol{x}+\boldsymbol{h},\boldsymbol{k})\boldsymbol{P}_x^k(1) \\ & +(v_a^h(\boldsymbol{x}-\boldsymbol{h},\boldsymbol{k})\boldsymbol{P}_x^k(2)+\boldsymbol{G}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{u}_r) \\ & +\sum_{\boldsymbol{k}'\neq\boldsymbol{k}}\boldsymbol{q}^2_{\boldsymbol{k}\boldsymbol{k}'}(\boldsymbol{u}_r)v_a^h(\boldsymbol{x},\boldsymbol{k}')+\rho\boldsymbol{K}\Big\} \qquad \boldsymbol{k}_1\in M_2 \end{aligned} \tag{14}$$

where $(\boldsymbol{U}^{\boldsymbol{h}}_{\boldsymbol{k}},\boldsymbol{U}^{\boldsymbol{h}}_{\boldsymbol{k},\boldsymbol{r}})$ is the discrete feasible control space or the so-called control grid and the other terms used in equations (13) and (14) are defined as follows:

$$\boldsymbol{Q}_h^{k_i}(\boldsymbol{u}_r) = |\frac{\boldsymbol{u}-\boldsymbol{z}}{h}| + |\boldsymbol{q}^i_{\boldsymbol{k}_i\boldsymbol{k}_i}(\boldsymbol{u}_r)| \qquad \boldsymbol{i}=1,2$$

$$\boldsymbol{P}_x^k(1) = \begin{cases} \frac{\boldsymbol{u}-\boldsymbol{z}}{h} & \text{if } \boldsymbol{u}-\boldsymbol{z}\geq 0 \\ 0 \text{ otherwise} \end{cases}$$

$$\boldsymbol{P}_x^k(2) = \begin{cases} \frac{\boldsymbol{z}-\boldsymbol{u}}{h} & \text{if } \boldsymbol{u}-\boldsymbol{z} < 0 \\ 0 \text{ otherwise} & \end{cases}$$

The system of equations (13) and (14) can be interpreted as the infinite horizon dynamic programming equation of a discrete-time, discrete-state decision process as in [2], [10], for capacity expansion, production and maintenance planning problems. The obtained discrete event dynamic programming can be solved using either policy improvement or successive approximation methods.

The next theorem shows that $(v^h(\boldsymbol{x},\boldsymbol{k})$ and $(v_a^h(\boldsymbol{x},\boldsymbol{k})$ are approximations to $(v(\boldsymbol{x},\boldsymbol{k})$ and $(v_a(\boldsymbol{x},\boldsymbol{k})$for small step size h.

4.1 THEOREM

Let $(v^h(\boldsymbol{x},\boldsymbol{k})$ and $(v_a^h(\boldsymbol{x},\boldsymbol{k})$ denote a solution to HJB equations (13) and (14). Assume that there are constants C_g and K_g, C_g^a and K_g^a such that

$$0 \leq (v^h(\boldsymbol{x},\boldsymbol{k}) \leq C_g(1+|\boldsymbol{x}|^{K_g})$$

$$0 \leq (v_a^h(\boldsymbol{x},\boldsymbol{k}) \leq C_g^a(1+|\boldsymbol{x}|^{K_g^a})$$

then

$$\lim_{h\to 0} v^h(\boldsymbol{x},\boldsymbol{k}) = v(\boldsymbol{x},\boldsymbol{k})$$
$$\lim_{h\to 0} v_a^h(\boldsymbol{x},\boldsymbol{k}) = v_a(\boldsymbol{x},\boldsymbol{k})$$

PROOF: The proof of this theorem can be obtained by extending the one presented in [17].

In this paper, we use the policy improvement technique, given by the following algorithm, to obtain a solution of the approximating optimization problem.
Step 1: Initialisation
Choose $\delta \in \mathbb{R}^+$, set $\boldsymbol{n} := 1$, $(v^h(\boldsymbol{x},\boldsymbol{k}))^n := 0$, $(v_a^h(\boldsymbol{x},\boldsymbol{k}))^n := 0$, $\forall \boldsymbol{k} \in M_{1,2}, \forall \boldsymbol{x} \in G_x^h$
Step 2: Compute

$$(v^h(\boldsymbol{x},\boldsymbol{k}))^{n-1} := (v^h(\boldsymbol{x},\boldsymbol{k}))^n \qquad \forall \boldsymbol{k} \in M_1, \forall \boldsymbol{x} \in G_x^h$$
$$(v_a^h(\boldsymbol{x},\boldsymbol{k}))^{n-1} := (v_a^h(\boldsymbol{x},\boldsymbol{k}))^n \qquad \forall \boldsymbol{k} \in M_2, \forall \boldsymbol{x} \in G_x^h$$

Step 3: Compute the correspondent value function to obtain the control policy:$(\tau, \boldsymbol{u}(.), \boldsymbol{u}_r(.))$

Step 4: Test the convergence

$$\begin{aligned}
\boldsymbol{c}_1^- &= \min_{x\in \boldsymbol{G}_x^h}((v^h(\boldsymbol{x},\boldsymbol{k}))^n - (v^h(\boldsymbol{x},\boldsymbol{k}))^{n-1})\\
\boldsymbol{c}_1^+ &= \max_{x\in \boldsymbol{G}_x^h}((v^h(\boldsymbol{x},\boldsymbol{k}))^n - (v^h(\boldsymbol{x},\boldsymbol{k}))^{n-1})\\
\boldsymbol{c}_2^- &= \min_{x\in \boldsymbol{G}_x^h}((v_a^h(\boldsymbol{x},\boldsymbol{k}))^n - (v_a^h(\boldsymbol{x},\boldsymbol{k}))^{n-1})\\
\boldsymbol{c}_2^+ &= \max_{x\in \boldsymbol{G}_x^h}((v_a^h(\boldsymbol{x},\boldsymbol{k}))^n - (v_a^h(\boldsymbol{x},\boldsymbol{k}))^{n-1})\\
\boldsymbol{c}_1^{\min} &= \frac{\rho}{1-\rho}\boldsymbol{c}_1^-, \qquad \boldsymbol{c}_1^{\max} = \frac{\rho}{1-\rho}\boldsymbol{c}_1^+\\
\boldsymbol{c}_2^{\min} &= \frac{\rho}{1-\rho}\boldsymbol{c}_2^-, \qquad \boldsymbol{c}_2^{\max} = \frac{\rho}{1-\rho}\boldsymbol{c}_2^+
\end{aligned}$$

If $\min(|\boldsymbol{c}_1^{\max} - \boldsymbol{c}_1^{\min}|, |\boldsymbol{c}_2^{\max} - \boldsymbol{c}_2^{\min}|) \le \delta$, then stop; else $\boldsymbol{n} := \boldsymbol{n} + 1$ and go to step 2

If $\min(|\boldsymbol{c}_1^{\max} - \boldsymbol{c}_1^{\min}|, |\boldsymbol{c}_2^{\max} - \boldsymbol{c}_2^{\min}|) = |\boldsymbol{c}_1^{\max} - \boldsymbol{c}_1^{\min}|$, then there is no capacity expansion, else there is a capacity expansion.

with $\boldsymbol{G}_x^h$, the state space grid related to the surplus $\boldsymbol{x}$ and a given value of $\boldsymbol{h}$. The boundary conditions presented in [17] are used here with the previous algorithm to solve the optimality conditions given by equations (13) and (14).

5 Numerical example and sensitivity analysis

Let us consider a firm with a two states Markov process $\boldsymbol{k}_1 \in M_1 = \{0,1\}$ describing the capacity of the system before expansion. After capacity expansion, a three states Markov process $\boldsymbol{k}_2 \in M_2 = \{0,1,2\}$ describes the capacity of the system. The discrete dynamic programming equations (13) and (14) give the equations (15) and (15) before capacity purchase:

$$\begin{aligned}
v^h(\boldsymbol{x},0) \quad = \quad & \min_{\boldsymbol{u}_r\in \boldsymbol{U}_{r0}} \frac{1}{\rho + \boldsymbol{Q}_h^{01}(\boldsymbol{u}_r)}\Big\{\Big((v^h(\boldsymbol{x}+\boldsymbol{h},0)\boldsymbol{P}_x^0(1)\\
& +(v^h(\boldsymbol{x}-\boldsymbol{h},0)\boldsymbol{P}_x^0(2) + \boldsymbol{G}(\boldsymbol{x},0,\boldsymbol{u}_r) + \boldsymbol{q}_{01}^1(\boldsymbol{u}_r)v^h(\boldsymbol{x},1)\Big\}
\end{aligned}$$

$$\begin{aligned}
v^h(\boldsymbol{x},1) \quad = \quad & \frac{1}{\rho + \boldsymbol{Q}_h^{11}} \min_{\boldsymbol{u}_1\in \boldsymbol{U}_1} \Big\{\Big((v^h(\boldsymbol{x}+\boldsymbol{h},1)\boldsymbol{P}_x^1(1)\\
& +(v^h(\boldsymbol{x}-\boldsymbol{h},1)\boldsymbol{P}_x^1(2) + \boldsymbol{G}(\boldsymbol{x},\boldsymbol{u}_1,0) + \boldsymbol{q}_{10}^1(\boldsymbol{u}_r)v^h(\boldsymbol{x},0)\Big\}
\end{aligned}$$

and equations (15) to (17) after capacity purchase:

$$v_a^h(\boldsymbol{x},0) = \min_{\boldsymbol{u}_r\in\boldsymbol{U}_{r0}} \frac{1}{\rho+\boldsymbol{Q}_h^{02}(\boldsymbol{u}_r)}\Big\{\Big((v_a^h(\boldsymbol{x}+\boldsymbol{h},0)\boldsymbol{P}_x^0(1) + (v_a^h(\boldsymbol{x}-\boldsymbol{h},0)\boldsymbol{P}_x^0(2) + \boldsymbol{G}(\boldsymbol{x},0,\boldsymbol{u}_r) + \boldsymbol{q}_{01}^2(\boldsymbol{u}_r)v_a^h(\boldsymbol{x},1) + \boldsymbol{q}_{02}^2(\boldsymbol{u}_r)v_a^h(\boldsymbol{x},2) + \rho\boldsymbol{K}\Big\} \quad (15)$$

$$v_a^h(\boldsymbol{x},1) = \min_{\boldsymbol{u}_r\in\boldsymbol{U}_{r1}} \Big\{\frac{1}{\rho+\boldsymbol{Q}_h^{12}(\boldsymbol{u}_r)} \min_{\boldsymbol{u}_1\in\boldsymbol{U}_1}\Big\{\Big((v_a^h(\boldsymbol{x}+\boldsymbol{h},1)\boldsymbol{P}_x^1(1) + (v_a^h(\boldsymbol{x}-\boldsymbol{h},1)\boldsymbol{P}_x^1(2) + \boldsymbol{G}(\boldsymbol{x},\boldsymbol{u}_1,\boldsymbol{u}_r) + \boldsymbol{q}_{10}^2 v_a^h(\boldsymbol{x},0) + \boldsymbol{q}_{12}^2(\boldsymbol{u}_r)v_a^h(\boldsymbol{x},2) + \rho\boldsymbol{K}\Big\}\Big\} \quad (16)$$

$$v_a^h(\boldsymbol{x},2) = \frac{1}{\rho+\boldsymbol{Q}_h^{21}(\boldsymbol{u}_r)} \min_{\boldsymbol{u}_2\in\boldsymbol{U}_2}\Big\{\Big((v_a^h(\boldsymbol{x}+\boldsymbol{h},2)\boldsymbol{P}_x^2(2) + (v_a^h(\boldsymbol{x}-\boldsymbol{h},2)\boldsymbol{P}_x^2(2) + \boldsymbol{G}(\boldsymbol{x},\boldsymbol{u}_2,0) + \boldsymbol{q}_{20}^2 v_a^h(\boldsymbol{x},0) + \boldsymbol{q}_{21}^2(\boldsymbol{u}_r)v_a^h(\boldsymbol{x},1) + \rho\boldsymbol{K}\Big\} \quad (17)$$

We use the following computational domain:

$$\boldsymbol{G}_x^h = \{\boldsymbol{x} : -5 \le \boldsymbol{x} \le 25, \quad \boldsymbol{x}(i) = -5 + (i-1)\boldsymbol{h}, \quad i = 1,2,...\}$$

Other parameters of the considered manufacturing system are given in table (1)

Table 1: Parameters of the considered manufacturing system

c^+	c^-	c_r	$\boldsymbol{U}_2^{max}$	$\boldsymbol{U}_1^{max}$	z	$\boldsymbol{K}$	ρ	q_{10}^1	$q_{01}^{1\min}$
1	15	100	0.2	0.4	0.12	50 000	0.001	0.05	0.4

h	$q_{01}^{1\max}$	$q_{01}^{2\min}$	$q_{01}^{2\max}$	q_{02}^2	q_{10}^2	$q_{12}^{2\min}$	$q_{12}^{2\max}$	q_{20}^2	q_{21}^2
0.1	0.6	0.4	0.6	0	0.05	0.05	0.1	0	0.05

The policy iteration technique is used to solve the optimality conditions related to equations ((15) to (17)). Obtained results are presented in figures (1) and (2).

Figure 1 gives simultaneously the optimal purchase time (associated to $\boldsymbol{X}_{op}$) and the production rate according to the initial inventory level. It is interesting to note from figure 1 that if the initial inventory level is less than $\boldsymbol{X}_{op} = -2.10$, then the optimal purchasing time will be at initial production time, meaning that $\tau = 0$. Backlog values under $\boldsymbol{X}_{op}$, corresponding to the initial inventory of the products used for the firm to absorb the excessive demand for a few initial periods, as stated in section 1. Thus, the optimal control policy suggest to purchase a new capacity when the initial surplus (i.e., backlog if negative) is under $\boldsymbol{X}_{op}$.

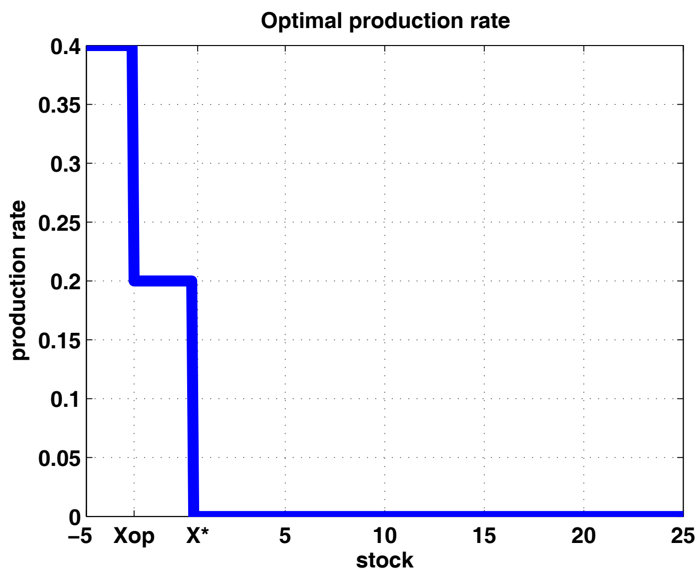

Figure 1: Optimal production rate

After a purchase of a new capacity, the production rate of each machine (the existing and the new machines) is set to its maximal value to satisfy unmet demands (backlog) and to built the safety stock described by the threshold value $\boldsymbol{X}^*$. The production control policy obtained is an extension to the so-called hedging point policy given that the previous behavior respect the structure presented in [1] for production without maintenance and capacity expansion. The obtained modified hedging point policy, characterized by the switching trend illustrated by figure 1, is given by the following equation:

$$\boldsymbol{u}(\boldsymbol{t}, \boldsymbol{x}, \boldsymbol{k}) = \begin{cases} \boldsymbol{U}_2^{max} & \text{if } \boldsymbol{x} \leq \boldsymbol{X}_{op} \\ \boldsymbol{U}_1^{max} & \text{if } \boldsymbol{X}_{op} < \boldsymbol{x} < \boldsymbol{X}^* \\ 0 & \text{if } \boldsymbol{x} \geq \boldsymbol{X}^* \end{cases} \tag{18}$$

The corrective maintenance policy, plotted in figure 2, divides the computational domain $\boldsymbol{G}_x^h$ into two regions after capacity expansion (i.e., $[-5, \boldsymbol{X}_{r_2}[$ and $[\boldsymbol{X}_{r_1}, \boldsymbol{X}_{r_2}[$) and another two regions before capacity expansion (i.e., and $[\boldsymbol{X}_{r_1}, \boldsymbol{X}_r^*[$ and $[\boldsymbol{X}_r^*, 25])$

with capacity expansion, if the stock level is under $\boldsymbol{X}_{r_2}$ and a failure occurs, one have to repair the failed machine at the maximal repair rate q_{23}^{max} while the

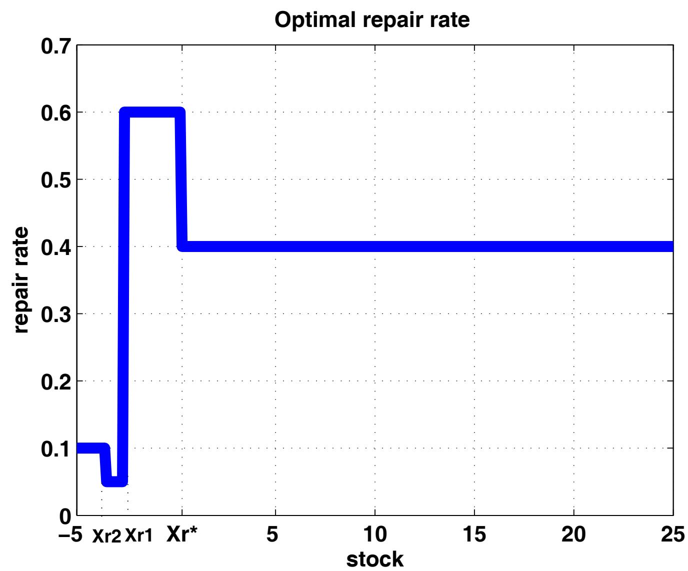

Figure 2: Optimal repair rate

minimal repair rate q_{23}^{min} is applied in $[\boldsymbol{X}_{r_1}, \boldsymbol{X}_{r_2}[$. Without capacity expansion, if the stock level is under $\boldsymbol{X}_r^*$ and a failure occurs, one have to repair the failed machine at the maximal repair rate q_{12}^{max} while the minimal repair rate q_{12}^{min} is applied in $[\boldsymbol{X}_r^*, 25]$. The corrective maintenance policy, illustrated in figure 2, is then summarized as in equation (19).

For this example, $\boldsymbol{X}_r^* = 0.20$, $\boldsymbol{X}_{r_1} = -2, 10$ and $\boldsymbol{X}_{r_2} = -3.60$

$$\boldsymbol{u}_r(\boldsymbol{x}, \boldsymbol{k}) = \begin{cases} \boldsymbol{q}_{23}^{max} & \text{if } \boldsymbol{x} \leq \boldsymbol{X}_{r_2} \\ \boldsymbol{q}_{23}^{min} & \text{if } \boldsymbol{X}_{r_2} < \boldsymbol{x} < \boldsymbol{X}_{r_1} \\ \boldsymbol{q}_{12}^{max} & \text{if } \boldsymbol{X}_{r_1} < \boldsymbol{x} < \boldsymbol{X}^* \\ \boldsymbol{q}_{12}^{min} & \text{if } \boldsymbol{x} \geq \boldsymbol{X}^* \end{cases} \tag{19}$$

The control policy described by equations (18)-(19) is completely defined for given values of parameters in the case of two-identical machine as in the example presented in this section.

5.1 Sensitivity analysis and comparative study

We performed a couple of experiments using the numerical example presented previously. A set of analysis have then been considered to illustrate the sensitivity of the obtained control policy with respect to capacity purchase, inventory, backlog, maintenance costs and machines availability. The results presented in table 2 illustrate four different situations used to show the variation of the production and new capacity purchasing parameters when the purchase cost increase and when the repair rate is controlled or not.

Table 2: Impact of the corrective maintenance policy on purchase time and production policy

-	$\boldsymbol{K}$	c_r	c_-	$\boldsymbol{X}_{op}$	$\boldsymbol{X}^*$
Without control of machine repair rate	5 000	-	15	0	0.7
With control of machine repair rate	5 000	100	15	0.1	0.1
Without control of machine repair rate	80 000	-	15	-3.9	0.7
With control of machine repair rate	80 000	100	15	-4.6	0.4

Table 2 shows that controlling machines repair rates (corrective maintenance) reduces the optimal purchase time and production threshold level. The proposed joint optimization of capacity expansion, production and maintenance activities significantly reduces the overall incurred cost compared to separate optimization models as shown in figure 3.

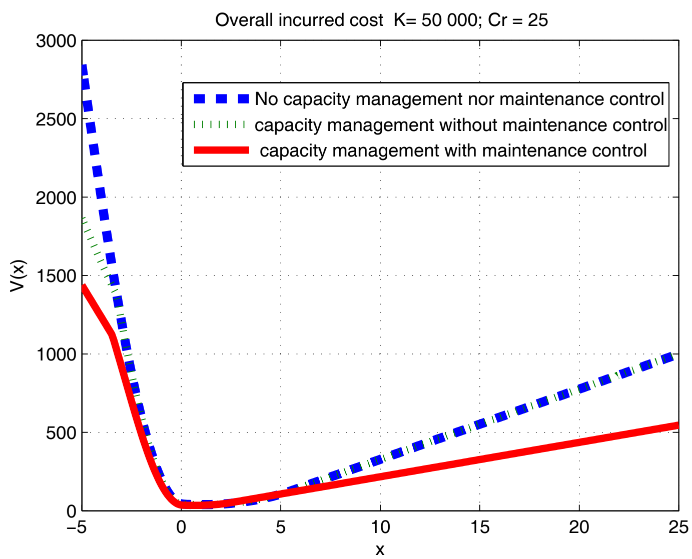

Figure 3: Overall incurred cost

As one can observe from figure 3, there is a significant difference between the system performances in three situations. Such situations are:
- Production planning with no capacity expansion and no corrective maintenance as in [1]
- Production planning with capacity expansion and no corrective maintenance as in [16]
- Production planning with capacity expansion and corrective maintenance as in this paper for the control policy given by equation (18)-(19).
For a comparative purpose, we used the algorithm presented in section 4 to solve the optimality conditions or HJB equations of the above three situations

and to obtain results presented in figure 3. Through the observations made from figure 3, it clearly appears that the proposed approach, based on a simultaneous control of capacity expansion, production and corrective maintenance rates, provides interesting results in the context of manufacturing systems under uncertainties, for given initial inventory levels.

For a m-identical machines manufacturing system producing one part type, the capacity expansion, the production and corrective maintenance policies could be defined by $2\boldsymbol{m}+1$ parameters or input factors. The experimental design approach, combined to simulation and analytical models could be used to determine the effects of considered factors on the incurred costs and to determine their optimal values. Details on experimental design and simulation modeling could be find in [8] and references therein.

6 Conclusion

In this paper, we develop a capacity expansion model for one-product, multiple-machine manufacturing systems with constant demand. We develop an effective solution approach to determine the optimal capacity purchase time, production and maintenance decisions over time. The introduction of the maintenance planning increases the availability of the production system, which guarantees the improvement of the system's productivity. Through a computational study, we show the effectiveness of the proposed models in terms of contribution in the control theory area and investigate the impact of capacity cost, maintenance control and other important parameters on the control policy.

In this work, we have only considered the model with a single capacity purchase, constant demand, equipment purchase and capacity expansion, mainly for simplicity, with-out loosing the generality of the proposal. An extension of the proposed models could significantly reduce the overall incurred cost if it incorporates models with any finite number of capacity purchases, stochastic demand or capacity reduction. In addition to machine purchase, the use of other resources, define as any part of the system that is not consumed or transformed during the production process, for capacity expansion could be considered.

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