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The Application of Stuart-Maxwell Test in Determining the Identically Distributed Correct Choice

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Abstract

In Comparison to Macnemar test, the Stuart-Maxwell test is considered to be a useful tool in determining the homogeneity of qualitative variables. In examing the answer keys to the final tests at Payme Noor University, one of the important problems raised was the identical distribution of successive questions. In this article, we use Stuart-Maxwell test to determine this problem.

Keywords: Chi-square test; Homogeneity; Mcnemar test; Independence; Stuart-Maxwell test

1 Introduction

Distance education system has been chosen as the medium of instruction at Payame Noor University. The evaluation methods in this system are in the

form of either multiple choice (MC) questions or MC-writing, or only writing questions. The final tests of each semester are written by faculty members from various fields of study at the university. In this study, we focus on the existence of possible patterns in the answer keys of the tests given to the students in the form of MC questions. The most important problem lies in the fact that because the test writers themselves provide the answer keys manually, the correct choice goes in a position that can be recognized from the appearance of the answer key. For example, test writers are more willing to choose "c" position for the correct choice. Despite the fact that we know the positions of two successive correct answers are interrelated, we study the identically distributed positions of correct choices in successive questions.

The Stuart-Maxwell test ([2], [3], and [7]) tests marginal homogeneity for all categories simultaneously. In next section, with the help of this test, two identically distributed successive questions are evaluated.

2 Preliminary Notes

Suppose the two factors A and B from a $k \times k$ frequency table.

	A_1	A_2	$\cdots$	A_k	$n_{i.}$
B_1	n_{11}	n_{12}	$\cdots$	n_{1k}	$n_{1.}$
B_2	n_{21}	n_{22}	$\cdots$	n_{2k}	$n_{2.}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
B_k	n_{k1}	n_{k2}	$\cdots$	n_{kk}	$n_{k.}$
$n_{.j}$	$n_{.1}$	$n_{.2}$	$\cdots$	$n_{.k}$	n

Suppose that the vector $\mathbf{d}$ contains each $k-1$ amount from the amounts of $d_1, d_2, \ldots, d_{k-1}$ in which

$$d_i = n_{i.} - n_{.i}, \qquad i = 1, 2, \ldots, k.$$

Also, the matrix $\mathbf{S} = [\mathbf{s_{ij}}]$ with the dimension of $(k-1) \times (k-1)$ which is the covariance matrix of $\mathbf{d}$ can be defined as follows:

$$\begin{aligned} s_{ii} &= n_{i.} + n_{.i} - 2n_{ii}, \\ s_{ij} &= -(n_{ij} + n_{ji}). \end{aligned}$$

The Stuart-Maxwell statistic is calculated from the following formula:

$$\chi^2 = \mathbf{d'S^{-1}d}.$$

If $k = 2$, this statistic will be the same as Mcnemar test statistic([4], [5], and [6]). To do the homogeneity hypothesis, we compare the above amount with Chi-square distribution of $k-1$ freedom degrees. Note that if $n_{i.} = n_{.i}$ is omitted from the vector component $\mathbf{d}$, there will be no change in the freedom degree. See [1] for details.

3 Application

In one educational semester all the answer keys to the multiple choice tests were analyzed and studied. From the conclusion of two successive questions, the following table was derived (The rows of table related to second question and the columns of table related to first question).

	a	b	c	d	sum	percent
a	732	1602	1672	1385	5391	0.25077
b	1524	837	1600	1484	5445	0.25328
c	1575	1554	841	1524	5494	0.25556
d	1577	1437	1363	791	5168	0.24039
sum	5408	5430	5476	5184	21498	
percent	0.25156	0.25258	0.25472	0.24114		

We expect there is no relationship between the correct choice for the first and second questions. The amount of independent statistic $\chi^2 = 1223.3$ will be rejected by the comparison of chi-square of 9 freedom degrees of the hypothesis.

With respect to the appearance of small amounts on the main diagonal and symmetry around this diagonal, the following hypothesis will be tested:

$$H_0 : p_{ij} = p_{ji} \quad vs \quad H_1 : p_{ij} \neq p_{ji},$$

where p_{ij} is the probability of observation belong to (A_i, B_j) cell.

The qualities defined in Maxwell-Stuart test will be as the following:

$$\begin{aligned} \mathbf{d} &= (-17 \;\; 15 \;\; 4131), \\ \mathbf{S} &= \begin{pmatrix} 9335 & -3126 & 3247 \\ -3126 & 9201 & -3154 \\ -3247 & -3154 & 5175 \end{pmatrix}. \end{aligned}$$

The amount of the test is equal to $\chi^2 = 12018.4795$ which rejects the above hypothesis. In other words, successive questions are not identically distributed.

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