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A Heuristic Approach for Solving Serially Distributed Storage Depots under Power-of-Two Policy

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Abstract

In this study, we present the problem of shipping a single-product from a single supply origin, through storage depots, to a single demand destination by trucks, ships, etc. and aim to minimize the sum of ordering, inventory and transportation costs over an infinite time horizon. We formulate the problem consisting of power-of-two (PoT), stationary and nested replenishment policies for the multistage inventory-distribution system. We then present the cost structure analysis for this serially distributed storage depot problem ($SDSDP$) and develop a heuristic method to obtain satisfactory results. On average, the average cost deviations between our proposed heuristic and $LINGO$ software are within 1.2%.

Mathematics Subject Classification: 90B05, 90B06, 90B40

Keywords: Multistage inventory control, Serial inventory-distribution systems, Power-of-two policy

1 Introduction

The serially distributed storage depot problem under PoT policy ($SDSDP\_PoT$) is investigated in this study. Maxwell and Muckstadt [10] and Jackson et al. [6] assumed that a basic period times general integers or power-of-two integers as the shipping frequencies for products. Love [9] and Muckstadt and Roundy [11] analyzed the serial logistic systems, i.e. multistage production systems, and introduced the concept of nested policies for more practical concerns. The so-called stationary policy is the one where each facility receives a constant ordering batch in equally spaced time. A nested policy is the one if storage depot i orders, each of its successors would be triggered to place orders. Gupta

[5] coped with a lot-size model in which the explicitly discrete and fixed transportation cost is incurred due to the ordering quantity delivered by trucks whether those are partial or fully loaded. Vroblefski et al. [16] is motivated by serially distributed warehouse logistics at Western New York, a large paint manufacturing company. The company avoids difficulties in tracking and cost management, i.e. a warehouse receiving from several higher-echelon facilities is excluded, and re-designs their logistic system as the streams of serial distribution centers. They developed a cluster algorithm and a hypercube method to decide the ordering quantity for each serially located warehouse with one or multiple transportation freight rates, respectively. Speranza and Ukovich [15] pointed out that most of the long distance shipments (international and overseas) fit in the single link case and the single link is an important building block to approach more complex problems. Likewise, Bertazzi and Speranza [1] also present the same single link problem and discuss about the worst-case performance of the full load policy applied in the capacity of the vehicles. Romeijn et al. [13] stated that a modern distribution network design model needed to take a number of factors into consideration. Those factors include (1) location and associated operating cost of distribution center; (2) total transportation costs; (3) storage holding and replenishment costs at the distribution center and retail outlets; (4) stockouts; (5) capacity concerns. Gallego and Özer [4] provided a dynamic programming (DP) formulation based on the idea of optimally allocating a given echelon-inventory level between the upstream stage and the downstream serial system. Shang and Song [14] develop a closed-form approximation for optimal base-stock levels for serial inventory systems. Eynan and Kropp [2] and Rao [12] studied the single-stage (R, nT) inventory model, where T is the fixed reorder interval, n is a positive integer, and R is the order-up-to base-stock level. Lately, Feng and Rao [3] studied an (R, nT) policy in a two-stage serial inventory system with stochastic demand. Lee and Wen [8] discussed the issues about the $SDSDP$ under the general-integer inventory replenishment policy. This paper continues focusing on the $SDSDP$ with stationary, nested and PoT replenishment policies and differs from the previous studies in two aspects: (1) although Vroblefski et al. [16] dealt with the problems consisting of serially distributed warehouses, they set the base lot size needed to be transported as a known parameter. However, most of the previous literature as we cited above mention that PoT replenishment policies consider the base lot size as a variable due to the practical implementation. Here, we deal with the base lot size as a variable and put more emphasis on developing analytical insights. For this purpose, we explore the cost-curve structure for the mathematical model so that an efficient search heuristic can be developed accordingly.(2) We do not employ the best integer approximation to the cost break mentioned in Vroblefski et al. [16] as our focused factor because the role of the base lot size is different from that in the original paper.

2 Problem Statement

Vroblefski et al. [16] consider a serial warehousing system designed for shipping only one product from a single supply source to a single retailer or customer destination, which is arranged in the order of $\{n,\ (n-1),\ \ldots\ ,\ 1\}$ where n is denoted as the number of warehouses. This serial system is represented as shown in Figure 1.

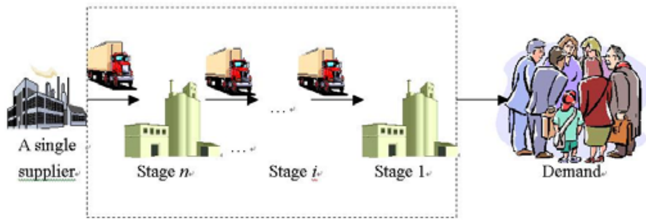

Figure 1: The serial inventory-distribution system

Two-level or multi-level transportation cost problem means that one or multiple common shipment discount volumes occur in all of warehouses, respectively. The $SDSDP\_PoT$ also takes two-level and multi-level cases for each storage depot into account. Assumptions made in the $SDSDP\_PoT$ are as follows: (1) shortages are not allowed; (2) transportation and replenishment occur instantaneously, right after an order is placed; (3) the planning horizon is infinite; (4) the unit transportation cost for each storage depot is assumed to be nonincreasing over the quantity shipped; and (5) the transportation discount quantity occurs at the same transportation volume for all storage depots. Accordingly, a mathematical model presents the above system under both PoT policy and two-level transportation cost condition denoted as $P_{2,TL}$, which is described as follows:

$P_{2,TL}$:

$$\text{Minimize} \sum_{i=1}^{n} \frac{K_i \lambda}{q_i} + \sum_{i=1}^{n} \frac{h_i q_i}{2} + \sum_{i=1}^{n} \lambda \left\{ P'_{i1} y_{i1} + P_{i0} \right\} \tag{1}$$

subject to

$$y_{i1} \in \{0, 1\} \qquad \forall i = 1, 2, \ldots , n, \tag{1a}$$

$$y_{i1}b - q_i \leq 0 \quad \forall i = 1, 2, \ldots, n, \tag{1b}$$

$$q_i > 0 \qquad \forall i = 1, 2, \ldots, n, \tag{1c}$$

$$q_i = 2^{\alpha_i} q_L \qquad \forall i = 1, 2, \ldots, n, \tag{1d}$$

$$\alpha_i \geq 0, \text{ integer} \quad \forall i = 1, 2, \ldots, n, \tag{1e}$$

$$q_i \geq q_{i-1} \qquad \forall i = 2, 3, \ldots, n. \tag{1f}$$

Another case, the mathematical model of multi-level transportation cost with PoT policy can be presented below and denoted as $P_{2,ML}$:

$P_{2,ML}$:

$$\text{Minimize} \sum_{i=1}^{n} \frac{K_i \lambda}{q_i} + \sum_{i=1}^{n} \frac{h_i q_i}{2} + \sum_{i=1}^{n} \lambda \left\{ \sum_{j=1}^{m} P'_{ij} y_{ij} + P_{i0} \right\} \tag{1'}$$

subject to

$$y_{ij} \in \{0, 1\} \quad \forall i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, m, \tag{1a'}$$

$$y_{ij} b_j - q_i \leq 0 \quad \forall i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, m, \tag{1b'}$$

with the constraints (1c), (1d), (1e), and (1f).

The corresponding parameters are organized as follows:

- i= an index of the storage depot, $i = 1, 2, \ldots, n$.
- K_i= a fixed ordering cost per order charged for each storage depot i, $i = 1, 2, \ldots, n$.
- λ= a known, external, continuous and constant demand rate for a single product at the #1 storage depot.
- h_i= the echelon inventory holding cost per unit time at each storage depot i, $i = 1, 2, \ldots, n$.
- P_{i0}= the unit transportation cost for dispatching a single consignment product from storage depot $(i + 1)$ to the next successive storage depot i while the reorder or shipped quantities for each storage depot q_i is less than some transportation volume in which the transportation price discount occurs, denoted as b, $i = 1, 2, \ldots, n$.

P_{i1}= the unit transportation cost when q_i is greater than or equal to b. P_{i0} and P_{i1} would result in $y_{i1} = 0$ and $y_{i1} = 1$, respectively, $i = 1, 2, \ldots, n$.

P'_{i1}= $(P_{i1} - P_{i0}) < 0$ and denote it as the transportation cost difference in two-level cases, $i = 1, 2, \ldots, n$.

q_L= the base lot size which is one of our decision variables.

2^{α_i}= k_i (= 1, 2, 4, 8, ... for $\alpha_i = 0, 1, 2, \ldots$), is one of our decision variables.

j= an index of the different transportation cost breakpoints and . $j = 0, 1, 2, \ldots., m$.

b_j= the transportation cost breakpoint, $j = 0, 1, 2, \ldots, m$. where $b_0 = 0$ and $b_{m+1} = \infty$.

P_{ij}= the unit transportation cost, $i = 1, 2, \ldots, n$ and $j = 0, 1, 2, \ldots., m$. Note that $b_j < b_k$ and $P_{ij} > P_{ik}$, $\forall k > j$.

P'_{ij} = $(P_{ij} - P_{ij-1}) < 0$ and denote it as the transportation cost difference in multi-level cases, $i = 1, 2, \ldots, n$ and $j = 0, 1, 2, \ldots., m$.

The objective of this paper is to investigate the problems of $P_{2,TL}$ and $P_{2,ML}$ by proposing a simple and efficient heuristic approach.

3 The Cost-Curve Structure Analysis

Lee and Yao [7] discussed the cost-curve structure on the *joint replenishment problem*. In this section, we attempt to illustrate the cost-curve properties for the $SDSDP\_PoT$. In order to solve mathematical models, we plot the curves to investigate the cost functions. We denote those curves as "cost-curves" and analyze on their basic structure such that we could further develop our heuristic approach. Because of constraint (1d), we must consider the positive PoT integer k_i and the base lot-size q_L simultaneously for each storage depot i. The objective function values from the models $P_{2,TL}$ and $P_{2,ML}$ are denoted as $TC_{2,TL}$ and $TC_{2,ML}$, respectively. Here, we first discuss some theoretical results to provide insights into the $TC_{2,TL}$ and $TC_{2,ML}$ functions in terms of the cost-curve properties.

3.1 Some Insights into Two-Level Transportation Cost Problems

We conducted an analysis by going through some examples. A numerical example of two-level transportation costs that Vroblefski et al. [16] used to illustrate their viewpoints is summarized in Table 1. Other parameters included in this example are the external demand at #1 storage depot, $\lambda = 5000$ units/year, and the ordering quantities at which the transportation cost break, $b = 500$ units.

Table 1. A summary of the parameter settings listed by Vroblefski et al.[16]

Storage depot i	1	2	3	4
Ordering Cost($/order)	K_1= 75	K_2=19	K_3= 150	K_4= 80
Holding Cost($/unit/year)	h_1= 10	h_2= 5	h_3= 3	h_4= 5
The Unit Transportation	P_{10}= 0.25	P_{20}= 0.2	P_{30}= 0.1	P_{40}= 0.2
Cost ($/unit)	P_{11}= 0.2	P_{21}= 0.075	P_{31}= 0.075	P_{41}= 0.175

For a storage depot i, its ordering and inventory costs, defined as $TC_i(k_i, q_L)$:

$$TC_i(k_i, q_L) = \frac{K_i \lambda}{k_i q_L} + \frac{h_i k_i q_L}{2}. \quad (2)$$

According to the above equation, the cost-curve of each storage depot can be represented in two-dimensional axes, which are q_L and $TC_i(k_i, q_L)$. The $TC_2(k_2, q_L)$ is given in Figure 2.

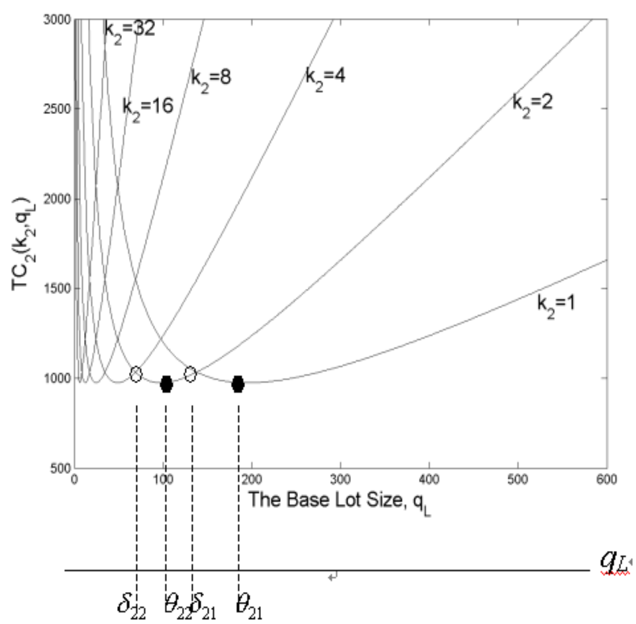

Figure 2: The ordering and inventory cost function of #2 storage depot

For any given value of q_L, we can obtain the PoT positive integer k_i so as to minimize eq. (2). We denote it as $\overline{\overline{TC}}_i(q_L)$, the minimal cost function with respect to q_L for the storage depot i.

$$\overline{\overline{TC}}_i(q_L) = \min_{k_i} \{TC_i(k_i, q_L)\} \tag{3}$$

Remark 3.1 *The* $\overline{\overline{TC}}_i(q_L)$ *function is piece-wise convex with respect to* q_L.

For the $\overline{\overline{TC}}_i(q_L)$ function, we denote a point at which two curves join as a junction point (JP), δ_{ir} , where $r = 1, 2, 3, \ldots$ is the JP counter for each storage depot i. The JP in the storage depot i plays an important role, because it leads to a change in the curve on its right-hand side, that is $k_i = 2^{\alpha_i}$, to the $k_i = 2^{(\alpha_i+1)}$, next and successive curve on its left-hand side. To be precise, the location of JP on the q_L-axis can be computed as follows:

$$\delta_{ir}(k_i) = (\frac{1}{k_i})\sqrt{\frac{K_i\lambda}{h_i}} \tag{4}$$

On the other hand, by setting the first-order derivation with respect to q_L from eq. (2) as zero, we acquire the conventional EOQ formula times a reciprocal of a PoT positive integer. For each strictly convex curve, there exists only one minimal cost point (MCP), denoted as θ_{iu}, for $i = 1, 2, \ldots, n$ and $u = 1, 2, 3, \ldots$ is the MCP counter.

$$\theta_{iu}(k_i) = (\frac{1}{k_i})\sqrt{\frac{2K_i\lambda}{h_i}} \tag{5}$$

In addition, the transportation cost is added into eq. (2) as follows:

$$TTC_i(k_i, q_L) = \frac{K_i\lambda}{k_i q_L} + \frac{h_i k_i q_L}{2} + \lambda\{P'_{i1}y_{i1} + P_{i0}\}. \tag{6}$$

Denote TTC_i as total cost for each storage depot i. Figure 3 demonstrates that each depicted k_i curve is convex but decreasing and discontinuous at a certain point called the PoT transportation-cost breakpoint ($PoT - TCB$), defined as ρ_{k_ij} for $k_i = 2^{\alpha_i}$, $\alpha_i = 0, 1, 2, \ldots \forall i$ and $j = 1, 2, 3, \ldots m$. In the cases of $P_{2,TL}$, $m = 1$. Notice that all storage depots have the same values of ρ_{k_ij} .

$$\rho_{k_i j} = \frac{b_j}{k_i} \tag{7}$$

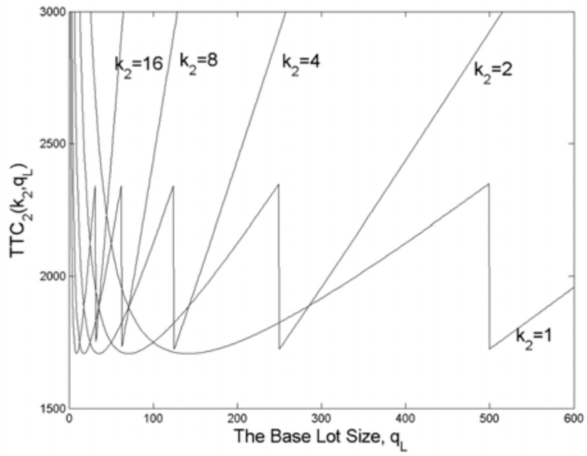

Figure 3: The total cost function of #2 storage depot

It is of interest that some changes would happen to JP between two curves because of the incorporation of transportation cost. We define those points as ξ_{iv} where $v = 1, 2, 3, \ldots$ is the new JP counter for all i. Concerning the positions where the JPs are located, we suggest one remark be taken into considerations. Some notations should be defined in advance:

$\check{C}_{2^{\alpha_i}}$: the corresponding curve with ($k_i = 2^{\alpha_i}$).
$\check{C}_{2^{(\alpha_i+1)}}$: the corresponding curve with ($k_i = 2^{(\alpha_i+1)}$).
$\check{C}_{2^{\alpha_i},sg0}$: the first segment of $\check{C}_{2^{\alpha_i}}$ separated by $\rho_{2^{\alpha_i}1}$.
$\check{C}_{2^{\alpha_i},sg1}$: the second segment of $\check{C}_{2^{\alpha_i}}$ separated by $\rho_{2^{\alpha_i}1}$.
$\check{C}_{2^{(\alpha_i+1)},sg0}$: the first segment of $\check{C}_{2^{(\alpha_i+1)}}$ separated by $\rho_{2^{(\alpha_i+1)}1}$.
$\check{C}_{2^{(\alpha_i+1)},sg1}$: the second segment of $\check{C}_{2^{(\alpha_i+1)}}$ separated by $\rho_{2^{(\alpha_i+1)}1}$.

Remark 3.2 *There are three intersecting conditions for $\check{C}_{2^{\alpha_i}}$ and $\check{C}_{2^{(\alpha_i+1)}}$:*

(1) $\check{C}_{2^{\alpha_i},sg0}$ *intersects* $\check{C}_{2^{(\alpha_i+1)},sg0}$ *at the point* $\xi_{iv} = (1/2^{\alpha_i})\sqrt{K_i\lambda/h_i}$. *(Similar solution is also obtained in eq. (4).) We need to check if* $\xi_{iv} \in (0, \rho_{2^{(\alpha_i+1)}1})$. *If* ξ_{iv} *is in the interval, we keep it; otherwise, ignore it.*

(2) $\check{C}_{2^{\alpha_i},sg0}$ *intersects* $\check{C}_{2^{\alpha_i},sg1}$ *at the point*

.

$$\xi_{iv} = \frac{\left\{\lambda(P_{i0} - P_{i1}) + \sqrt{\lambda\left[\lambda(P_{i0} - P_{i1})^2 + h_i K_i\right]}\right\}}{h_i k_i}. \tag{8}$$

We need to check if $\xi_{iv} \in [\rho_{2^{(\alpha_i+1)}1}\ ,\ \rho_{2^{\alpha_i}1}\ )$. *If* ξ_{iv} *is in the interval, we keep it; otherwise, ignore it.*

(3) $\check{C}_{2^{\alpha_i},sg1}$ *intersects* $\check{C}_{2^{(\alpha_i+1)},sg1}$ *at the point* $\xi_{iv} = (1/2^{\alpha_i})\sqrt{K_i\lambda/h_i}$. *We need to check if* $\xi_{iv} \in [\ \rho_{2^{\alpha_i}1}\ , \infty)$. *If* ξ_{iv} *is in the interval, we keep it; otherwise, ignore it.*

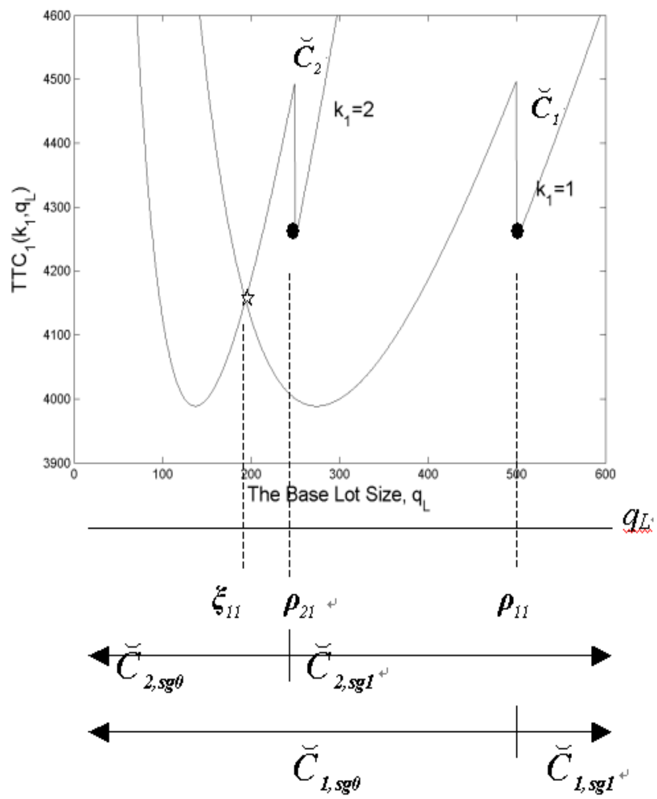

Figure 4: The criteria illustrated in Remark 3.2 are used for judging the position of the JP

Figure 5 shows total cost-curve structure of this two-level demonstrated problem. All kinds of points that we define here are going to be the fundamental elements of our proposed heuristic approach.

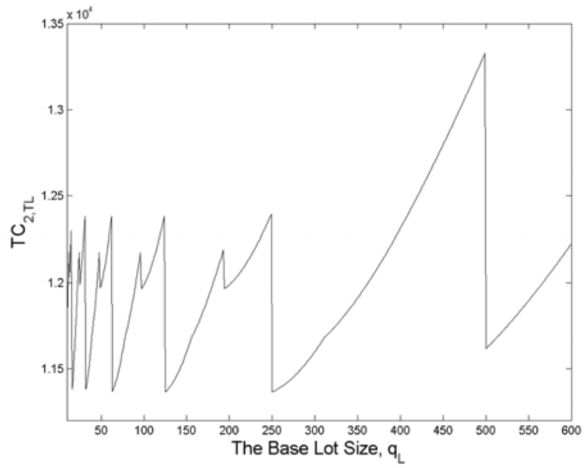

Figure 5: The cost structure of the demonstrated example with two-level transportation costs

3.2 Some Insights into Multi-Level Transportation Cost Problems

In this section, we extend the results obtained from the problem with two-level (a single breakpoint) transportation costs to that with multi-level and more complicated transportation costs.

We have discussed cases involving only one breakpoint in the previous section. Now we are concerned with the conditions with $2, 3, \ldots,$ or m breakpoints. We adopt cases with two breakpoints as an example, while maintaining the parameters in Table 1 with minor modification. We need to have the extra parameters P_{i2} and b_2 (if we set b_1 to be equal to b) for the newly added 1 breakpoint. Continuing the previous example of #2 storage depot, we set $P_{22} = 0.045$ and $b_2 = 1,000$. The corresponding cost-curve function for #2 storage depot is shown in Figure 6 below:

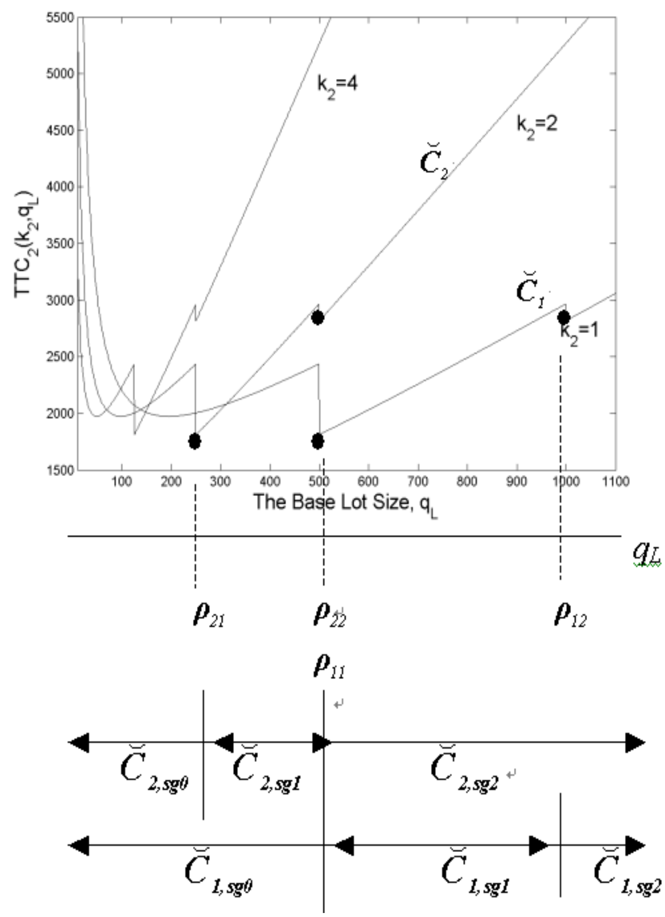

Figure 6: A demonstrated example for determining segments under the multi-level transportation cost condition

The determination process for the JP, ξ_{iv}, as stated in Remark 3.3, can be adopted to multi-level problems.

Remark 3.3 *Sort 2m breakpoints that are related to curves $\check{C}_{2^{\alpha_i}}$ and $\check{C}_{2(\alpha_i+1)}$ in ascending order and check the individual interval. If indices js of two segments are the same when they are considered at the same time (for instance, $\check{C}_{2^{\alpha_i},sg0}$ vs. $\check{C}_{2(\alpha_i+1),sg0}$), then the JP can be computed by eq. (4); otherwise, by the following formula:*

$$\xi_{iv} = \left\{ \lambda(P_{i\check{C}_{2^{\alpha_i},sgx}} - P_{i\check{C}_{2(\alpha_i+1),sgy}}) + \sqrt{\lambda \left[\lambda(P_{i\check{C}_{2^{\alpha_i},sgx}} - P_{i\check{C}_{2(\alpha_i+1),sgy}})^2 + h_i K_i \right]} \right\} / h_i k_i, \tag{9}$$

where $x, y \in j$ (= 0, 1, 2, ..., m) and $x \neq y$. Note that we need to further check if ξ_{iv} belongs to the interval. All the consecutive points can be determined by utilizing the same method, for each storage depot.

Except setting $P_{22} = 0.045$ and $b_2 = 1,000$, we also add the new parameters $P_{12} = 0.1$, $P_{32} = 0.06$ and $P_{42} = 0.15$ to forming a problem $P_{2,ML}$ with three-level transportation costs. The cost-curve structure of that problem is presented in Figure 7. All different kinds of points we discussed in this section are the foundation for the heuristic approach.

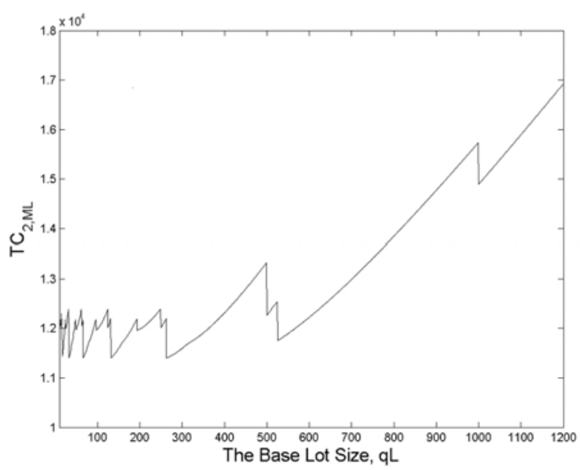

Figure 7: The cost structure of the demonstrated example with three-level transportation costs

4 A Proposed Heuristic Approach

Recall that our proposed heuristic method is based on searching the previous defined points, including the JP, MCP and $PoT - TCB$. Then, the steps of the heuristic for determining the best solution to problem $P_{2,TL}$ is as follows:

Step 1: Compute JPs by eq. (4) or (8) and $MCPs$ by eq. (5), for each storage depot. Employ eq. (7) to get $PoT - TCBs$.

Step 2: Sort each kind of the searched points in descending order. We define those searched points as v_c, where $c = 1, 2, 3, \ldots$ is a counter for the searched points.

Step 3: As each searched interval means the region where two adjacent points JPs (δ_{ir} or ξ_{iv}) are located, find the searched point $v_c(\delta_{ir}$ or ξ_{iv}, $MCPs$ and $PoT - TCBs)$ in each interval and record their corresponding k_i-values, where $i = 1, 2, \ldots, n$. The first searched point, v_c, is the point with the largest value of q_L and its k_i-values,denoted as K_c, are expressed as a $1 \times n$ unit vector with $(1, 1, 1, \ldots, 1)$ at the beginning of the search process. The 1^{st} element in the vector K_c means the k_i-value that minimizes $TTC_i(k_i, q_L)$ at the searched point for #1 storage depot, and the 2^{nd} element in the vector K_c means the k_i-value that minimizes $TTC_i(k_i, q_L)$ at the searched point for #2 storage depot, etc.

Step 4: Once the v_c is chosen, record its related storage depot i and k_i-value. As the searched point is selected from JPs (ξ_{iv}), k_i-value has to be changed from k_i to $2k_i$ for a certain storage depot i. After deciding each k_i-value for all storage depots, we must check k_i values again from storage depot 1 to n for meeting the constraint (1f) and save them into K_c.

Step 5: Pick out the best or minimum value of $TC_{2,TL}$ and set it as $TC^*_{2,TL}$. Select the best solution $(K^*_c, v^*_c)=\underset{c}{\operatorname{argmin}}\left\{TC^*_{2,TL}(K_c,\ v_c)\right\}$.

This search procedure can also be applied to solving multi-level problems with the change of the process of obtaining JPs to equations (4) or (9).

4.1 An Illustrative Example

The search procedure of solving problem $P_{2,TL}$ with the data in Table 1 is illustrated step by step:

Step 1: The JPs, $MCPs$ and $PoT - TCBs$ are computed and summarized as shown in Tables 2 to 4, respectively. Each result in the Tables 2 and 3 means the different kinds of searched points obtained from different k_i-values . at different storage depots

Step 2: Two adjacent points JPs in each searched interval are selected from Tables 2(a) and 2(b) as shown in the 1^{st} column of Table 5. Gather and sort searched points from Tables 2, 3 and 4 in descending order as presented in the 2^{nd} column of Table 5.

Step 3: The first searched point is 707.11, and it is chosen from $MCPs$ in Table 3 with the storage depot $i = 3$ and $k_3 = 1$. Since they have not had any searched point so far in the storage depots $1, 2$, and 4, $k_1 = 1$, $k_2 = 1$, and $k_4 = 1$, therefore, $K_1 = (1,\ 1, 1, 1)$ at $v_1 = 707.11$.

Step 4: Next searched point is $v_2 = 500$. It appears at JPs (δ_{31}) in Table 2(a), $k_3 = 1$, JPs(ξ_{31}) in Table 2(b), $k_3 = 1$ and at $PoT - TCBs$ in Table 4, $k_i = 1\ \forall i$. Hence, $K_2 = (1,\ 1, 1, 1)$ at $v_2 = 500$. But that point is chosen from JPs (ξ_{31}) at a certain storage depot, #3, we need to change k_3-value from 1 to 2 before searching the next searched point v_3.Accordingly, $v_3 = 400$ is selected from $MCPs$, we now have $(1, 1, 2, 1)$ due to $k_4 = 1$. Because of the constraint (1f), we need to change k_4-value from 1 to 2 such that $K_3 = (1,\ 1, 2, 2)$. The adjacent and next searched point $v_4 = 353.55$ is selected from $MCPs$ with $k_3 = 2$ such that $K_4 = (1,\ 1, 2, 2)$. v_5 appears at $JP(\xi_{21}\ )$,

i.e. 311.08 with $i = 2$ and $k_2 = 1$, and its k_i-values are $K_5 = (1,\ 1, 2, 2)$. Before finding v_6, we must change k_2-value from 1 to 2 such that $(1, 2, 2, 2)$. Next JP(ξ_{41}) point, i.e. 308.95, $K_6 = (1,\ 2, 2, 2)$ meets the constraint (1f) although $k_4 = 1$ is obtained from Table 2(b). Similarly, k_4-value needs to be 2 before keeping on the search process.

Step 5: Based on our proposed heuristic, we can obtain the best solution, $v_c^* = 250$ (the base lot-size $q_L^* = 250$), four PoT positive integer k_i^*-values $(1, 2, 2, 2)$ and an ultimate total average cost of $TC^*_{2,TL}$= \$11,365, to this numerical example as shown in Table 5. Hence, the PoT replenishment policy (q_1^*, q_2^*, q_3^*, q_4^*) would be of the quantities $(250, 500, 500, 500)$.

Table 2. The JPs for the model $P_{2,TL}$

(a) JPs (δ_{ir})

Storage depot i	1	2	3	4
$k_i = 1$	193.65	137.84	500.00	282.84
$k_i = 2$	96.82	68.92	250.00	141.42
$k_i = 4$	48.41	34.46	125.00	70.71
$k_i = 8$	24.21	17.23	62.50	35.36
$k_i = 16$	12.10	8.62	31.25	17.68
$k_i = 32$	6.05	4.31	15.63	8.84

(b) JPs (ξ_{iv})

Storage depot i	1	2	3	4
$k_i = 1$	193.65	311.08	500.00	308.95
$k_i = 2$	96.82	155.54	250.00	154.47
$k_i = 4$	48.41	77.77	125.00	77.24
$k_i = 8$	24.21	38.88	62.50	38.62
$k_i = 16$	12.10	19.44	31.25	19.31
$k_i = 32$	6.05	9.72	15.63	9.65

Table 3. The $MCPs$ for the model $P_{2,TL}$

Storage depot i	1	2	3	4
$k_i = 1$	273.86	194.94	707.11	400.00
$k_i = 2$	136.93	97.47	353.55	200.00
$k_i = 4$	68.47	48.73	176.78	100.00
$k_i = 8$	34.23	24.37	88.39	50.00
$k_i = 16$	17.12	12.18	44.19	25.00
$k_i = 32$	8.56	6.09	22.10	12.50

Table 4. The $PoT - TCBs$ for the model $P_{2,TL}$

$k_i = 1, \forall i$	$\rho_{11} = 500$
$k_i = 2, \forall i$	$\rho_{21} = 250$
$k_i = 4, \forall i$	$\rho_{41} = 125$
$k_i = 8, \forall i$	$\rho_{81} = 62.5$
$k_i = 16, \forall i$	$\rho_{16,1} = 31.25$
$k_i = 32, \forall i$	$\rho_{32,1} = 15.625$

Table 5. The detailed search procedure of solving the model $P_{2,TL}$

A searched interval	v_c	k_1	k_2	k_3	k_4	$TC_{2,TL}$
$[500, \infty)$	707.11	1	1	1	1	13,047.75
	500	1	1	1	1	11,615.00
		1	1	2	1	
$[311.08, 500)$	400	1	1	2	2	12,312.50
	353.55	1	1	2	2	11,935.78
	311.08	1	1	2	2	11,680.99
		1	2	2	2	
$[308.95, 311.08)$	308.95	1	2	2	2	11,664.74
		1	2	2	2	
$[282.84, 308.95)$	282.84	1	2	2	2	11,578.69
$[250, 282.84)$	273.86	1	2	2	2	11,560.65
	250	1	2	2	2	11,365.00
		1	2	4	2	
$[193.65, 250)$	200	1	2	4	4	11,987.50
	194.94	1	2	4	4	11,966.89
	193.65	1	2	4	4	12,201.30
		2	2	4	4	
$[155.54, 193.65)$	176.78	2	2	4	4	11,935.78
	155.54	2	2	4	4	11,680.99
		2	4	4	4	
$[154.47, 155.54)$	154.47	2	4	4	4	11,664.74
		2	4	4	4	
$[141.42, 154.47)$	141.42	2	4	4	4	11,578.69
$[137.84, 141.42)$	137.84	2	4	4	4	11,563.67
	136.93	2	4	4	4	11,560.65
	125	2	4	4	4	11,365.00

4.2 Computational and Comparative Results

To generate data randomly, we refer to the experiments in Vroblefski et al. [16]. Levels of factors, including EOQ turnover level, transportation cost break turnover level, transportation cost differentials, and relative measure of transportation to ordering and holding costs, are our referencing bases in both two-level and multi-level experiments. We randomly generate 360 instances (36 combinations and each combination has 10 examples) in two-level experiment and another 360 examples in multi-level experiment. We try to compare the results of our heuristic with those of the $LINGO$ software in terms of average run times and average cost deviations. In two-level and multi-level experiments, we use a computer $P4 - 1.8GHz, 512MB\ RAM$ to run both experiments. A comparison of average run times in both two-level and multi-level cases is demonstrated in Table 6. From these experimental results, we can conclude that the proposed heuristic methods can solve the problems very efficiently and within 1.006 CPU seconds. Tables 7 and 8 present average cost deviations, the percentage of the term $(TC_{2,TL}(Heuristic) - TC_{2,TL}\ (LINGO))/TC_{2,TL}\ (LINGO)$ and $(TC_{2,ML}(Heuristic) - TC_{2,ML}\ (LINGO))/\ TC_{2,ML}\ (LINGO)$, in both two-level and multi-level experiments. In general, the heuristic method performed satisfactorily, yielding solution within 3% of the optimal cost. In particular, the two-level cases performed very well, yielding solution within 0.66%. Besides, we found that the cost deviations tend to enlarge in both two-level and multi-level experiments in EOQ turnover level factor. Finally, Table 8 shows that on average, the average cost deviations between our proposed heuristic and $LINGO$ software are within 1.2%.

Table 6. Comparative results of average run time
(a) two-level experiments

n (storage depots)	m (breakpoints)	$LINGO$ (seconds)	$Heuristic$ (seconds)
5	1	15.278	49.252
10	1	0.088	0.468

(b) multi-level experiments

n (storage depots)	m (breakpoints)	$LINGO$ (seconds)	$Heuristic$ (seconds)
5	5	26.328	0.104
5	10	92.764	0.154
10	5	111.389	0.619
10	10	342.403	1.006

Table 7. The average cost deviations of each level
(a) two-level experiments

Variable	Level	$n = 5$, $m = 1$	$n = 10$, $m = 1$
EOQ turnover level	[5, 10]	0.046%	0.016%
	[20, 30]	0.037%	0.022%
	[5, 40]	0.660%	0.545%
Transportation cost break	[5, 7]	0.348%	0.131%
turnover level	[9, 11]	0.164%	0.258%
Transportation cost	[0.1, 0.3]	0.365%	0.142%
differential	[0.4, 0.6]	0.110%	0.244%
	[0.7, 0.9]	0.292%	0.193%
Transportation / ordering	[0.2, 0.3]	0.333%	0.212%
and holding costs	[0.7, 0.8]	0.185%	0.173%

(b) multi-level experiments

Variable	Level	$n = 5$, $m = 5$	$n = 5$, $m = 10$	$n = 10$, $m = 5$	$n = 10$, $m = 10$
EOQ turnover level	[5, 10]	0.092%	0.565%	0.050%	0.115%
	[20, 30]	0.496%	0.583%	0.330%	0.383%
	[5, 40]	0.901%	0.905%	2.960%	2.559%
Transportation cost break	[5, 7]	0.571%	0.837%	1.336%	1.050%
turnover level	[9, 11]	0.422%	0.531%	0.891%	0.989%
Transportation cost	[0.1, 0.3]	0.424%	0.953%	1.059%	0.930%
differential	[0.4, 0.6]	0.656%	0.716%	1.056%	0.736%
	[0.7, 0.9]	0.409%	0.384%	1.224%	1.392%
Transportation / ordering	[0.2, 0.3]	0.415%	0.500%	0.987%	0.914%
and holding costs	[0.7, 0.8]	0.579%	0.868%	1.239%	1.124%

Table 8. The average cost deviations of each generated example

n (storage depots)	m (breakpoints)	Average TC Deviation
5	1	0.248%
5	5	0.497%
5	10	0.684%
10	1	0.194%
10	5	1.113%
10	10	1.019%

5 Concluding Remarks

Based on the comprehensive analysis on cost properties of the serial distributed storage depot problem under *PoT* policy, we first solve the sort of problems with multiple storage depots and a single transportation cost breakpoint, then demonstrate the performance of the heuristic method with those of *LINGO* package. Next, we expand the problems to multiple storage depots and multi-level differential transportation costs. The results obtained from computational experience show that the proposed heuristic method is efficient and reliable in solving practical inventory-distribution problems.

The contributions of this paper are in two aspects. First, this study considers about more complicated problems than those enunciated in Vroblefski et al. [16]. Also, our study presents several important results on the cost structures of the single link problems under *PoT* policy. For instance, we have discussed the properties of both the junction points and the breakpoints that are the crucial searched elements. Second, we develop a new search heuristic and obtain the satisfactory outcomes efficiently. The proposed heuristic is the first solution approach in the literature to solve the problems analyzing those with cost-curve properties.

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