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LMI Approach to Stability Analysis of Discrete-Time BAM Neural Networks with Time-Varying Delays[1]

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Abstract

In this paper, a discrete-time bidirectional associative memory (BAM) neural network with time-varying delays is considered. The description of the activation functions is more general than the recently commonly used Lipschitz conditions. By using appropriate Lyapunov-Krasovskii functional and linear matrix inequality (LMI) technique, a delay-dependent sufficient condition is obtained to guarantee the global exponential stability of the addressed neural network. The condition is a LMI, hence the stability of the neural network can be checked readily by resorting to the Matlab LMI toolbox. In addition, the proposed stability criterion does not require the monotonicity and differentiability of the activation functions, and a impose condition on the time-varying delays in recent publication is removed. A simulation example is given to show the effectiveness and less conservatism of the obtained condition.

Mathematics Subject Classification: 34D20; 37C75; 93D05

Keywords: Discrete-time BAM neural network; exponential stability; time-varying delays; Lyapunov-Krasovskil functional; linear matrix inequality

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1 Introduction

Bidirectional associative memory (BAM) neural network, as an extension of the traditional single layer neural network model, was first introduced in 1987 by Kosko [1] and since then, the two-layer interassociative neural network has been extensively studied due to its potential for pattern recognition, signal and image processing, solving optimization problems and automatic control engineering [2]-[8]. In such applications, the stability of networks play an important role.

As is well known, in both biological and man-made neural networks, time delays occur due to finite switching speed of the amplifiers and communication time. The delays are usually time-varying, and sometimes vary violently with time. They slow down the transmission rate and can influence the stability of designed neural networks by creating oscillatory or unstable phenomena [9, 10]. So it is more in accordance with this fact to study the BAM neural networks with time-varying delays. The circuits diagram and connection pattern implementing for the delayed BAM neural networks can be found in [4]. In recent years, many useful results on the stability of the equilibrium point and periodic solutions for the delayed BAM neural networks have been given, for example, see [2]-[15] and references therein.

Note that, up to now, most BAM neural networks have been assumed to act in a continuous-time manner. However, in implementing the continuous-time BAM neural network for computer simulation, experimental or computational purposes, it is essential to formulate a discrete-time system which is an analogue of the continuous-time BAM neural network. Certainly, the discrete-time analogue inherits the dynamical characteristics of the continuous-time BAM neural network under mild or no restriction on the discretization step-size, and also remain functional similarity to the continuous-time BAM neural network and any physical or biological reality that the continuous-time BAM neural network has [16]-[22]. Unfortunately, as pointed out in [17], the discretization can not preserve the dynamics of the continuous-time counterpart even for a small sampling period. Therefore, there is a crucial need to study the dynamics of discrete-time neural networks.

Recently, the stability analysis of discrete-time neural networks without time delays and with time delays has received considerable research interests, and various stability criteria have been proposed, for example, see [16]-[25] and references therein. In [17, 21, 22, 23], the global exponential stability has been investigated for discrete-time delayed Hopfield neural networks, cellular neural networks and recurrent neural networks, several sufficient conditions for checking global exponential stability of equilibrium point were obtained. In [20], the global robust stability problem was considered for a general class of discrete-time interval neural networks which contain time-invariant uncertain parameters with their values being unknown but bounded in given compact sets, three sufficient conditions ensuring global robust stability were given. In [24, 25], authors studied the stability and bifurcation for discrete-time cellular neural network and Cohen-Grossberg neural network. In [16, 18, 19], discrete-time BAM neural network was considered, several sufficient conditions were derived to ensure the existence, uniqueness and global exponential stability of the equilibrium point for discrete-time BAM neural networks with constant and variable delays. In [26]-[30], authors investigated the existence and global exponential stability of periodic solutions for discrete-time Hopfield neural networks, cellular neural networks and BAM neural networks, and gave some sufficient conditions for checking the existence and global exponential stability of periodic solutions.

It should be pointed out that, in all the papers concerning discrete-time BAM neural networks with delay mentioned above, the activation functions are assumed to satisfy the Lipschitz conditions, and the derived stability criteria are conservative. There is still room for improvement, for example, reducing the conservatism under milder constraints.

Motivated by the above discussions, the objective of this paper is to study the exponential stability of discrete-time BAM neural network with time-varying delays by employing a new Lyapunov-Krasovskii functional and using a unified linear matrix inequality (LMI) approach. Under more general description on the activation functions, we obtain a sufficient condition, which can be checked numerically using the effective LMI toolbox in MATLAB. A simulation example is given to show the effectiveness and less conservatism of the proposed criterion.

Notations: The notations are quite standard. Throughout this paper, R^n and $R^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes matrix transposition. The notation $X \geq Y$ (respectively, $X > Y$) means that X and Y are symmetric matrices, and that $X - Y$ is positive semidefinite (respectively, positive definite). $\|\cdot\|$ is the Euclidean norm in R^n. If A is a matrix, denote by $\|A\|$ its operator norm, i.e., $\|A\| = \sup\{\|Ax\| \,:\, \|x\| = 1\} = \sqrt{\lambda_{\max}(A^TA)}$, where $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) means the largest (respectively, smallest) eigenvalue of A. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise. For integers a, b, and $a < b$, $N[a, b]$ denotes the discrete interval given $N[a, b] = \{a, a+1, \cdots, b-1, b\}$. $C(N[-\tau, 0], R^n)$ denotes the set of all functions ϕ: $N[-\tau, 0] \to R^n$.

2 Model description and preliminaries

In this paper, we consider the following model

$$\begin{cases} x(k+1) = Cx(k) + A\widetilde{f}(y(k)) + B\widetilde{f}(y(k-\tau(k))) + I, \\ y(k+1) = Dy(k) + W\widetilde{g}(x(k)) + H\widetilde{g}(x(k-\sigma(k))) + J \end{cases} \tag{1}$$

for $k = 1, 2, \cdots$, where $x(k) = (x_1(k), x_2(k), \cdots, x_n(k))^T \in R^n$, $y(k) = (y_1(k), y_2(k), \cdots, y_m(k))^T \in R^m$, $x_i(k)$ and $y_j(k)$ are the state of the ith neurons from the neural field F_X and the jth neurons from the neural field F_Y at time k, respectively; $\widetilde{f}(y(k)) = (\widetilde{f_1}(y_1(k)), \cdots, \widetilde{f_m}(y_m(k)))^T$, $\widetilde{g}(x(k)) = (\widetilde{g_1}(x_1(k)), \cdots, \widetilde{g_n}(x_n(k)))^T$, $\widetilde{f}_j$, $\widetilde{g}_i$ denote the activation functions of the jth neurons from F_Y and the ith neurons from F_X at time k, respectively; $I = (I_1, I_2, \cdots, I_n)^T \in R^n$, $J = (J_1, J_2, \cdots, J_m)^T \in R^m$, I_i and J_j denote the external inputs on the ith neurons from F_X and the jth neurons from F_Y, respectively; the positive integer $\tau(k)$ and $\sigma(k)$ correspond to the transmission delays and satisfy $\tau \leq \tau(k) \leq \widetilde{\tau}$ and $\sigma \leq \sigma(k) \leq \widetilde{\sigma}$ ($\tau \geq 0$, $\widetilde{\tau} \geq 0$, $\sigma \geq 0$ and $\widetilde{\sigma} \geq 0$ are known integers); $C = diag(c_1, c_2, \cdots, c_n)$ and $D = diag(d_1, d_2, \cdots, d_m)$, where $0 \leq c_i < 1$ and $0 \leq d_j < 1$ represent the rate with which the ith neuron from F_X and the jth neurons from F_Y will reset their potential to the resting state in isolation when disconnected from the networks and external inputs, respectively; $A = (a_{ij})_{n\times m}$ and $W = (w_{ji})_{m\times n}$ are the connection weight matrix, $B = (b_{ij})_{n\times m}$ and $H = (h_{ji})_{m\times n}$ are the delayed connection weight matrix.

The initial conditions associated with model (1) are given by

$$\begin{cases} x_i(s) = \phi_{x_i}(s), & s \in N[-\widetilde{\sigma}, 0], \quad i = 1, 2, \cdots, n, \\ y_j(s) = \phi_{y_j}(s), & s \in N[-\widetilde{\tau}, 0], \quad j = 1, 2, \cdots, m. \end{cases} \tag{2}$$

Throughout this paper, we make the following assumptions:

(H1) The activation functions are continuous and bounded.

(H2) There exist constants F_j^-, F_j^+, G_i^- and G_i^+ ($j = 1, 2, \cdots, m; i = 1, 2, \cdots, n$) such that

$$F_j^- \leq \frac{\widetilde{f}_j(\alpha_1) - \widetilde{f}_j(\alpha_2)}{\alpha_1 - \alpha_2} \leq F_j^+, \quad G_i^- \leq \frac{\widetilde{g}_i(\alpha_1) - \widetilde{g}_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq G_i^+$$

for all $\alpha_1 \neq \alpha_2$.

Since activation functions are bounded, by employing the well-known Brouwer's fixed point theorem, one can easily prove that there exists an equilibrium point for model (1). In the sequel we shall analyze the global exponential stability of the equilibrium point, which in turn implies the uniqueness of the equilibrium point.

To simplify the stability analysis of model (1), we let $(x^*, y^*)^T$ be the equilibrium point of model (1), and shift the intended equilibrium point $(x^*, y^*)^T$

to the origin by letting $u(k)=x(k)-x^*$ and $v(k)=y(k)-x^*$. Thus model (1) can be transformed into:

$$\begin{cases} u(k+1)=Cu(k)+Af(v(k))+Bf(v(k-\tau(k))), \\ v(k+1)=Dv(k)+Wg(u(k))+Hg(u(k-\sigma(k))) \end{cases} \tag{3}$$

for $k=1,2,3,\cdots$, where $f(v(k))=(f_1(v_1(k)),\cdots,f_m(v_m(k)))^T$, $g(u(k))=(g_1(u_1(k)),\cdots,g_n(u_n(k)))^T$, $f_j(v_j(k))=\widetilde{f}_j(v_j(k)+y_j^*)-\widetilde{f}_j(y_j^*)$, $g_i(u_i(k))=\widetilde{g}_i(u_i(k)+x_i^*)-\widetilde{g}_i(x_i^*)$.

It follows from assumption (**H2**) that

$$F_j^- \le \frac{f_j(v_j)}{v_j} \le F_j^+, \quad G_i^- \le \frac{g_i(u_i)}{u_i} \le G_i^+, \quad j=1,2,\cdots,m; i=1,2,\cdots,n. \tag{4}$$

Definition 1. *The equilibrium point $(0,0)^T$ of model (3) is said to be globally exponentially stable, if there exist two positive constants $M>0$ and $0<\varepsilon<1$ such that every solution $(u(k),v(k))^T$ of model (3) satisfies*

$$\|u(k)\|^2+\|v(k)\|^2 \le M\varepsilon^k(\sup_{s\in N[-\widetilde{\sigma},0]}\|u(s)\|^2+\sup_{s\in N[-\widetilde{\tau},0]}\|v(s)\|^2)$$

for all $k=1,2,\cdots$.

3 Main result

In this section, we shall establish our stability criterion based on the LMI approach.

For presentation convenience, in the following, we denote

$$F_1 = diag(F_1^-F_1^+, F_2^-F_2^+, \cdots, F_m^-F_m^+),$$
$$F_2 = diag(\frac{F_1^-+F_1^+}{2}, \frac{F_2^-+F_2^+}{2}, \cdots, \frac{F_m^-+F_m^+}{2}), \tag{5}$$
$$G_1 = diag(G_1^-G_1^+, G_2^-G_2^+, \cdots, G_n^-G_n^+),$$
$$G_2 = diag(\frac{G_1^-+G_1^+}{2}, \frac{G_2^-+G_2^+}{2}, \cdots, \frac{G_n^-+G_n^+}{2}). \tag{6}$$

Theorem 1. *Under assumptions (**H1**) and (**H2**), the equilibrium point $(0,0)^T$ of model (3) is globally exponentially stable if there exist four symmetric positive definite matrices P, Q, R and S, and four positive diagonal matrices Λ, Γ, Υ and Θ such that the following LMI holds:*

$$\Omega = \begin{pmatrix} \Omega_1 & 0 & G_2\Lambda & G_2\Gamma & 0 & 0 & CPA & CPB \\ 0 & -R & 0 & 0 & 0 & 0 & 0 & 0 \\ \Lambda G_2 & 0 & W^TQW-\Lambda & W^TQH & W^TQD & 0 & 0 & 0 \\ \Gamma G_2 & 0 & H^TQW & H^TQH-\Gamma & H^TQD & 0 & 0 & 0 \\ 0 & 0 & DQW & DQH & \Omega_2 & 0 & F_2\Upsilon & F_2\Theta \\ 0 & 0 & 0 & 0 & 0 & -S & 0 & 0 \\ A^TPC & 0 & 0 & 0 & \Upsilon F_2 & 0 & A^TPA-\Upsilon & A^TPB \\ B^TPC & 0 & 0 & 0 & \Theta F_2 & 0 & B^TPA & B^TPB-\Theta \end{pmatrix} < 0, \tag{7}$$

where $\Omega_1 = CPC - P + (1 + \widetilde{\sigma} - \sigma)R - G_1\Lambda - G_1\Gamma$, $\Omega_2 = DQD - Q + (1 + \widetilde{\tau} - \tau)S - F_1\Upsilon - F_1\Theta$.

Proof. Consider the following Lyapunov-Krasovskii functional candidate for model (3) as

$$V(k) = V_1(k) + V_2(k) + V_3(k), \tag{8}$$

where

$$V_1(k) = u^T(k)Pu(k) + v^T(k)Qv(k), \tag{9}$$

$$V_2(k) = \sum_{i=k-\sigma(k)}^{k-1} u^T(i)Ru(i) + \sum_{j=k-\tau(k)}^{k-1} v^T(j)Sv(j), \tag{10}$$

$$V_3(k) = \sum_{l=k-\widetilde{\sigma}+1}^{k-\sigma} \sum_{i=l}^{k-1} u^T(i)Ru(i) + \sum_{l=k-\widetilde{\tau}+1}^{k-\tau} \sum_{j=l}^{k-1} v^T(j)Sv(j). \tag{11}$$

Calculating the difference of $V_1(k)$ along the trajectories of model (3), we obtain

$$\begin{aligned}
\Delta V_1(k) &= V_1(k+1) - V_1(k)\\
&= \Big(Cu(k) + Af(v(k)) + Bf(v(k-\tau(k)))\Big)^T P\Big(Cu(k) + Af(v(k)) + Bf(v(k-\tau(k)))\Big)\\
&\quad +\Big(Dv(k) + Wg(u(k)) + Hg(u(k-\sigma(k)))\Big)^T Q\Big(Dv(k) + Wg(u(k)) + Hg(u(k-\sigma(k)))\Big)\\
&\quad -u^T(k)Pu(k) - v^T(k)Qv(k)\\
&= u^T(k)(CPC - P)u(k) + 2u^T(k)CPAf(v(k)) + 2u^T(k)CPBf(v(k-\tau(k)))\\
&\quad +f^T(v(k))A^TPAf(v(k)) + 2f^T(v(k))A^TPBf(v(k-\tau(k)))\\
&\quad +f^T(v(k-\tau(k)))B^TPBf(v(k-\tau(k)))\\
&\quad +v^T(k)(DQD - Q)v(k) + 2v^T(k)DQWg(u(k)) + 2v^T(k)DQHg(u(k-\sigma(k)))\\
&\quad +g^T(u(k))W^TQWg(u(k)) + 2g^T(u(k))W^TQHg(u(k-\sigma(k)))\\
&\quad +g^T(u(k-\sigma(k)))H^TQHg(u(k-\sigma(k))).
\end{aligned} \tag{12}$$

Evaluating the difference of $V_2(k)$, we get

$$\begin{aligned}
\Delta V_2(k) &= V_2(k+1) - V_2(k)\\
&= \sum_{i=k+1-\sigma(k+1)}^{k} u^T(i)Ru(i) + \sum_{j=k+1-\tau(k+1)}^{k} v^T(j)Sv(j)\\
&\quad - \sum_{i=k-\sigma(k)}^{k-1} u^T(i)Ru(i) - \sum_{j=k-\tau(k)}^{k-1} v^T(j)Sv(j)\\
&= \sum_{i=k+1-\sigma(k+1)}^{k-\sigma} u^T(i)Ru(i) + \sum_{i=k-\sigma+1}^{k-1} u^T(i)Ru(i) + u^T(k)Ru(k)\\
&\quad + \sum_{j=k+1-\tau(k+1)}^{k-\tau} v^T(j)Sv(j) + \sum_{j=k-\tau+1}^{k-1} v^T(j)Sv(j) + v^T(k)Sv(k)
\end{aligned}$$

$$
\begin{aligned}
&- \sum_{i=k-\sigma(k)+1}^{k-1} u^T(i)Ru(i) - u^T(k-\sigma(k))Ru(k-\sigma(k))\\
&- \sum_{j=k-\tau(k)+1}^{k-1} v^T(j)Sv(j) - v^T(k-\tau(k))Sv(k-\tau(k))\\
\leq\ & \sum_{i=k+1-\sigma(k+1)}^{k-\sigma} u^T(i)Ru(i) + u^T(k)Ru(k) - u^T(k-\sigma(k))Ru(k-\sigma(k))\\
&+ \sum_{j=k+1-\tau(k+1)}^{k-\tau} v^T(j)Sv(j) + v^T(k)Sv(k) - v^T(k-\tau(k))Sv(k-\tau(k))\\
\leq\ & \sum_{i=k+1-\widetilde{\sigma}}^{k-\sigma} u^T(i)Ru(i) + u^T(k)Ru(k) - u^T(k-\sigma(k))Ru(k-\sigma(k))\\
&+ \sum_{j=k+1-\widetilde{\tau}}^{k-\tau} v^T(j)Sv(j) + v^T(k)Sv(k) - v^T(k-\tau(k))Sv(k-\tau(k)). \quad (13)
\end{aligned}
$$

Calculating the difference of $V_3(k)$, we have

$$
\begin{aligned}
\Delta V_3(k) &= V_3(k+1) - V_3(k)\\
&= \sum_{l=k-\widetilde{\sigma}+2}^{k+1-\sigma} \sum_{i=l}^{k} u^T(i)Ru(i) + \sum_{l=k-\widetilde{\tau}+2}^{k+1-\tau} \sum_{j=l}^{k} v^T(j)Sv(j)\\
&\quad - \sum_{l=k-\widetilde{\sigma}+1}^{k-\sigma} \sum_{i=l}^{k-1} u^T(i)Ru(i) - \sum_{l=k-\widetilde{\tau}+1}^{k-\tau} \sum_{j=l}^{k-1} v^T(j)Sv(j)\\
&= \sum_{l=k-\widetilde{\sigma}+1}^{k-\sigma} \sum_{i=l+1}^{k} u^T(i)Ru(i) + \sum_{l=k-\widetilde{\tau}+1}^{k-\tau} \sum_{j=l+1}^{k} v^T(j)Sv(j)\\
&\quad - \sum_{l=k-\widetilde{\sigma}+1}^{k-\sigma} \sum_{i=l}^{k-1} u^T(i)Ru(i) - \sum_{l=k-\widetilde{\tau}+1}^{k-\tau} \sum_{j=l}^{k-1} v^T(j)Sv(j)\\
&= \sum_{l=k-\widetilde{\sigma}+1}^{k-\sigma} \Big(u^T(k)Ru(k) - u^T(l)Ru(l)\Big) + \sum_{l=k-\widetilde{\tau}+1}^{k-\tau} \Big(v^T(k)Sv(k) - v^T(l)Sv(l)\Big)\\
&= (\widetilde{\sigma}-\sigma)u^T(k)Ru(k) - \sum_{l=k-\widetilde{\sigma}+1}^{k-\sigma} u^T(l)Ru(l) + (\widetilde{\tau}-\tau)v^T(k)Sv(k) - \sum_{l=k-\widetilde{\tau}+1}^{k-\tau} v^T(l)Sv(l) \quad (14)
\end{aligned}
$$

It follows from (8), (12)-(14) that

$$
\begin{aligned}
\Delta V(k) \leq\ & u^T(k)\Big(CPC - P + (1+\widetilde{\sigma}-\sigma)R\Big)u(k) + 2u^T(k)CPAf(v(k))\\
&+2u^T(k)CPBf(v(k-\tau(k))) + f^T(v(k))A^TPAf(v(k))\\
&+2f^T(v(k))A^TPBf(v(k-\tau(k))) + f^T(v(k-\tau(k)))B^TPBf(v(k-\tau(k)))\\
&+v^T(k)\Big(DQD - Q + (1+\widetilde{\tau}-\tau)S\Big)v(k) + 2v^T(k)DQWg(u(k))
\end{aligned}
$$

$$\begin{aligned}
&+2v^T(k)DQHg(u(k-\sigma(k)))+g^T(u(k))W^TQWg(u(k))\\
&+2g^T(u(k))W^TQHg(u(k-\sigma(k)))+g^T(u(k-\sigma(k)))H^TQHg(u(k-\sigma(k)))\\
&-u^T(k-\sigma(k))Ru(k-\sigma(k))-v^T(k-\tau(k))Sv(k-\tau(k))\\
=\ &\xi^T(k)\Pi\xi(k), \qquad (15)
\end{aligned}$$

where $\xi(k)=(u(k)\quad u(k-\sigma(k))\quad g(u(k))\quad g(u(k-\sigma(k)))\quad v(k)\quad v(k-\tau(k))\quad f(v(k))\quad f(v(k-\tau(k))))^T$,

$$\Pi=\begin{pmatrix}
\Pi_1 & 0 & 0 & 0 & 0 & 0 & CPA & CPB\\
0 & -R & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & W^TQW & W^TQH & W^TQD & 0 & 0 & 0\\
0 & 0 & H^TQW & H^TQH & H^TQD & 0 & 0 & 0\\
0 & 0 & DQW & DQH & \Pi_2 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -S & 0 & 0\\
A^TPC & 0 & 0 & 0 & 0 & 0 & A^TPA & A^TPB\\
B^TPC & 0 & 0 & 0 & 0 & 0 & B^TPA & B^TPB
\end{pmatrix},$$

with $\Pi_1=CPC-P+(1+\widetilde{\sigma}-\sigma)R$, $\Pi_2=DQD-Q+(1+\widetilde{\tau}-\tau)S$.

From (4), we have

$$\Big(g_i(u_i(k))-G_i^-u_i(k)\Big)\Big(g_i(u_i(k))-G_i^+u_i(k)\Big)\le 0,\quad i=1,2,\cdots,n,$$

$$\Big(g_i(u_i(k-\sigma(k)))-G_i^-u_i(k-\sigma(k))\Big)\Big(g_i(u_i(k-\sigma(k)))-G_i^+u_i(k-\sigma(k))\Big)\le 0,\quad i=1,2,\cdots,n,$$

$$\Big(f_j(v_j(k))-F_j^-v_j(k)\Big)\Big(f_j(v_j(k))-F_j^+v_j(k)\Big)\le 0,\quad j=1,2,\cdots,m,$$

$$\Big(f_j(v_j(k-\tau(k)))-F_j^-v_j(k-\tau(k))\Big)\Big(f_j(v_j(k-\tau(k)))-F_j^+v_j(k-\tau(k))\Big)\le 0,\quad j=1,2,\cdots,m,$$

which are equivalent to

$$\begin{pmatrix} u(k)\\ g(u(k))\end{pmatrix}^T\begin{pmatrix} G_i^-G_i^+e_ie_i^T & -\frac{G_i^-+G_i^+}{2}e_ie_i^T\\ -\frac{G_i^-+G_i^+}{2}e_ie_i^T & e_ie_i^T\end{pmatrix}\begin{pmatrix} u(k)\\ g(u(k))\end{pmatrix}\le 0, \qquad (16)$$

$$i=1,2,\cdots,n,$$

$$\begin{pmatrix} u(k)\\ g(u(k-\sigma(k)))\end{pmatrix}^T\begin{pmatrix} G_i^-G_i^+e_ie_i^T & -\frac{G_i^-+G_i^+}{2}e_ie_i^T\\ -\frac{G_i^-+G_i^+}{2}e_ie_i^T & e_ie_i^T\end{pmatrix}\begin{pmatrix} u(k)\\ g(u(k-\sigma(k)))\end{pmatrix}\le 0, \qquad (17)$$

$$i=1,2,\cdots,n,$$

$$\begin{pmatrix} v(k)\\ f(v(k))\end{pmatrix}^T\begin{pmatrix} F_j^-F_j^+e_je_j^T & -\frac{F_j^-+F_j^+}{2}e_je_j^T\\ -\frac{F_j^-+F_j^+}{2}e_je_j^T & e_je_j^T\end{pmatrix}\begin{pmatrix} v(k)\\ f(v(k))\end{pmatrix}\le 0, \qquad (18)$$

$$j=1,2,\cdots,m,$$

$$\begin{pmatrix} v(k) \\ f(v(k-\tau(k))) \end{pmatrix}^T \begin{pmatrix} F_j^- F_j^+ e_j e_j^T & -\frac{F_j^- + F_j^+}{2} e_j e_j^T \\ -\frac{F_j^- + F_j^+}{2} e_j e_j^T & e_j e_j^T \end{pmatrix} \begin{pmatrix} v(k) \\ f(v(k-\tau(k))) \end{pmatrix} \leq 0, \tag{19}$$

$$j = 1, 2, \cdots, m,$$

where e_r denotes the unit column vector having 1 element on its rth row and zeros elsewhere.

Let $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_n)$, $\Gamma = diag(\gamma_1, \gamma_2, \cdots, \gamma_n)$, $\Upsilon = (\eta_1, \eta_2, \cdots, \eta_m)$, $\Theta = diag(\delta_1, \delta_2, \cdots, \delta_m)$, then it follows from (15)-(19) that

$$\begin{aligned}
\Delta V(k) \leq\ & \xi^T(k)\Pi\xi(k) - \sum_{i=1}^{n} \lambda_i \begin{pmatrix} u(k) \\ g(u(k)) \end{pmatrix}^T \begin{pmatrix} G_i^- G_i^+ e_i e_i^T & -\frac{G_i^- + G_i^+}{2} e_i e_i^T \\ -\frac{G_i^- + G_i^+}{2} e_i e_i^T & e_i e_i^T \end{pmatrix} \begin{pmatrix} u(k) \\ g(u(k)) \end{pmatrix} \\
& - \sum_{i=1}^{n} \gamma_i \begin{pmatrix} u(k) \\ g(u(k-\sigma(k))) \end{pmatrix}^T \begin{pmatrix} G_i^- G_i^+ e_i e_i^T & -\frac{G_i^- + G_i^+}{2} e_i e_i^T \\ -\frac{G_i^- + G_i^+}{2} e_i e_i^T & e_i e_i^T \end{pmatrix} \begin{pmatrix} u(k) \\ g(u(k-\sigma(k))) \end{pmatrix} \\
& - \sum_{j=1}^{m} \eta_j \begin{pmatrix} v(k) \\ f(v(k)) \end{pmatrix}^T \begin{pmatrix} F_j^- F_j^+ e_j e_j^T & -\frac{F_j^- + F_j^+}{2} e_j e_j^T \\ -\frac{F_j^- + F_j^+}{2} e_j e_j^T & e_j e_j^T \end{pmatrix} \begin{pmatrix} v(k) \\ f(v(k)) \end{pmatrix} \\
& - \sum_{j=1}^{m} \delta_j \begin{pmatrix} v(k) \\ f(v(k-\tau(k))) \end{pmatrix}^T \begin{pmatrix} F_j^- F_j^+ e_j e_j^T & -\frac{F_j^- + F_j^+}{2} e_j e_j^T \\ -\frac{F_j^- + F_j^+}{2} e_j e_j^T & e_j e_j^T \end{pmatrix} \begin{pmatrix} v(k) \\ f(v(k-\tau(k))) \end{pmatrix} \\
=\ & \xi^T(k)\Pi\xi(k) - \begin{pmatrix} u(k) \\ g(u(k)) \end{pmatrix}^T \begin{pmatrix} G_1\Lambda & -G_2\Lambda \\ -G_2\Lambda & \Lambda \end{pmatrix} \begin{pmatrix} u(k) \\ g(u(k)) \end{pmatrix} \\
& - \begin{pmatrix} u(k) \\ g(u(k-\sigma(k))) \end{pmatrix}^T \begin{pmatrix} G_1\Gamma & -G_2\Gamma \\ -G_2\Gamma & \Gamma \end{pmatrix} \begin{pmatrix} u(k) \\ g(u(k-\sigma(k))) \end{pmatrix} \\
& - \begin{pmatrix} v(k) \\ f(v(k)) \end{pmatrix}^T \begin{pmatrix} F_1\Upsilon & -F_2\Upsilon \\ -F_2\Upsilon & \Upsilon \end{pmatrix} \begin{pmatrix} v(k) \\ f(v(k)) \end{pmatrix} \\
& - \begin{pmatrix} v(k) \\ f(v(k-\tau(k))) \end{pmatrix}^T \begin{pmatrix} F_1\Theta & -F_2\Theta \\ -F_2\Theta & \Theta \end{pmatrix} \begin{pmatrix} v(k) \\ f(v(k-\tau(k))) \end{pmatrix} \\
=\ & \xi^T(k)\Omega\xi(k) \\
\leq\ & -\lambda_{min}(-\Omega)(\|u(k)\|^2 + \|v(k)\|^2).
\end{aligned} \tag{20}$$

From the definition of $V(k)$, it is easy to verify that

$$\begin{aligned}
V(k) \leq\ & \lambda_{max}(P)\|u(k)\|^2 + (1 + \widetilde{\sigma} - \sigma)\lambda_{max}(R) \sum_{i=k-\widetilde{\sigma}}^{k-1} \|u(i)\|^2 \\
& + \lambda_{max}(Q)\|v(k)\|^2 + (1 + \widetilde{\tau} - \tau)\lambda_{max}(S) \sum_{i=k-\widetilde{\tau}}^{k-1} \|v(i)\|^2.
\end{aligned} \tag{21}$$

For any scalar $\alpha > 1$, it follows from (20) and (21) that

$$\begin{aligned}
\alpha^{j+1}V(j+1) - \alpha^j V(j) =\ & \alpha^{j+1}\Delta V(j) + \alpha^j(\alpha - 1)V(j) \\
\leq\ & \Big[\alpha^j(\alpha - 1)\lambda_{max}(P) - \alpha^{j+1}\lambda_{min}(-\Omega)\Big]\|u(j)\|^2 \\
& + \Big[\alpha^j(\alpha - 1)\lambda_{max}(Q) - \alpha^{j+1}\lambda_{min}(-\Omega)\Big]\|v(j)\|^2
\end{aligned}$$

$$+\alpha^j(\alpha-1)(1+\widetilde{\sigma}-\sigma)\lambda_{max}(R)\sum_{i=j-\widetilde{\sigma}}^{j-1}\|u(i)\|^2$$
$$+\alpha^j(\alpha-1)(1+\widetilde{\tau}-\tau)\lambda_{max}(S)\sum_{i=j-\widetilde{\tau}}^{j-1}\|v(i)\|^2. \quad (22)$$

Summing up both sides of (22) from 0 to $k-1$ with respect to j, we have

$$\begin{aligned}\alpha^k V(k)-V(0) \leq\ & \Big[(\alpha-1)\lambda_{max}(P)-\alpha\lambda_{min}(-\Omega)\Big]\sum_{j=0}^{k-1}\alpha^j\|u(j)\|^2 \\ & +\Big[(\alpha-1)\lambda_{max}(Q)-\alpha\lambda_{min}(-\Omega)\Big]\sum_{j=0}^{k-1}\alpha^j\|v(j)\|^2 \\ & +(\alpha-1)(1+\widetilde{\sigma}-\sigma)\lambda_{max}(R)\sum_{j=0}^{k-1}\sum_{i=j-\widetilde{\sigma}}^{j-1}\alpha^j\|u(i)\|^2 \\ & +(\alpha-1)(1+\widetilde{\tau}-\tau)\lambda_{max}(S)\sum_{j=0}^{k-1}\sum_{i=j-\widetilde{\tau}}^{j-1}\alpha^j\|v(i)\|^2. \quad (23)\end{aligned}$$

It is easy to compute that

$$\begin{aligned}\sum_{j=0}^{k-1}\sum_{i=j-\widetilde{\sigma}}^{j-1}\alpha^j\|u(i)\|^2 \leq\ & \Big(\sum_{i=-\widetilde{\sigma}}^{-1}\sum_{j=0}^{i+\widetilde{\sigma}}+\sum_{i=0}^{k-1-\widetilde{\sigma}}\sum_{j=i+1}^{i+\widetilde{\sigma}}+\sum_{i=k-\widetilde{\sigma}}^{k-1}\sum_{j=i+1}^{k-1}\Big)\alpha^j\|u(i)\|^2 \\ \leq\ & \widetilde{\sigma}\sum_{i=-\widetilde{\sigma}}^{-1}\alpha^{i+\widetilde{\sigma}}\|u(i)\|^2+\widetilde{\sigma}\sum_{i=0}^{k-1-\widetilde{\sigma}}\alpha^{i+\widetilde{\sigma}}\|u(i)\|^2+\widetilde{\sigma}\sum_{i=k-1-\widetilde{\sigma}}^{k-1}\alpha^{i+\widetilde{\sigma}}\|u(i)\|^2 \\ \leq\ & \widetilde{\sigma}\alpha^{\widetilde{\sigma}}\sup_{s\in N[-\widetilde{\sigma},0]}\|u(s)\|^2+\widetilde{\sigma}\alpha^{\widetilde{\sigma}}\sum_{i=0}^{k-1}\alpha^i\|u(i)\|^2. \quad (24)\end{aligned}$$

Similarly, we have

$$\sum_{j=0}^{k-1}\sum_{i=j-\widetilde{\tau}}^{j-1}\alpha^j\|v(i)\|^2 \leq \widetilde{\tau}\alpha^{\widetilde{\tau}}\sup_{s\in N[-\widetilde{\tau},0]}\|v(s)\|^2+\widetilde{\tau}\alpha^{\widetilde{\tau}}\sum_{i=0}^{k-1}\alpha^i\|v(i)\|^2. \quad (25)$$

It follows from (23)-(25) that

$$\begin{aligned}\alpha^k V(k) \leq\ & V(0)+(\alpha-1)(1+\widetilde{\sigma}-\sigma)\widetilde{\sigma}\alpha^{\widetilde{\sigma}}\lambda_{max}(R)\sup_{s\in N[-\widetilde{\sigma},0]}\|u(s)\|^2 \\ & +(\alpha-1)(1+\widetilde{\tau}-\tau)\widetilde{\tau}\alpha^{\widetilde{\tau}}\lambda_{max}(S)\sup_{s\in N[-\widetilde{\tau},0]}\|v(s)\|^2 \\ & +\Big[(\alpha-1)\lambda_{max}(P)-\alpha\lambda_{min}(-\Omega)+(\alpha-1)(1+\widetilde{\sigma}-\sigma)\widetilde{\sigma}\alpha^{\widetilde{\sigma}}\lambda_{max}(R)\Big]\sum_{j=0}^{k-1}\alpha^j\|u(j)\|^2 \\ & +\Big[(\alpha-1)\lambda_{max}(Q)-\alpha\lambda_{min}(-\Omega)+(\alpha-1)(1+\widetilde{\tau}-\tau)\widetilde{\tau}\alpha^{\widetilde{\tau}}\lambda_{max}(S)\Big]\sum_{j=0}^{k-1}\alpha^j\|v(j)\|^2. \quad (26)\end{aligned}$$

Let $\rho = \max\{\lambda_{max}(P), (1+\widetilde{\sigma}-\sigma)\lambda_{max}(R)\}$, $\omega = \max\{\lambda_{max}(Q), (1+\widetilde{\tau}-\tau)\lambda_{max}(S)\}$, from (21), we have

$$V(0) \leq \rho\widetilde{\sigma} \sup_{s\in N[-\widetilde{\sigma},0]} \|u(s)\|^2 + \omega\widetilde{\tau} \sup_{s\in N[-\widetilde{\tau},0]} \|v(s)\|^2. \tag{27}$$

Let

$$\varphi(\alpha) = (\alpha-1)\lambda_{max}(P) - \alpha\lambda_{min}(-\Omega) + (\alpha-1)(1+\widetilde{\sigma}-\sigma)\widetilde{\sigma}\alpha^{\widetilde{\sigma}}\lambda_{max}(R),$$

$$\psi(\alpha) = (\alpha-1)\lambda_{max}(Q) - \alpha\lambda_{min}(-\Omega) + (\alpha-1)(1+\widetilde{\tau}-\tau)\widetilde{\tau}\alpha^{\widetilde{\tau}}\lambda_{max}(S),$$

then $\varphi(1) = 0$ and $\psi(1) = 0$. By the continuity of functions $\varphi(\alpha)$ and $\psi(\alpha)$, we can choose a scalar $\beta > 1$ such that $\varphi(\beta) \leq 0$ and $\psi(\beta) \leq 0$. That is

$$\begin{cases} (\beta-1)\lambda_{max}(P) - \beta\lambda_{min}(-\Omega) + (\beta-1)(1+\widetilde{\sigma}-\sigma)\widetilde{\sigma}\beta^{\widetilde{\sigma}}\lambda_{max}(R) \leq 0 \\ (\beta-1)\lambda_{max}(Q) - \beta\lambda_{min}(-\Omega) + (\beta-1)(1+\widetilde{\tau}-\tau)\widetilde{\tau}\beta^{\widetilde{\tau}}\lambda_{max}(S) \leq 0 \end{cases}. \tag{28}$$

It follows from (26)-(28) that

$$\begin{aligned} \beta^k V(k) \leq & \Big[\rho\widetilde{\sigma} + (\beta-1)(1+\widetilde{\sigma}-\sigma)\widetilde{\sigma}\beta^{\widetilde{\sigma}}\lambda_{max}(R)\Big] \sup_{s\in N[-\widetilde{\sigma},0]} \|u(s)\|^2 \\ & + \Big[\omega\widetilde{\tau} + (\beta-1)(1+\widetilde{\tau}-\tau)\widetilde{\tau}\beta^{\widetilde{\tau}}\lambda_{max}(S)\Big] \sup_{s\in N[-\widetilde{\tau},0]} \|v(s)\|^2. \end{aligned} \tag{29}$$

From the definition of $V(k)$, we get

$$V(k) \geq \lambda_{min}(P)\|u(k)\|^2 + \lambda_{min}(Q)\|v(k)\|^2. \tag{30}$$

Let

$$\kappa = \max\{\rho\widetilde{\sigma} + (\beta-1)(1+\widetilde{\sigma}-\sigma)\widetilde{\sigma}\beta^{\widetilde{\sigma}}\lambda_{max}(R), \omega\widetilde{\tau} + (\beta-1)(1+\widetilde{\tau}-\tau)\widetilde{\tau}\beta^{\widetilde{\tau}}\lambda_{max}(S)\},$$

$$\varepsilon = \frac{1}{\beta}, \quad \mu = \min(\lambda_{min}(P), \lambda_{min}(Q)), \quad M = \frac{\kappa}{\mu},$$

then $0 < \varepsilon < 1$, and

$$\|u(k)\|^2 + \|v(k)\|^2 \leq M\varepsilon^k \Big(\sup_{s\in N[-\widetilde{\sigma},0]} \|u(s)\|^2 + \sup_{s\in N[-\widetilde{\tau},0]} \|v(s)\|^2 \Big)$$

for all $k = 1, 2, \cdots$. The proof is completed. □

Remark 1. Discrete-time BAM neural network (1) is a discrete analog of the well-known continuous-time BAM neural network of the form

$$\begin{cases} \frac{dx_i(t)}{dt} = -c_i x_i(t) + \sum_{j=1}^{m} a_{ij} f_j(y_j(t)) + \sum_{j=1}^{m} b_{ij} f_j(y_j(t-\tau(t))) + I_i, \quad i = 1, 2, \cdots, n, \\ \frac{dy_j(t)}{dt} = -d_j y_j(t) + \sum_{i=0}^{n} w_{ji} g_i(x_i(t)) + \sum_{i=1}^{n} h_{ji} g_i(x_i(t-\sigma(t))) + J_j, \quad j = 1, 2, \cdots, m \end{cases}$$

for $t \geq 0$, which has been investigated intensively in recent years, for example, see [4, 8, 9, 12, 13] and references therein.

Remark 2. In assumption (**H2**) of this paper, the constants F_j^-, F_j^+, G_i^- and G_i^+ $(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$ are allowed to be positive, negative or zero. Hence, assumption (**H2**) is weaker than the following assumptions (**H3**) and (**H4**):

(**H3**) There exists positive constant F_j^+ and G_i^+ $(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$ such that

$$0 \leq \frac{f_j(u_1) - f_j(u_2)}{u_1 - u_2} \leq F_j^+, \quad 0 \leq \frac{g_i(u_1) - g_{ij}(u_2)}{u_1 - u_2} \leq G_i^+$$

for all $u_1 \neq u_2$.

(**H4**) There exists positive constant F_j^+ and G_i^+ $(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m)$ such that

$$|f_j(u_1) - f_j(u_2)| \leq F_j^+|u_1 - u_2|, \quad |g_i(u_1) - g_i(u_2)| \leq G_i^+|u_1 - u_2|$$

for all $u_1, u_2 \in R$. Assumptions (**H3**) and (**H4**) were mostly used in literature [1]-[19]. Obviously, the activation functions such as sigmoid type and piecewise linear type are also the special case of the function satisfying assumption (**H2**).

Remark 3. In [16, 18], the given stability criteria for discrete-time BAM neural network with constant delays were based upon certain diagonal dominance or M-matrix conditions on weight matrices of the networks, which only depend on absolute values of the weights and ignore the signs of the weights, and hence are somewhat conservative.

Remark 4. In [19], authors studied respectively the exponential stability for discrete-time BAM neural network with time-varying delays under assumption (**H3**) and assumption (**H4**). However, two kinds of methods in [19] can be unified by the method of this paper. In addition, this paper has also removed the imposed conditions $1 < \sigma(k+1) < 1 + \sigma(k)$ and $1 < \tau(k+1) < 1 + \tau(k)$ in [19].

4 An Example

Consider a discrete-time BAM neural network (1), where

$$C = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix}, A = \begin{pmatrix} 0.2 & -0.1 & 0.1 \\ -0.3 & 0.1 & -0.2 \end{pmatrix}, B = \begin{pmatrix} -0.2 & 0.3 & -0.2 \\ 0.1 & 0.2 & -0.1 \end{pmatrix}, I = \begin{pmatrix} 0.2 \\ -0.1 \end{pmatrix},$$

$$D = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, W = \begin{pmatrix} 0.1 & -0.1 \\ 0.3 & 0.2 \\ -0.1 & -0.3 \end{pmatrix}, H = \begin{pmatrix} 0.1 & -0.2 \\ -0.1 & 0.3 \\ 0.2 & -0.1 \end{pmatrix}, J = \begin{pmatrix} -0.5 \\ 0.7 \\ 1.1 \end{pmatrix},$$

$$f_1(y) = \tanh(0.2y), f_2(y) = \tanh(-0.4y), f_3(y) = \tanh(-0.2y),$$

$$g_1(x) = \tanh(-0.2x), g_2(x) = \tanh(0.2x),$$

$$\tau(k) = 3 + \sin(\frac{k\pi}{2}), \quad \sigma(k) = 4 + \cos(k\pi).$$

It can be verified that assumptions (**H1**) and (**H2**) are satisfied with $F_1^- = 0$, $F_1^+ = 0.2$, $F_2^- = -0.4$, $F_2^+ = 0$, $F_3^- = -0.2$, $F_3^+ = 0$, $G_1^- = -0.2$, $G_1^+ = 0$, $G_2^- = 0$, $G_1^+ = 0.2$, and $\tau = 2$, $\widetilde{\tau} = 4$, $\sigma = 3$, $\widetilde{\sigma} = 5$. Thus,

$$F_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, F_2 = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & -0.2 & 0 \\ 0 & 0 & -0.1 \end{pmatrix}, G_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, G_2 = \begin{pmatrix} -0.1 & 0 \\ 0 & 0.1 \end{pmatrix}.$$

By the Matlab LMI Control Toolbox, we find a solution to the LMI in (7) as follows:

$$P = \begin{pmatrix} 2.6813 & 0.5353 \\ 0.5353 & 9.1009 \end{pmatrix}, \quad Q = \begin{pmatrix} 16.8938 & 1.5285 & -1.5991 \\ 1.5285 & 12.3769 & 3.1933 \\ -1.5991 & 3.1933 & 13.7147 \end{pmatrix}, \quad R = \begin{pmatrix} 0.1484 & 0.0994 \\ 0.0994 & 1.3683 \end{pmatrix},$$

$$S = \begin{pmatrix} 3.6134 & 0.4138 & -0.4594 \\ 0.4138 & 2.7602 & 0.9247 \\ -0.4594 & 0.9247 & 3.2971 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} 5.2633 & 0 \\ 0 & 6.0038 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 4.1780 & 0 \\ 0 & 5.9303 \end{pmatrix},$$

$$\Upsilon = \begin{pmatrix} 4.6292 & 0 & 0 \\ 0 & 7.5728 & 0 \\ 0 & 0 & 4.8229 \end{pmatrix}, \quad \Theta = \begin{pmatrix} 3.6240 & 0 & 0 \\ 0 & 5.3464 & 0 \\ 0 & 0 & 4.5600 \end{pmatrix}.$$

Therefore, by Theorem 1, we know that model (1) with above given parameters is globally exponentially stable, which is further verified by the simulation given in Figure 1. It should be pointed out that the condition in [19] cannot be applied to this example since it requires $1 < \sigma(k+1) < 1 + \sigma(k)$ and $1 < \tau(k+1) < 1 + \tau(k)$.

5 Conclusions

In this paper, the global exponential stability has been investigated for the discrete-time BAM neural network with time-varying delays. The description of the activation functions was more general than the recently commonly used Lipschitz conditions. By employing a appropriate Lyapunov-Krasovskii functional and LMI technique, a delay-dependent sufficient condition has been obtained to guarantee the global exponential stability of the addressed neural network. The condition is a LMI, hence the stability of the neural network can be checked readily by resorting to the Matlab LMI toolbox. In addition, the proposed stability criterion has not required the monotonicity and differentiability of the activation functions, and a impose condition on the time-varying delays in recent publication has been removed. A simulation example is given to show the effectiveness and less conservatism of the obtained condition.

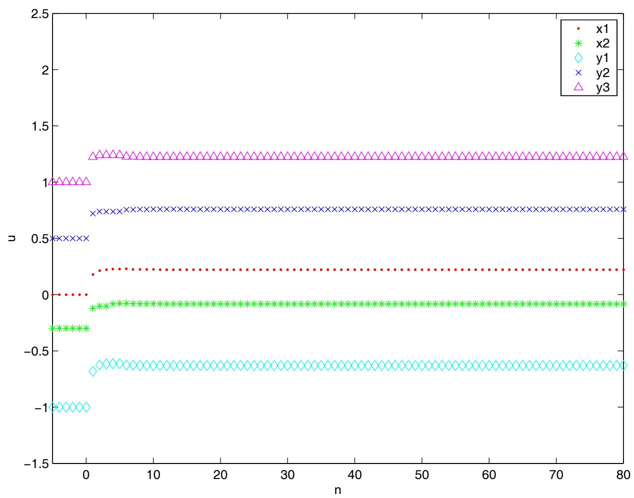

Figure 1: State responses of the discrete-time BAM neural network with initial conditions $(x_1(s), x_2(s), y_1(s), y_2(s), y_3(s))^T = (0, -0.3, -1, 0.5, 1)^T$.

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