Inventory Management of Time Dependent Deteriorating Items with Salvage Value

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Abstract

A mathematical model is developed for items which deteriorate with respect to time. The salvage value is incorporated to this deteriorated units . The sensivity analysis is carried out to study the effect of salvage value and other parameters using numerical example.

Keywords: Deterioration Weibull distributions, Salvage value, Economic order Quantity (EOQ)

1. Introduction

Since long time, researchers are engaged in analyzing inventory models for deteriorating items such as volatile liquids, blood, medicines, electronic components, fashion goods, fruits and vegetables etc. Whitin (1957) studied deterioration at the end of the storage period, for example, for the fashion goods industry. Berrotoni (1962) observed that both the leakage failure of the dry batteries and life expectancy of ethical drugs could be expressed in terms of Weibull distribution, in discussing the problem of fitting empirical data to mathematical distributions. In both cases, the rate of deterioration increases with time or the longer the time remains unused, the higher rate at which they fail. Ghare and Schrader (1963) first formulated a mathematical model with a constant deterioration rate they classified the phenomena of inventory deterioration into three types, viz direct spoilage, physical depletion and deterioration. Covert and

Philip (1973) derived on EOQ model for items with Weibull distribution deterioration.

Since then Misra (1975), Shah (1977), Dave and patel (1981), Holler Mak (1983), Heng et al. (1991), Hariga (1996) and Wee (1995) on deteriorating inventory systems. The review articles Rattat (1991), Shah & Shah (2000) and Goyal and Giri (2001) give a complete and up-to-date survey of published literature for the deteriorating inventory models. The most of the addressed articles assume that deterioration of units is the complete loss to the inventory system. In this paper an attempt has been made to develop a mathematical model for obtaining optimal purchase quantity for time dependent deteriorating items with some salvage value associated to the deteriorated units during the cycle time. The sensitivity analysis is carried out to study the effect of various parameters on the decision variables and objective function.

2. Assumptions and notations

The proposed model is derived under the following assumptions and notations:

- Demand of R units per time unit is deterministic and constant.
- The replenishement rate is infinite.
- The lead time is zero and shortages are not allowed.
- The purchase cost – C / units, the inventory holding cost – h / unit / time unit and the ordering cost- A / order are known during the cycle time under consideration.
- The rate of units in inventory follows the Weibull distribution function given by $\theta(t) = \alpha \beta t^{(\beta - 1)}$, $0 \le t \le T$
  (2.1)
  where α $(0 \le \alpha \le 1)$ denotes scale parameter β $(\beta \ge 1)$ denotes shape parameter and t (t > 0) is time to deterioration.
- The salvage value γC $(0 \le \gamma < 1)$ is associated to deteriorated units during the cycle time.
- The deteriorated units can not be repaired or replaced during the period under review.

3   Mathematical Model

Let Q(t) be the on-hand inventory at any instant of time t $(0 \le t \le T)$. The depletion of units inventory is due to demand and due to deterioration. The instantaneous state of Q(t) at any instant of time is governed by the differential equation.

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = - R, \quad 0 \le t \le T \tag{3.1}$$

with initial condition Q(0) = Q and boundary condition Q(T) = 0, where θ(t) is given by equation (1.1). Taking series expansion and ignoring second and higher powers of α (assuming α very small), the solution of differential equation (2.1) using boundary condition Q(T) = 0 is

$$Q(t) = R\left[T - t + \frac{\alpha T}{\beta + 1}(T^{\beta} - (1 + \beta)t^{\beta}) + \frac{\alpha \beta T^{\beta+1}}{\beta + 1}\right] \qquad (3.2)$$

Using Q(0) = Q , we get

$$Q(t) = R\left[T + \frac{\alpha T^{\beta+1}}{\beta + 1}\right] \qquad (3.3)$$

The number of units that deteriorated; D(T) during one cycle time is given by

$$D(T) = Q - RT = \frac{R\alpha T^{\beta+1}}{\beta + 1} \qquad (3.4)$$

The average inventory , $I_1(T) = \int_0^T Q(t)dt$

$$= R\left[\frac{T^2}{2} + \frac{\alpha \beta T^{\beta+2}}{(\beta+1)(\beta+2)}\right] \qquad (3.5)$$

The total cost per time unit , K(T) comprises of following cost :

a) Inventory holding cost per time unit IHC = $hR\left[\dfrac{T}{2} + \dfrac{\alpha \beta T^{\beta+1}}{(\beta+1)(\beta+2)}\right]$

b) Ordering cost per order, OC = $\dfrac{A}{T}$

c) Cost due to deterioration per time unit, CD = $\dfrac{C\gamma RT^{\beta+1}}{\beta+1}$

d) Salvage value of deteriorated items per time unit , SV = $\left[\dfrac{\gamma CR\alpha T^{\beta+1}}{\beta+1}\right]$

Thus , K(T) = IHC + OC + CD − SV $\qquad (3.6)$

The necessary condition for K(T) to be minimum is $\dfrac{\partial K(T)}{\partial T}$ =0 and solving it for

T by a suitable mathematical software For obtained T , K(T) is minimum only if

$$\frac{\partial^2 K}{\partial T^2} = hR\left[1 + \frac{\alpha \beta T^{\beta+2}(\beta+2)}{T^2(\beta+1)} - \frac{\alpha \beta T^{\beta+2}}{T^2(\beta+1)}\right] - 2hR\frac{\left[T + \frac{\alpha \beta T^{\beta+2}}{T(\beta+1)}\right]}{T^2} + 2h$$

$$\frac{\left[\frac{T^2}{2} + \frac{\alpha \beta T^{\beta+2}}{(\beta+1)(\beta+2)}\right]}{T^3} + \frac{CR\alpha T^{\beta+1}(\beta+1)}{T^3} - \frac{3CR\alpha T^{\beta+1}}{T^3} + \frac{2CR\alpha T^{\beta+1}}{(\beta+1)T^3} + \frac{2A}{T^3} -$$

$$\frac{\gamma CR\alpha T^{\beta+1}(\beta+1)}{T^3} + \frac{3\gamma CR\alpha T^{\beta+1}}{T^3} - \frac{2\gamma CR\alpha T^{\beta+1}}{(\beta+1)T^3} \quad > 0 \text{ for all T} > 0.$$

Special Cases : When $\gamma = 0$ and $\beta = 1$ i.e . So salvage value is associated with deteriorated units , the derived model reduces to that of Ghare and Scharder (1963) . When $\alpha = \theta$ and $\beta = 1$, the model reduces to that of Aggarwal and Jaggi (1996) .

4. Theorems

Theorem 4.1 : The total cost is increasing function of scale parameter α.
Proof: Clearly,

$$\frac{\partial K}{\partial \alpha} = \frac{hR\beta T^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{CRT^{\beta+1}}{(\beta+1)T} - \frac{\gamma CRT^{\beta+1}}{(\beta+1)} \qquad > \quad 0$$

Theorem 4.2 : The total cost is decreasing function of shape parameter β.
Proof: Clearly,

$$\frac{\partial K}{\partial \beta} = hR\left[\frac{\alpha T^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha\beta T^{\beta+2}\ln(T)}{(\beta+2)(\beta+1)} - \frac{\alpha\beta T^{\beta+2}}{(\beta+1)^2(\beta+2)} - \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)^2}\right]$$

$$+ \frac{CR\alpha T^{\beta+1}\ln(T)}{(\beta+1)T} - \frac{CR\alpha T^{\beta+1}}{(\beta+1)^2 T} - \frac{\gamma CR\alpha T^{\beta+1}\ln(T)}{(\beta+1)T} + \frac{\gamma CR\alpha T^{\beta+1}}{(\beta+1)^2 T} \qquad < \quad 0$$

Theorem 4.3 : The total cost is a decreasing function of salvage value γ.

Proof: Clearly, $\dfrac{\partial K}{\partial \gamma} = -\dfrac{CR\alpha T^{\beta+1}}{(\beta+1)T} \qquad < \quad 0$

5. Numerical Example

Consider an inventory system with the following parametric values in proper units:

$$[R , C , h , A] = [10{,}000 , 20 , 2 , 200]$$

Table 1 : Sensitivity analysis of the scale parameter α

α	T	Q	D(T)	TC
0.10	0.1205	1206	16.73171	3167.65
0.15	0.1136	1138	22.9730	3312.32
0.20	0.1079	1082	28.3545	3445.57
0.25	0.1033	1036	33.2009	3569.67

Table heading: $\beta = 1.5, \quad \gamma = 0.1$

Table 2 : Sensivity analysis of the shape parameter β

α = 0.2, γ = 0.1

β	T	Q	D(T)	TC
1.5	0.1079	1082	28.3545	3445.57
2.0	0.1240	1241	10.2506	3038.69
2.5	0.1332	1333	3.7001	2900.65
3.0	0.1378	1379	1.3083	2853.14

Table 3 : Sensitivity analysis of the salvage value γ

α = 0.20, β =1.5

γ	T	Q	D(T)	TC
0.1	0.1079	1082	28.3545	3445.57
0.3	0.1128	1131	30.3077	3328.28
0.5	0.1186	1190	32.6751	3202.41
0.7	0.1258	1262	35.6952	3065.84

6. Conclusions

The economic purchase quantity for time dependent deteriorated units with associated salvage value is analysed in this paper. It is observed that shape parameter β increases number of units to be procured increases and total cost of an inventory system decreases. Increase in shape parameter α decreases procurement quantity and increases total cost of an inventory system. Increase in salvage value γ decreases number of units to be procured and total cost of an inventory system.

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