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The Physical Problem and the Approximate Solutions of the Sheath in Collisional Plasma

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Abstract

In this paper we have studied the behaviour of sheath structure in plasma with collisions using an analytical approach. The effects of ion and dust collisionality on the plasma sheath are revealed by a fluid model. We have derived expressions that give the sheath thickness and the ion impact energy for collisionally dominated sheath. The ion impact energy at the wall for the two cases; with dust and without dust is evaluated.

Mathematics Subject Classification: 74H10, 82D10

Keywords: sheath thickness, collisional parameter, impact energy, ion, dust

1 Introduction

The study of dusty plasmas represents one of the most rapidly growing branches of plasma physics The dust grains in plasma acquire a large negative charge because they are bombarded much more rapidly by the fast electrons than by the slower positive ions. When a dust grain is exposed to plasma, it acquires a charge by collecting electrons and ions, or through photo-electron or secondary-electron emission.

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Dusty plasma is an ionized gas containing small particles of solid matter, which either grow in the plasma or are introduced externally. The particles acquire a large electric charge by collecting electrons and ions from the plasma. Interest in dusty plasmas arises from several scientific disciplines and industrial applications, including contamination of materials during plasma processing, nanoparticle, manufacturing, astrophysics and basic science.

Dusty plasmas are of great interest in astrophysics because of the ubiquitous presence of dust particles in interstellar [1] and interplanetary space[2], planetary rings [3], comet tails, protoplanetary disks, and moon and asteroid surfaces [4]. In addition to numerous theoretical and observational works, there have been experimental attempts to synthesize cosmic dust in the laboratory[5, 6]

A plasma sheath is the localized electric field that separates plasma from a material boundary. It confines the more mobile species in the plasma and accelerates the less mobile species out of the plasma and toward the walls. For the typical case where the electrons are more mobile than both the positively charged ions and the negatively charged dusts, the electric field in the sheath points toward the boundary.

Understanding sheaths is perhaps one of the oldest problems in plasma physics. [7] The basic problem of plasma flowing into a wall is important in many aspects of plasma physics. Because of this, many models have been developed to describe sheaths. They include, for example, the theory of Langmuir probes,[8] and models of divertor plates in tokamaks [9] These models range from simple analytical expressions, such as Child's law, [10] to complex kinetic simulations[11].

The sheath is composed of ions, atoms, electrons and dust particles, where we have a one dimensional model, the vector valued functions like the velocity only have a component following the ox axis, and for the sake of simplicity, we denote $v_i$and v_d respectively the ion and the dust velocity following the ox component. In the plasma both the density of electrons n_e and the density of ions n_i are equal to the plasma density n_0. The potential in the sheath is ϕ, and the wall is held at a negative potential ϕ_w .Consequently, a sheath forms to separate the plasma from the wall. Ions enter the sheath as a cold beam with a velocity v_0 and strike the wall with a velocity v_w . Ions experience a collisional drag inside the sheath [12]. The boundary between the plasma and the sheath is at $x = 0$, and the sheath thickness is D, That is, the wall is at $x = D$, finally in order to determine the sheath thickness we introduce T_e the temperature of electrons in plasma.

In order to estimate the sheath characteristics, we adopt a one-dimensional model of sheath. The sheath surface is represented by the oyz plane, and the ox axis is perpendicular to the surface. We assume that all the physical variables (density, velocity, potential...) depend only the x coordinate and we suppose a steady process (i.e. the variables do note depend on time).

2 Sheath model of a collisional plasma without dust

The electrons are characterized simply by a prescribed uniform temperature T_e and a density given by the Boltzmann relations :

$$n_e = n_0 \exp(\frac{e\phi}{k_B T_e}) \tag{1}$$

Where:
e = elementary charge
k_B = Boltzmann's constant
The ion number density is governed by the continuity equation

$$\nabla(n_i v_i) = 0 \tag{2}$$

And the ion mean velocity obeys

$$m_i(v_i \nabla)v_i = -e\nabla\phi - F_{ci} \tag{3}$$

Here m_i, n_i and v_i are respectively the mass , density and velocity ion particle.
The ion fluid travels through the sheath it experiences a drag force:

$$F_{ci} = m_i n_n v_i^2 \sigma \tag{4}$$

n_n and σ are respectively the neutral gas density and the momentum transfer cross section for collisions between ions and neutrals:

$$\sigma(v_i) = \sigma_s(\frac{v_i}{C_s})^\gamma \tag{5}$$

where C_s is the ion acoustic speed ($C_s = \sqrt{\frac{k_B T_e}{m_i}}$), σ_s is the cross section, and γ is a dimensionless parameter ranging from 0 to -1.
The system is closed by Poisson's equation:

$$\varepsilon_0 \nabla^2\phi = -e(n_i - n_e) \tag{6}$$

Combining Eqs. (1)-(5) with Poisson's equation, we find two differential equations:

$$v_i \frac{d}{dx} v_i = -\frac{e}{m_i} \frac{d}{dx} \phi - n_n \sigma_s \frac{v_i^{2+\gamma}}{C_s^{\gamma}} \tag{7}$$

$$\frac{d^2}{dx^2} \phi = -\frac{e}{\varepsilon_0} n_0 (\frac{v_0}{v_i} - \exp(\frac{e\phi}{k_B T_e})) \tag{8}$$

Here ε_0 is the permittivity constant.

The governing equations can be made dimensionless by an appropriate choice of variables. Like, v_i, x, ϕ and D are scaled by :

$$u_i = \frac{v_i}{Cs}; \quad \xi = \frac{x}{\lambda_D} \; ; \quad \eta = -\frac{e\phi}{k_B T_e} \text{ and } d = \frac{D}{\lambda_D}$$

The degree of collisionality in the sheath is parameterized by: $\alpha = \frac{\lambda_D}{\lambda_{mfp}}$

Where λ_D is the Debye length ($\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_0 e^2}}$) and $\lambda_{mfp} = \frac{1}{n_n \sigma_s}$ is the mean free path.

After the dimensionless variables in Eqs. (7) and (8), those equations become:

$$u_i u_i' = \eta' - \alpha u_i^{2+\gamma} \tag{9}$$

$$\eta'' = \frac{u_{i0}}{u_i} - \exp(-\eta) \tag{10}$$

Where the prime denotes differentiation with respect to the spatial coordinate ξ and hence η' is the dimensionless electric field. Here, Eqs. (9) represents the conservation of ion momentum and Eqs. (10) is the poisson's equation. These two equations, together with appropriate boundary conditions, provide the description of the collisional sheath.

To solve these equations boundary conditions must to be specified. At the wall $\xi = d$, $\eta(d) = \eta_w$ at the sheath plasma boundary $\xi = 0$ the boundary conditions are $\eta(0) = 0$; $\eta'(0) = 0$ and $u(0) = u_0$. In fact, these conditions are only an approximation to the conditions that actually hold at the sheath plasma interface. We have derived expressions that give the potential profile and the thickness for the collisionally dominated sheath.

In the limit of strong collisions, the collision parameter is large the equations of motion are simplified by neglecting the convective term on the left hand side. The equations thus become:

$$u^{2+\gamma} = \frac{\eta'}{\alpha} \tag{11}$$

By inserting equation (11) into the Poisson's equation, and neglecting the electron term $\exp(-\eta)$ we obtained:

$$\eta = \frac{3+\gamma}{5+2\gamma}(\frac{3+\gamma}{2+\gamma}u_0)^{\frac{2+\gamma}{3+\gamma}}\alpha^{\frac{1}{3+\gamma}}\xi^{\frac{5+2\gamma}{3+\gamma}} \tag{12}$$

The Sheath thickness, found by invoking the boundary condition $\eta(d) = \eta_w$, is:

$$d = \left(\frac{(5+2\gamma)^{3+\gamma}(2+\gamma)^{2+\gamma}}{(3+\gamma)^{5+2\gamma}}\frac{\eta_w^{3+\gamma}}{\alpha u_0^{2+\gamma}}\right)^{\frac{1}{5+2\gamma}} \tag{13}$$

From Eqs. (12), we find that the electric potential η varies not only with ξ and α but also with the energy dependence of the cross-section, characterized by γ.

For constant ion mobility ($\gamma = -1$) the results given above for the electric potential and the sheath thickness simplify to

$$\eta = 2^{\frac{3}{2}}3^{-1}(u_0)^{\frac{1}{2}}\alpha^{\frac{1}{2}}\xi^{\frac{3}{2}} \tag{14}$$

$$d = 3^{\frac{2}{3}}2^{-1}(u_0)^{\frac{-1}{3}}\alpha^{\frac{-1}{3}}\eta_0^{\frac{3}{2}} \tag{15}$$

The sheath thickness profile is shown in fig. 1. We note that the evolution of sheath width is correlated with the collisional parameter evolution. In the limit of strong collisions the decreases in the sheath thickness approach a limiting asymptote. We conclude that a sheath always develops if there are collisions, and also if $m_i v_i^2 \succ k_B T_e$ at some point. We show also that, sheath width increases when the electric potential increases

3 Sheath model of a collisional dusty plasma

3.1 Ions are in thermal equilibrium

Dust grains are one of the common of both space and laboratory plasma[15, 16, 17] . Dusty plasma consists of neutral gas, Ions, electrons, and micron size particles that have a negative charge. Since the dust particles are much heavier than both the electrons and ions, the latter can be assumed to in thermal equilibrium which dusts as a cold fluid. The neutrals are taken as immobile.

Here, since we have used fluid theory for dusts, we can ignore the variations in shape, size and charge separations among the individual dust particles. This

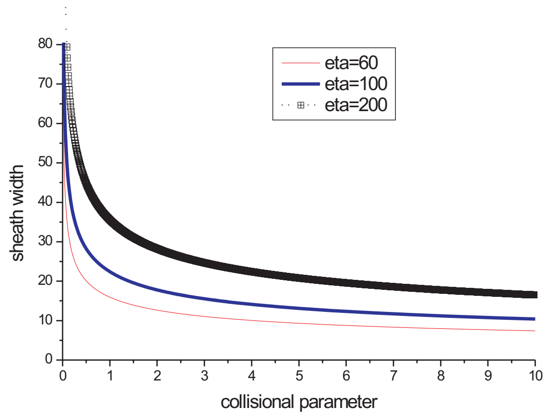

Figure 1: the approximate solutions of the dimensionless sheath thickness as the function of the collisional parameter α for various wall potentials. We show results for constant mobility (γ = -1).

is so because these variations are succulently small and that they can not be distinguished on the fluid element. Moreover though the dusts are massive with respect to the ions and electrons, due to their inertia and positive ion-dust interactions, the dusts will possess a drift velocity. For convenience, a steady state: $\left(\text{i.e.}\frac{\partial}{\partial t} = 0\right)$ plasma we assume here as one dimensional. The mass and charge of dust grains (both assumed constant, for simplicity)

The dynamics of the electrons and ions are treated as a neutral background in plasma and are assumed to follow respectively the following Boltzmann relations[13, 14]

$$n_e = n_0 \exp(\frac{e\phi}{k_B T_e}) \ ; \ \ n_i = n_{i0} \exp(\frac{e\phi}{k_B T_i}) \tag{16}$$

The cold dust fluid obeys the source free, steady state of continuity equation:

$$n_d v_d = n_{d0} v_{d0}$$

And momentum transfer equation

$$m_d(v_d \nabla) v_d = Z_d e \nabla\phi - F_{cd} \tag{17}$$

Where m_d, n_d, v_d and Z_d are respectively the dust particle mass , density, velocity and charge number and n_{d0}, v_{d0} are dust density and velocity at the sheath edge respectively.

The collisional effects between the dust and the neutrals are introduced. We use the collisional force term F_{cd}, which is given by[18]

$$F_{cd} = m_d n_n v_d^2 \sigma = m_d n_n v_d^2 \sigma_s \left(\frac{v_d}{C_d} \right)^{\gamma}$$

σ is the momentum transfer cross section for the collisions between the dust charged grains and neutrals.

$C_d = \sqrt{\frac{k_B T_e}{md}}$. The dust acoustic speed

The electron, ion and dust densities are then included in the Poisson equation

$$\nabla^2 \phi = -4\pi e(n_i - n_e - Z_d n_d) \tag{18}$$

For strong dust-neutral collisions the movements of dusts are mobility limited. There for, here we are interested only about the constant dust mobility case (i.e. $\gamma = -1$). Hence, we do not consider the case of constant dust mean free path (i.e. $\gamma = 0$).

Combining Eqs. (16) to (18), we find two coupled differentials equation describing the sheath structure as:

$$\begin{aligned} v_d \frac{d}{dx} v_d &= \frac{Z_d e}{m_d} \frac{d}{dx} \phi - n_n \sigma_s \frac{v_d^{2+\gamma}}{C_d^{\gamma}} && (19) \\ \nabla^2 \phi &= -4\pi e(n_i - n_e - Z_d n_d) && (20) \end{aligned}$$

In order to simplify the basic equations; the following non-dimensional parameters are defined: $u_d = \frac{v_d}{C_d}$ and $\alpha = \lambda_D n_n \sigma_s$

Based on these non-dimensional parameters the basic equations reduce to:

$$\begin{aligned} u_d u_d' &= -Z_d \eta' - \alpha u_d^{2+\gamma} && (21) \\ \eta'' &= \delta \exp(\eta\theta) - \exp(-\eta) + (1-\delta) \frac{u_{d0}}{u_d} && (22) \end{aligned}$$

Where:

$\delta = \frac{n_{i0}}{n_{e0}}$, $Zd \frac{n_{d0}}{n_{e0}} = \delta - 1$ and $\theta = \frac{T_e}{T_i}$

In the limit of strong collisions, the equations of motion thus become:

$$\eta' = -\left(\frac{\alpha}{Z_d} \right) u_d^{2+\gamma} \tag{23}$$

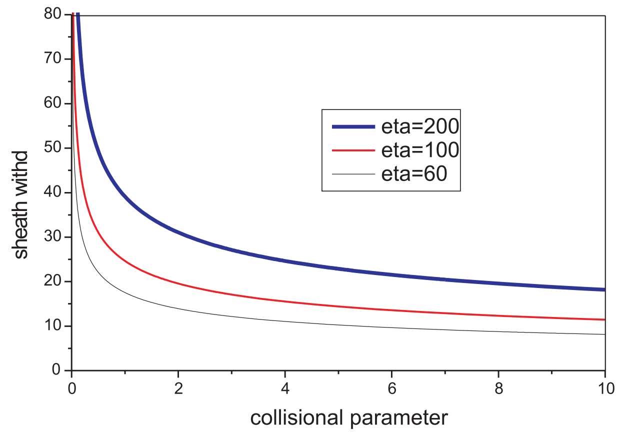

Figure 2: the approximate solutions of the dimensionless sheath thickness as the function of the collisional parameter α for various wall potentials. We show results for constant mobility (γ = -1).

By inserting equation (23) into the Poisson's equation, and neglecting the electron term we obtained that:

$$\eta = \frac{3+\gamma}{5+2\gamma}\left(\frac{3+\gamma}{2+\gamma}\left(1-\delta\right)u_0\right)^{\frac{2+\gamma}{3+\gamma}}\left(-\frac{\alpha}{Z_d}\right)^{\frac{1}{3+\gamma}}\xi^{\frac{5+2\gamma}{3+\gamma}}$$

The Sheath thickness, found by invoking the boundary condition $\eta(d) = \eta_w$, is:

$$d = \left(\frac{(5+2\gamma)^{3+\gamma}\,(2+\gamma)^{2+\gamma}}{(3+\gamma)^{5+2\gamma}}\frac{\eta_w^{3+\gamma}}{((1-\delta)\,u_0)^{2+\gamma}}\right)^{\frac{1}{5+2\gamma}}\left(-\frac{\alpha}{Z_d}\right)^{-\frac{1}{5+2\gamma}}$$

Note that d decreases with increasing collisionality .

The sheath thickness profile in dusty plasma is shown in fig. 2. we plot the sheath width d as functions of the collision parameter α and wall potential η_w.For large α the ion and the dust motion is collisionally dominated, The sheath thickness decrease and approach a limiting asymptote. We show also that, sheath width increases when the electric potential increases. We conclude also that a sheath always develops if there are collisions at some point.

3.2 Ions assumed in out thermal equilibrium.

We consider governing equations based on a two-fluid model. The electrons are thermalized so their density obeys the Boltzmann relation,

$$n_e = n_0 \exp\left(\frac{e\phi}{k_B T_e}\right) \tag{24}$$

The cold dust fluid obeys the source free, steady state of continuity equation:

$$\nabla .\,(n_d v_d) = 0 \tag{25}$$

And momentum transfer equation:

$$m_d(v_d\nabla)v_d = Z_d e\nabla\phi - F_{cd} \tag{26}$$

The collisional effects between the dust and the neutrals are introduced. We use the collisional force term F_{cd}, which is given by:

$$F_{cd} = m_d n_n v_d^2 \sigma = m_d n_n v_d^2 \sigma_s \left(\frac{v_d}{C_d}\right)^{\gamma} \tag{27}$$

The cold ions obey the source free, steady state of continuity equation:

$$\nabla(n_i v_i) = 0 \tag{28}$$

And momentum transfer equation:

$$m_i(v_i\nabla)v_i = -e\nabla\phi - F_{ci} \tag{29}$$

The collisional effects between the ion and the neutrals are introduced. We use the collisional force term F_{ci}, which is given by:

$$F_{ci} = m_i n_n v_i^2 \sigma = m_i n_n v_i^2 \sigma_s \left(\frac{v_i}{C_s}\right)^{\gamma} \tag{30}$$

The Poisson's equation now becomes:

$$\nabla^2\phi = -4\pi e(n_i - n_e - Z_d n_d) \tag{31}$$

Combining eqs (24) to (31) we find tree coupled, differential equation describing the sheath structure as:

$$\begin{aligned} v_i\frac{d}{dx}v_i &= -\frac{e}{m_i}\frac{d}{dx}\phi - n_d\sigma_s\frac{v_i^{2+\gamma}}{C_s^{\gamma}} && (32)\\ v_d\frac{d}{dx}v_d &= \frac{Z_d e}{m_d}\frac{d}{dx}\phi - n_i\sigma_s\frac{v_d^{2+\gamma}}{C_d^{\gamma}} && (33)\\ \nabla^2\phi &= -4\pi e(n_i - n_e - Z_d n_d) && (34) \end{aligned}$$

Based on non-dimensional parameters, the basic equations reduce to:

$$\begin{aligned} u_i u_i' &= \eta' - \alpha u_i^{2+\gamma} && (35)\\ u_d u_d' &= -Z_d \eta' - \alpha u_d^{2+\gamma} && (36)\\ \eta'' &= \delta \frac{u_{io}}{u_i} - \exp(-\eta) + (1-\delta)\frac{u_{d0}}{u_d} && (37) \end{aligned}$$

In the limit of strong collisions, the collision parameter is large the equations of motion are simplified by neglecting convective term on the left hand side. The equations thus become:

$$\begin{aligned} u^{2+\gamma} &= \frac{\eta'}{\alpha} && (38)\\ Z_d \eta' &= -\alpha u_d^{2+\gamma} && (39) \end{aligned}$$

We obtained that:

$$\begin{aligned} u_i &= \left(\frac{-1}{Z_d}\right)^{\frac{1}{2+\gamma}} u_d && (40)\\ \eta'' &= -\frac{\alpha}{Z_d}\,(2+\gamma)\, u_d^{1+\gamma} u_d' && (41) \end{aligned}$$

By neglecting the electron term $\exp(-\eta)$ into the Poisson's equation we again arrive at:

$$\eta'' = \delta \frac{u_{io}}{u_i} + (1-\delta)\frac{u_{d0}}{u_d} \qquad (42)$$

From eqs . (40), (41) and (42) we find:

$$u_{io}\delta \left(\frac{-1}{Z_d}\right)^{\frac{-1}{2+\gamma}} + (1-\delta)u_{d0} = -\frac{\alpha}{Z_d}\,(2+\gamma)\, u_d^{2+\gamma} u_d' \qquad (43)$$

Equation (43) leads to

$$u_d^{2+\gamma} = \left[\frac{-Z_d}{\alpha}\frac{3+\gamma}{2+\gamma}\left(u_{io}\delta\left(\frac{-1}{Z_d}\right)^{\frac{-1}{2+\gamma}} + (1-\delta)u_{d0}\right)\right]^{\frac{2+\gamma}{3+\gamma}} \xi^{\frac{2+\gamma}{3+\gamma}} \qquad (44)$$

Combining eqs (40) and (44)

$$\eta' = \frac{-\alpha}{Z_d}\left[\frac{-Z_d}{\alpha}\frac{3+\gamma}{2+\gamma}\left(u_{io}\delta\left(\frac{-1}{Z_d}\right)^{\frac{-1}{2+\gamma}} + (1-\delta)u_{d0}\right)\right]^{\frac{2+\gamma}{3+\gamma}} \xi^{\frac{2+\gamma}{3+\gamma}} \qquad (45)$$

Equation (45) leads to

$$\eta = \frac{-\alpha}{Z_d}\frac{3+\gamma}{5+2\gamma}\left[\frac{-Z_d}{\alpha}\frac{3+\gamma}{2+\gamma}\left(u_{io}\delta\left(\frac{-1}{Z_d}\right)^{\frac{-1}{2+\gamma}} + (1-\delta)u_{d0}\right)\right]^{\frac{2+\gamma}{3+\gamma}} \xi^{\frac{5+2\gamma}{3+\gamma}} \tag{46}$$

The sheath thickness, found by invoking the boundary condition $\eta = \eta_w$, $\xi = d$, is

$$d = \left(-\frac{Z_d}{\alpha}\frac{5+2\gamma}{3+\gamma}\right)^{\frac{3+\gamma}{5+2\gamma}}\left[\frac{-Z_d}{\alpha}\frac{3+\gamma}{2+\gamma}\left(u_{io}\delta\left(\frac{-1}{Z_d}\right)^{\frac{-1}{2+\gamma}} + (1-\delta)u_{d0}\right)\right]^{\frac{-2-\gamma}{5+2\gamma}} \xi^{\frac{3+\gamma}{5+2\gamma}} \tag{47}$$

We next wish to find the ion impact energy, which can be written using Eq.(38) as:

$$\varepsilon_{wi} = \frac{1}{2}u_{wi}^2 = \frac{1}{2}\left(\frac{\eta_w'}{\alpha}\right)^{\frac{2}{2+\gamma}} \tag{48}$$

Evaluating η_w' using Eq.(45) we find:

$$\varepsilon_{wi} = \frac{1}{2}\left(\frac{\eta_w}{\alpha^2}\frac{5+2\gamma}{2+\gamma}\right)^{\frac{2}{5+2\gamma}}(-Z_d)^{\frac{-2}{(5+2\gamma)(2+\gamma)}}\left(u_{io}\delta\left(\frac{-1}{Z_d}\right)^{\frac{-1}{2+\gamma}} + (1-\delta)u_{d0}\right)^{\frac{2}{5+2\gamma}} \tag{49}$$

The sheath thickness profile in dusty plasma in case of ions in out thermal equilibrium is shown in fig. 3. We represent the analytical solution for the average sheath width as function of the collisional parameter for various wall potentials. In this figure, d approaches a limiting asymptote

3.3 Comparison

The approximate solutions of the dimensionless sheath equations, with approximation, for constant mobility ($\gamma = -1$). It is found that the sheath width for two component plasma (without dust) is less than the collisional sheath width in multicomponent plasma (dusty plasma). We conclude that a presence of dust charged grains in plasma influenced the characteristic behavior of the plasma sheath. We show also that in dusty plasma the collisional sheath width is more in case of ions in out thermal equilibrium.

4 Conclusion

The present theoretical investigation deals with the interaction between a dusty plasma and a solid boundary. We have presented a fluid model for the collisional plasma sheath (without dust and with dust) that includes a power law

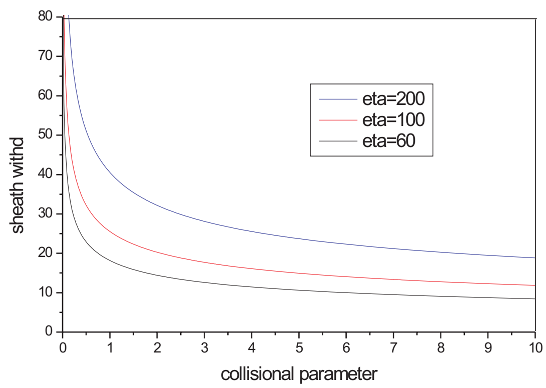

Figure 3: the approximate solutions of the dimensionless sheath thickness as the function of the collisional parameter α for various wall potentials. We show results for constant mobility (γ = -1).

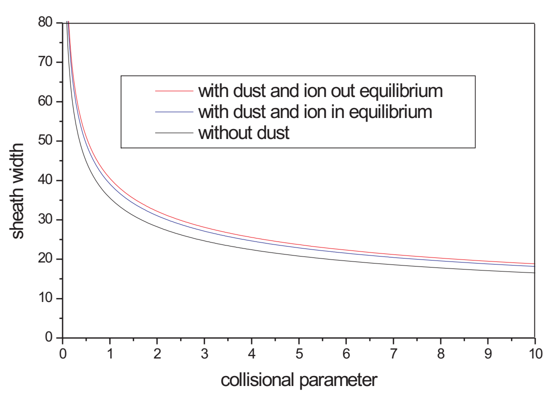

Figure 4: the approximate solutions of the dimensionless sheath thickness as the function of the collisional parameter.

dependence of the ion and the dust collision cross section on sheath width. Approximate solutions of this model appropriate for the collisionally dominated sheath were derived.

We note that the evolution of sheath width is correlated with the collisional parameter evolution. In the limit of strong collisions the decreases in the sheath thickness approach a limiting asymptote. We conclude that a sheath always develops if there are collisions; at some point and also that a presence of dust charged grains in plasma influenced the characteristic behavior of the plasma sheath

Our results show that the sheath width decreases and approaches a limiting asymptote, and also show that in dusty plasma the collisional sheath width is more in case of ions in out thermal equilibrium.

Our model can be applied to study the sheath in various material plasma processing techniques where negatively charged dust particles are usually found to be present

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