Applied Mathematical Sciences, Vol. 2, 2008, no. 1, 1 - 18

Cost Efficiency Measurement with Certain Price on Fuzzy Data and Application in Insurance Organization

G. R. Jahanshahloo, F. Hosseinzadeh Lotfi[1]

Dept. of Math. Science and Research Branch
Islamic Azad University, Tehran, Iran

M. Alimardani Jondabeh

Dept. of Math. Tehran-North Branch
Islamic Azad University, Tehran, Iran

Sh. Banihashemi

Dept. of Math. Karaj Branch
Islamic Azad University, Sama College, Karaj, Iran

L. Lakzaie

Dept. of Math. Science and Research Branch,
Islamic Azad University, Zahedan, Iran

Abstract

In this paper, a new approach for obtaining cost efficiency measurment with data set of fuzzy numbers are considered. These consist of situations where prices are fixed and known exactly at each decision making unit (DMU). By using a linear ranking function, each fuzzy number maps into the real number, hence, solution of linear programming problems with fuzzy numbers are real numbers. Method, is illustrated by solving a numerical example proposed by Camanho [3] for data set of fuzzy numbers.

Keywords: Data envelopment analysis (DEA), Cost Efficiency, Fuzzy data

[1]Corresponding author: F. Hosseinzadeh Lotfi, hosseinzadeh_lotfi@yahoo.com

1 Introduction

Cost efficiency (CE) evaluates the ability to produce current outputs at minimal cost. The concept of cost efficiency can be traced back to Farrell (1957), who originated many of the ideas underlying Data Envelopment Analysis (DEA). Following Farrell's concept of CE, its estimation requires input and output quantity data as well as exact knowledge of input prices at each decision making unit (DMU).

The first, considers that prices are fixed and known at each DMU and also data set are real or crisp [3]. In this case, the efficiency assessment can follow the approach described by Farrell (1957)[8] and operationalised by Fare et al. (1985)[7].

Then, considers that prices are fixed and known at each DMU and data set are Fuzzy numbers. The concept of decision making in fuzzy environment was first proposed by Bellman and Zadeh [1]. Subsequently, Tanaka et. al. [18] made use of this concept in mathematical programming. Fuzzy linear programming problem with fuzzy coefficients was proposed by Negoita [17]. Maleki et.al. [16] introduced a linear programming problem with fuzzy variables and proposed a method for solving it.

Maleki [15] used a certain ranking function to solve fuzzy linear programming problems. He also introduced a new method for solving linear programming problems with vagueness in constraints using linear ranking function.

The structure of this paper is as follows. Section 2 describes minimal cost model used for estimation of cost efficiency and Farrell CE with data set of real number.

In section 3 we review the fundamental notions of fuzzy sets and fuzzy numbers. In section 4 we consider a linear ranking function, similar to the ranking function proposed by N. Mahdavi-Amini et.al. [14], to order fuzzy numbers and fuzzy number linear programming.

Then we applied minimal cost model for estimation of cost efficiency with data set of fuzzy numbers two algorithm proposed in section (5). In section (6) there is example of insurance organization. Finally, conclusions are given in the last section.

2 Preliminaries

The underpinnings of efficiency measurement back to the work of Debreu (1951) [5] and Koopmans (1957)[13], Koopmans was the first to define the concept of technical efficiency. The measurment of TE as defined by Farrell (1957)[8] was operationalised and popularized by Charnes et.al. (1978)[4]. In 1978, Charnes, Cooper and Rhodes (CCR) developed a procedure for assessing in relative efficiency and inefficiency of decision making units (DMU) [4].

For assessing the relative efficiency of DMUo which is defined from production possibility set we have the following problem:

$$\begin{aligned} \text{Min} \quad & \theta \\ \text{s.t.} \quad & (\theta X_o, Y_o) \in T_c \end{aligned} \tag{2.1}$$

the emprical production possibility set T_c is defined as follows:

$$T_c = \{(X, Y)|X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0, j = 1, \ldots, n\}$$

Farrell (1957) extended and looked beyond technical efficiency, to purpose the measurment of cost efficiency which assums that prices are fixed and known, at though they may possibly be defferent between the DMUs. Following Farrell (1957) chose graphical illustration of the efficient concept, has become Comanho et.al. (2005) [3] illustrated the efficiency concepts with a small-scale example consisting of eight DMUs. The data set is reported in table 1, and the PPS is portrayed in Fig 1.

Table 1
Data set and prices at the DMUs

	X_1	X_2	Y	P_1	P_2
DMU_A	2	7	1	3	4
DMU_B	3	5	1	3	4
DMU_C	5	3	1	3	4
DMU_D	7	2	1	3	4
DMU_E	3	7	1	3	4
DMU_F	5	5	1	3	4
DMU_G	9	2	1	3	4
DMU_H	10	2.5	1	3	4

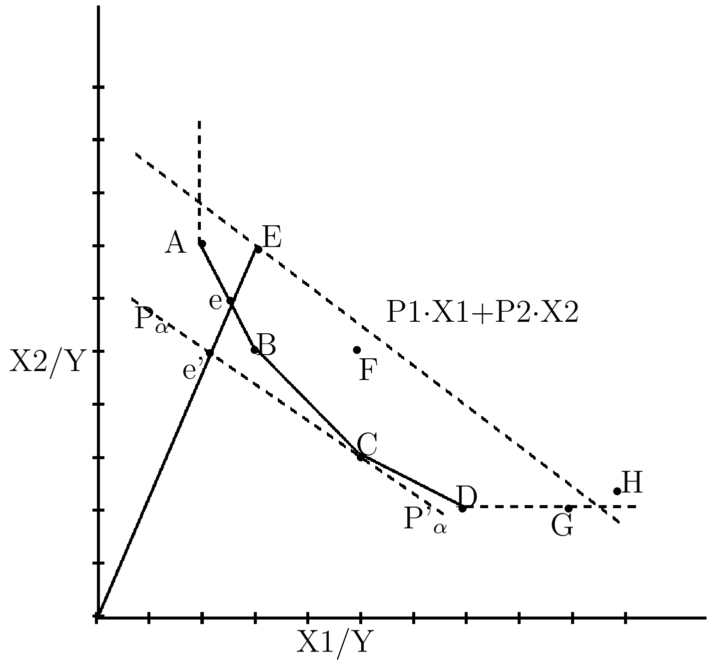


Fig 1. The technically farel frontier of data set and prices at the DMUs in Table 1.

DMUs A, B, C and D are the technically efficient frontier. DMU_E will be used to illustrate the efficiency concepts. It's technical efficiency (TE) is given by the ratio $\frac{Oe}{OE}$. In order to assess cost efficiency we have to specify an incest line $(P_1X_1 + P_2.X_2 = K)$. The total cost (K) associated with a given position of the incest line can be reduced by a parallel movement in a downward direction, until becomes tangent to the efficient frontier.

This is represented in Fig 1 by the broken line $P_\alpha P'_\alpha$. The point where this incest is tangent to the production frontier (point C) identifies the input combination corresponding to the minimal cost of output production for these prices, as further parallel movement of the incest towards the origin is associated with reduced output. The CE measure for DMU_E is obtained as the ratio of the minimal cost (associated with point C on the frontier of the PPS) to the observed cost (associated with point E within the PPS), both evaluated with the current prices at E (i.e., $P_1 = 3$ and $P_2 = 4$). Graphically the cost efficiency measure is given by the ratio Oe'/OE, where e' has the same cost as C with the current input prices $P_1 = 3$ and $P_2 = 4$. This measure indicates the extent to which the DMU_E is supporting its current level of output at minimum cost. In order to obtain a measure of cost efficiency for DMUs with multiple inputs and outputs, the minimum cost for the production of DMU's current outputs with existing input prices is obtained solving the following linear problem, as first formulated by Fare et al. (1985):

$$
\begin{array}{lll}
\text{Min} & \sum_{i=1}^{m} p_{io} x_i & \\
\text{s.t.} & \sum_{j=1}^{n} x_{ij}\lambda_j \leq x_i & i = 1, \ldots, m, \\
& \sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{ro}, & r = 1, \ldots, s, \\
& \lambda_j \geq 0, & j = 1, \ldots, n, \\
& x_i \geq 0, & i = 1, \ldots, m.
\end{array}
\tag{2.2}
$$

In the model above, p_{io} is the price of input i for the DMU under assessment. x_i is a variable that, at the optimal solution, gives the amount of input i to be employed by DMU_o in order to produce the current outputs at minimal cost, subject to the technological restrictions imposed by the existing PPS.

Cost efficiency is then obtained as the ration of minimum cost with current prices (i.e., the optimal solution to model (2.2)) to the current cost at DMU_o, as follows:

$$
\text{Cost efficiency} = \frac{\sum_{i=1}^{m} p_{io} x_i^*}{\sum_{i=1}^{m} p_{io} x_{io}}. \tag{2.3}
$$

3 Definitions and notations of fuzzy sets and fuzzy number

We review the fundamental notions of fuzzy set theory, initiated by Bellman and Zadeh [1], to be used throughout this note. Below, we give definitions and notations taken from Bezdek [2].

Definition 3.1. Let X be the universal set. $\tilde{A}$ is called a fuzzy set in X if $\tilde{A}$ is a set of ordered pairs

$$
\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\},
$$

where $\mu_{\tilde{A}}(x)$ is the membership value of x in $\tilde{A}$.

Remark 3.2. The membership function of A ($\mu_{\tilde{A}}$) shows the degree that x belongs to $\tilde{A}$.

Definition 3.3. The support of a fuzzy set $\tilde{A}$ is a set of elements in X for which $\mu_{\tilde{A}}(x)$ is positive, that is,

$$
\text{supp}\tilde{A} = \{x \in X | \mu_{\tilde{A}}(x) > 0\}.
$$

Definition 3.4. A fuzzy set $\tilde{A}$ is convex if

$$
\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}, \quad \forall x, y \in X \text{ and } \lambda \in [0,1].
$$

Definition 3.5. A convex fuzzy set $\tilde{A}$ on $\Re$ is a fuzzy number if the following conditions hold:

(a) Its membership function is piecewise continuous.

(b) There exist only x_0 that $\mu_A(x_0) = 1$.

Definition 3.6. A fuzzy number $\tilde{A}$ is called positive (Negative), if it membership function is such that $\mu_{\tilde{A}}(x) = 0 \ \forall x < 0 \ (\forall x > 0)$.

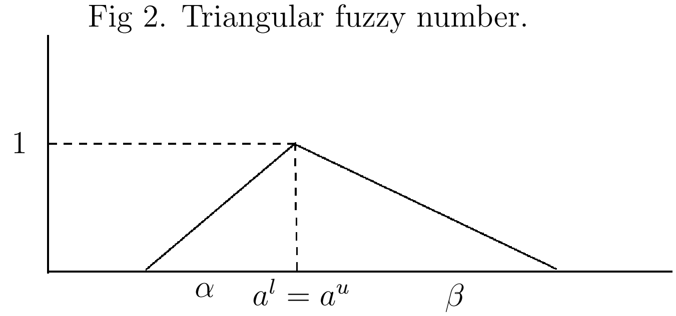
Definition 3.7. (L-R fuzzy number) A fuzzy number $\tilde{A}$ is of L-R type if there exit reference function L (L for left), (R for right) and scalers $\alpha > 0$, $\beta > 0$ with

$$\mu_{\tilde{A}}(x) = \begin{cases} L(\frac{m-x}{\alpha}) & x \leq m \\ R(\frac{x-m}{\beta}) & x > m \end{cases}$$

m, called the mien value of $\tilde{A}$, is real number, and α, β are called the left and right of $\tilde{A}$, is real number, and α, β are called the left and right expanse respectively, $\tilde{A}$ is denoted by $(m, \alpha, \beta)_{LR}$.

When $L(x) = R(x) = \begin{cases} 1 - x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$ then we have triangular fuzzy number.

Fig 2. Triangular fuzzy number.



We next define arithmetic on fuzzy numbers. Let $\tilde{a} = (m, \alpha, \beta)$ and $\tilde{b} = (n, \gamma, \theta)$ be two triangular fuzzy numbers. Define

$$c > 0, \quad c \in \Re; \quad c.\tilde{m} = (cm, c\alpha, c\beta), \tag{3.4}$$

$$c < 0, \quad c \in \Re; \quad c.\tilde{m} = (cm, -c\beta, -c\alpha), \tag{3.5}$$

$$\tilde{a} + \tilde{b} = (m + n, \alpha + \gamma, \beta + \theta), \tag{3.6}$$

Definition 3.8. The extension principle: [19]. One of the most basic concepts of fuzzy set theory that can be used generalize crisp mathematical concepts to fuzzy set is the extension principle. In its elementary form, it was already implied in Zadeh's first contribution [18]. Zadeh and Dubois and Prade [6], we define the extension principle as follows:

Let X be a cartesian product of universes $X = X_1 \times \cdots \times X_r$, and $\tilde{A}_1, \ldots, \tilde{A}_r$ be r fuzzy sets in $X_1, \ldots, X_r$, respectively, f is mapping form X to universe Y, $Y = f(x_1, \ldots, x_r)$. Then the extension principle allows us to define a fuzzy set $\tilde{B}$ in Y by $\tilde{B} = \{(Y, M_{\tilde{B}}(Y))|Y = f(x_1, \ldots, x_r), (x_1, \ldots, x_r) \in X\}$ where

$$\mu_{\tilde{B}}(y) = \begin{cases} \sup_{(x_1,\ldots,x_n)\in f^{-1}(y)} & \min\{\mu_{\tilde{A}_1}(x_1), \ldots, \mu_{\tilde{A}_r}(x_r) iff^{-1}(u) \neq \emptyset\} \\ 0 & \text{otherwise} \end{cases} \tag{3.7}$$

where f^{-1} is the inverse of f. for $r = 1$, the extension principle, of course, reduces to

$$\tilde{B} = f(\tilde{A}) = \{(Y, \mu_{\tilde{B}}(Y))|Y = f(x), x \in X\}$$

where

$$\mu_{\tilde{B}}(Y) = \begin{cases} \sup_{x\in f^{-1}(Y)} \mu_{\tilde{A}}(x), & iff^{-1}(Y) \neq \theta \\ 0 & \text{otherwise} \end{cases} \tag{3.8}$$

Theory 3.1: If $\tilde{A}, \tilde{B}$ are fuzzy numbers whose membership functions are continuous and surjective from R to $I = [0, 1]$, and $\circledast$ is continuous increasing (decreasing) binary operetation then $\tilde{A} \circledast \tilde{B}$ is a fuzzy number whose membership function is continuous and surjective from R to [0,1].

Definition 3.9. Extended Division. Division is also neither an increasing nor a decreasing operation if $\tilde{A}$ and $\tilde{B}$ are strictly positive fuzzy numbers, however (that is, $\mu_{\tilde{A}}(x) = 0$ and $\mu_{\tilde{B}}(x) = 0 \ \forall x \leq 0$), we obtain in analogy to the extended subtraction

$$\mu_{\tilde{A}\oslash\tilde{B}}(z) = \sup \min_{z=xy} \tag{3.9}$$

$$(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y))$$

$$= \sup \min_{z=xy}(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(\frac{1}{y}))$$

$$= \sup \min_{z=xy}(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}^{-1}(y))$$

$\tilde{B}^{-1}$ is a positive fuzzy number and also consider the following relation [16]:

$$(m, \alpha, \beta)_{LR}^{-1} \approx (m^{-1}, \beta m^{-2}, \alpha m^{-2})_{RL} \tag{3.10}$$

$$(m, \alpha, \beta)_{LR} \oslash (n, \delta, \gamma)_{RL} \approx (\frac{m}{n}, \frac{m\gamma + n\alpha}{n^2}, \frac{m\delta + n\beta}{n^2}) \tag{3.11}$$

4 Fuzzy Number Linear Programming

4.1 Ranking functions

There are several methods for solving fuzzy linear programming problems. One of the most convenient of these methods is based on the concept of comparison of fuzzy numbers by use of ranking functions [10, 17]. In fact, an efficient approach for ordering the elements of $F(\Re)$ is to define a ranking function $\tau : F(\Re) \longrightarrow \Re$ which maps each fuzzy number into the real line, where a natural order exists.

We define orders on $F(\Re)$ by

$$\tilde{a} \succeq \tilde{b} \quad \text{if and only if} \quad \tau(\tilde{a}) \geq \tau(\tilde{b}), \tag{4.12}$$

$$\tilde{a} \succ \tilde{b} \quad \text{if and only if} \quad \tau(\tilde{a}) > \tau(\tilde{b}), \tag{4.13}$$

$$\tilde{a} \cong \tilde{b} \quad \text{if and only if} \quad \tau(\tilde{a}) = \tau(\tilde{b}), \tag{4.14}$$

where $\tilde{a}$ and $\tilde{b}$ are in $F(\Re)$. Also we write $\tilde{a} \preceq \tilde{b}$ if and only if $\tilde{b} \succeq \tilde{a}$.

We restrict our attention to linear ranking functions, that is similar to the ranking function adopted, by N. Mahdavi-Amiri [13].

For a triangular fuzzy number $\tilde{a} = (m, \alpha, \beta)$, we use ranking function as follows:

$$\tau(\tilde{a}) = \frac{1}{2} \int_0^1 (\inf \tilde{a}_\alpha + \sup \tilde{a}_\alpha) d\alpha \longrightarrow \tau(\tilde{a}) = m + 1/4(\beta - \alpha). \tag{4.15}$$

4.2 Formulation of the fuzzy number linear programming problem

Definition 4.1. A fuzzy number linear programming problem (FNLPP) is defined as follows:

$$\begin{array}{lll} \text{Max} & z \cong \sum_{j=1}^n \tilde{c}_j x_j & \\ \text{s.t.} & \sum_{j=1}^n \tilde{a}_{ij} x_j \preceq \tilde{b}_i, & i = 1, \ldots, m, \\ & x_j \geq 0, & j = 1, \ldots, n, \end{array} \tag{4.16}$$

where $\cong$ and $\succeq$ mean equality and inequality with respect to the ranking function $\tau, \tilde{A} = (\tilde{a}_{ij})_{m \times n}$, $\tilde{c} = (\tilde{c}_1, \ldots, \tilde{c}_n)$, $\tilde{b} = (\tilde{b}_1, \ldots, \tilde{b}_m)^T$ and $\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_j \in F(\Re)$ for $i = 1, \ldots, m, j = 1, 2, \ldots, n$.

Definition 4.2. Any x which satisfies the set of constraints of FNLPP is called a feasible solution. Let Q be the set of all crisp feasible solutions of FNLPP. We say that $x^0 \in Q$ is an optimal feasible solution for FNLPP if $\tilde{c}x \preceq \tilde{c}x^0$ for all $x \in Q$.

Definition 4.3. We say that the real number a corresponds to the fuzzy number $\tilde{a}$, with respect to a given linear ranking function τ, if $a = \tau(\tilde{a})$.

The following theorem shows that any FNLPP can be reduced to a linear programming problem (see Maleki [11] and Maleki et al. [12]).

Theorem 4.4. The following linear programming problem (LPP) and the FNLPP in (4.16) are equivalent:

$$\begin{array}{lll} \text{Max} & z = \sum_{j=1}^{n} c_j x_j & \\ \text{s.t.} & \sum_{j=1}^{n} a_{ij} x_j \leq b_i, & i = 1, \ldots, m, \\ & x_j \geq 0, & j = 1, \ldots, n, \end{array} \tag{4.17}$$

where a_{ij}, b_i, c_j are real numbers corresponding to the fuzzy numbers $\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_j$ with respect to a given linear ranking function τ, respectively.

Remark 4.5. The above theorem shows that the set of all crisp feasible solutions of FNLPP and all feasible solutions of LPP are the same. Also if $\bar{x}$ is an optimal feasible solution for FNLPP, then $\bar{x}$ is an optimal feasible solution for LPP.

Corollary 4.6. If LPP does not have a feasible solution then FNLPP does not have a solution either.

5 Fuzzy Cost Efficiency

For obtaining fuzzy cost efficiency (FCE) we consider two algorithm for algorithm (1) we solve minimal cost model with data set of fuzzy numbers as follows:

$$\begin{array}{lll} \text{Min} & \sum_{i=1}^{m} p_{io} \tilde{x}_i & \\ \text{s.t.} & \sum_{j=1}^{n} \tilde{x}_{ij} \lambda_j \preceq \tilde{x}_i & i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \tilde{y}_{rj} \lambda_j \succeq \tilde{y}_{ro}, & r = 1, \ldots, s, \\ & \lambda_j \geq 0, & j = 1, \ldots, n, \\ & \tilde{x}_i \succeq 0, & i = 1, \ldots, m. \end{array} \tag{5.18}$$

Mahdavi et.al. [13] show that real number a corresponds to the fuzzy number $\tilde{a}$, with respect to a given linear ranking function τ, if $a = \tau(\tilde{a})$, then $x_{ij} = \tau(\tilde{x}_{ij})$ $y_{rj} = \tau(\tilde{y}_{rj})$ which $\tilde{x}_{ij}, \tilde{y}_{rj}$ are triangular fuzzy number and $\tilde{x}_i = (m, 0, 0)$. Then we define crisp model (2.2) which is equivalent to the fuzzy number linear programming problem.

For algorithm (2) $\tilde{x}_i$ is a fuzzy number that is $\tilde{x}_i = (m_i, \alpha_i, \beta_i)$, minimal cost model as follows:

$$\begin{array}{lll} \text{Min} & \sum_{i=1}^{m} p_{io}(m_i, \alpha_i, \beta_i) & \\ \text{s.t.} & \sum_{j=1}^{n} \tilde{x}_{ij}\lambda_j \preceq (m_i, \alpha_i, \beta_i) & i = 1, \dots, m, \\ & \sum_{j=1}^{n} \tilde{y}_{rj}\lambda_j \succeq \tilde{y}_{ro}, & r = 1, \dots, s, \\ & \lambda_j \geq 0, & j = 1, \dots, n, \\ & \beta_i, \alpha_i, m_i \geq 0 & \text{i=1, \dots ,m,} \\ & \text{m}_i - \alpha_i \geq 0 & \text{i=1, \dots ,m} \end{array} \quad (5.19)$$

because (m_i, α_i, β_i) is a fuzzy number then we have equivalent model follows:

$$\begin{array}{lll} \text{Min} & \sum_{i=1}^{m} p_{io}(m_i + \frac{1}{4}(\beta_i - \alpha_i)) & \\ \text{s.t.} & \sum_{j=1}^{n} \tilde{x}_{ij}\lambda_j \leq (m_i + \frac{1}{4}(\beta_i - \alpha_i)) & i = 1, \dots, m, \\ & \sum_{j=1}^{n} \tilde{y}_{rj}\lambda_j \geq \tilde{y}_{ro}, & r = 1, \dots, s, \\ & \lambda_j \geq 0, & j = 1, \dots, n, \\ & \beta_i, \alpha_i, m_i \geq 0 & \text{i=1, \dots ,m,} \\ & \text{m}_i - \alpha_i \geq 0 & \text{i=1, \dots ,m} \end{array} \quad (5.20)$$

Theorem 5.1. Models (5.18) , (2.2) are equivalent and all so (5.19) , (5.20) are equivalent (see Maleki [15] and Maleki et.al. [16]). Where x_{ij}, y_{rj} are real number corresponding to the fuzzy numbers $\tilde{x}_{ij}, \tilde{y}_{rj}$ with respect to a given linear ranking function τ, respectively.
Then, we develop two following algorithm for obtaining cost efficiency with data of fuzzy numbers:

Algorithm 1:

1. Solve model (5.18) for each DMU, by using ranking function we solve crisp model (2.2) which is equivalent(by Theorem (4.4)).
2. Then by using model (2.3), we obtain cost efficiency numerator of a fraction is a real number and, denominator is a fuzzy number, therefore by this fraction we obtain a fuzzy number. For obtaining, Fuzzy cost efficiency, we consider fraction as following, Fuzzy cost efficiency $FCE = \sum_{i=1}^{m} p_{io}x_i^* \frac{1}{\sum_{i=1}^{m} p_{io}x_{io}}$.
3. For obtaining $\frac{1}{\sum_{i=1}^{m} p_{io}\tilde{x}_{io}}$, we apply relation (2.4), (2.6) and then relation (3.10) and because the result of fraction is a positive fuzzy number, hence theorem (3.10) can now be applied.

4. Then by using relation (2.4), ($p_i > 0$ and $x_i > 0$ ($i = 1, \dots, m$) are respectively, cost and data of obtained DMU by model (2.2), and we know data set of Data Envelopment Analysis (DEA) are positive, also c in model (2.4) is positive and real), hence we yield a positive fuzzy number for Fuzzy Cost Efficiency (FCE).

5. After that we obtain $\tau(FCE)$ and it is Cost Efficiency with fuzzy data.

Algorithm 2:

1. Solve model (5.19) for each DMU, by using ranking function we solve crisp model (5.20) which is equivalent pp (5.19).

2. Then by using model (2.3), we obtain cost efficiency. Numerator and denominator of fraction is a fuzzy number therefore fraction is a fuzzy number.

3. For obtaining $FCE = \frac{\sum_{i=1}^{m} p_{io} x_i^*}{\sum_{i=1}^{m} p_{io} \tilde{x}_{io}}$, we apply relation (2.4), (2.6) and then relation (3.11) for division of fuzzy numbers.

4. After that we obtain $\tau(FCE)$ and it is Cost Efficiency with fuzzy data.

For an illustration of proposed algorithm consider an example used in insurance organization data set of the triangular fuzzy numbers.

6 Methodology and examples

We evaluate 30 branches of Tehran Social Security Insurance Organization at this section. Each branch uses of four inputs in order to produce four outputs. The labels of inputs and outputs are presented in under table.

	Input	Output	Cost
1	The number of personals	The total number of insured persons	350000
2	The total number of computers	The number of insured persons' agreements	400000
3	The area of the branch	The total number of life-pension receivers	500000
4	Administrative expenses	The receipt total sum (Income)	1

Table 2: The labels of inputs and outputs.

The total triangular Fuzzy data has been viewed in tables (3), and (4).We have input 3 as crisp data in table (3). It is considered that "M"as number middle, "α"left expanse. "β"as right expanse. For example if $\tilde{a} = (m, \alpha, \beta)$ denote a triangular fuzzy number then we use ranking function as follows:

$$\tau(\tilde{a}) = m + \frac{1}{4}(\beta - \alpha)$$

After using the ranking function $\tau(\tilde{a})$, the data is given as crisp numbers and then with applying explained method on the essay, the results Fuzzy Cost Efficiency and Defuzzy of them by Algorithm(1) are presented in table (5) and the results Fuzzy Cost Efficiency and Defuzzy of them by Algorithm (2) are presented in table (6). We see that DMU_3,DMU_9,DMU_{11}, and DMU_{18} are Efficient in table (6), but in table (5) we don't have Efficient DMU. It shows that Algorithm (2) is better than Algorithm (1). because the results in table (6) are nearest to real.

	$Im1$	$I\alpha_1$	$I\beta_1$	$Im2$	$I\alpha_2$	$I\beta_2$	$I3$	$Im4$	$I\alpha_4$	$I\beta_4$
1	94.83	1.83	2.16	84.5	0.5	2.5	4000	78262041.33	25392261.33	69544898.67
2	77	2	2	93	2	2	2565	73241407	18700853	65837545
3	76.5	1.5	1.5	87	1343	0	0	71482960	27879871	61941109
4	92.83	0.83	1.16	93	1500	0	0	497692042.2	440112343.2	2095132858
5	90.33	2.33	1.66	85.66	2.66	1.33	1680	82962450.83	2055 5596.83	57375613.17
6	102.33	1.33	2.66	97	0	0	3750	83979470.83	75553970.83	55503766.17
7	94.5	0.5	0.5	90.5	0.5	0.5	3313	124490390	38894251	64118743
8	86.33	3.33	2.66	92.33	0.33	0.66	1500	77394251.33	18133979.33	26530914.67
9	102.83	0.83	3.16	92	0	0	1600	136743469.3	52591813.33	119377644.7
10	102.5	0.5	0.5	96.33	1.33	0.66	1725	71533303.67	322922 03.67	57999041.33
11	95.5	2.5	0.5	79	0	0	1920	93931521.5	32759865.5	59851928.5
12	78.16	2.16	0.83	91	0	0	4433	80608666.67	32905666.67	7894.33
13	104.66	1.66	2.33	104.33	1.33	0.66	2500	81318447	40482291	70233678
14	88.5	2.5	1.5	95	0	0	2800	58841917.83	19884666.83	23593549.17
15	81.66	3.66	2.33	93.83	1.83	1.16	1630	67658394.67	21270151.67	54420348.33
16	89	2	2	85.33	0.33	0.66	1127	65516 280.33	32077593.33	57141322.67
17	91.5	1.5	1.5	104	0	0	3400	125551611.2	75560965.17	163604623.8
18	114.33	3.33	2.66	94.33	2.33	0.66	1304	78379833	46487464	52020714
19	96.33	2.33	2.66	98	0	0	4206	105183991.8	50050519.83	159270923.2
20	86.66	1.66	1.33	01	0	0	1340	84203387.33	35893023.33	33078687.67
21	69.16	1.16	1.83	90.16	1.1667	0.83	1393	63480967.83	25531087.83	42072685.17
22	113.33	5.33	5.66	123	1	1	2191	1070 27548.2	29761595.17	36600655.83
23	79	1	1	100	0	0	2140	136266225	42751019	149072659
24	87.33	1.33	1.66	93.5	0.5	0.5	1231	67564034.33	62464054.33	35970829.67
25	98.33	1.33	1.66	90	0	0	1960	89732 318.33	29169845.33	43037370.67
26	74.66	1.66	2.33	85	3	1	3375	72153562.17	27478172.17	4 1472834.83
27	104.33	3.33	3.66	102.5	1.5	0.5	2540	92589909.17	31236367.17	94422444.83
28	99.16	2.16	2.83	97.5	1.5	0.5	1603	452222971.8	381361846.8	479307836.2
29	73.83	4.83	2.16	79	2	2	2300	83384457.33	23299737.33	27313481.67
30	89.5	2.5	2.5	92	0	0	2930	78813784.17	21245567.17	26894182.83

Table 3: The triangular fuzzy inputs for 30 branches of insurance organization.

	Om1	$O\alpha_1$	$O\beta_1$	$Om2$	$O\alpha_2$	$O\beta_2$	$Om3$	$O\alpha_3$	$O\beta_3$	$Om4$	$O\alpha_4$	$O\beta_4$
1	58449.33	781.33	1153.66	49.16	17.16	2	1136.16	19.16	11.83	211.16	22.16	64.83
2	37044.83	122.83	134.16	21.83	7.83	14	8795.83	160 .83	123.16	230.83	55.83	68.16
3	34438.33	9078.33	5010.66	31.66	11.66	7	6599.16	11.16	4.83	427.66	112.66	157.33
4	36651.83	404.83	371.16	41	20	0	9406	1326	1415	234.5	93.5	94.5
5	46389.33	10018.33	9692.66	40.5	12.5	19	9940.16	242.16	103.83	278.16	63.16	105.83
6	71808.66	2737.66	1196.33	15.83	15.83	7	8010.66	142.66	109.33	382.5	160.5	261.5
7	38667.66	1191.66	871.33	104.83	31.83	26	13781.83	660.83	1482.16	344.5	0.5	0.5
8	50189.16	2095.16	1954.83	20	8	5	1565.33	12 .33	9.66	391.5	130.5	242.5
9	88309.5	3778.5	4446.5	68.16	68.16	45	11802.5	1012.5	3599.5	425.33	26.33	190.66
10	49309	2352	4248	30.16	11.16	10	7617.5	753.5	7 61.5	226.33	39.33	81.66
11	35972	4418	3369	212.66	42.66	89	12639	297	239	254.83	127.83	110.16
12	31291.16	4279.16	8319.83	26	5	10	7793.5	241.5	202.5	181	21	64
13	61404.5	1823.5	1547.5	47	16	0	7409.33	231.33	22 2.66	326.66	72.66	93.33
14	90320.66	9895.66	3621.33	45.83	9.83	1	717.66	44.66	30.33	229.83	57.83	82.16
15	48643.83	4338.83	2321.16	20.66	3.66	1	10290.16	4.16	2.83	134.83	55.83	137.16
16	43741.5	3944.5	6113.5	31.5	5.5	11	7851.33	354.33	345.66	191.66	16.66	40.33
17	77586.66	23987.66	5336.33	24	10	0	5081.33	122.33	123.66	206.33	56.33	78.66
18	79290.66	6737.66	6534.33	28.33	9.33	6	4953.16	674.16	335.83	215.66	100.66	108.33
19	73663	26766	13700	23.33	10.33	0	1083.33	258.33	522.66	238	107	292
20	32189.16	3334.16	1447.83	59.66	36.66	0	14785.83	641.83	436.16	360.83	59.83	83.16
21	28340.83	512.83	400.16	27.5	12.5	15	953.16	15.16	14.83	241.5	81.5	18.5
22	105355.5	2112.5	3121.5	58	17	10	2658.5	9.5	9.5	362.66	107.66	105.33
23	34310.66	2051.66	1894.33	36.33	6.33	18	2273	67	59	463	40	126
24	58240.83	4306.83	3519.16	62.16	9.16 67	18	10337.5	112.5	124.5	201	33	77
25	83197.66	8395.66	3504.33	65.83	22.83	0	4772.66	204.66	237.33	286	75	72
26	44457	572	939	31.16	10.16	10	617.16	30.16	40.83	306.16	81.16	124.83
27	82569	482	579	55.16	14.16	15	9240.83	471. 83	318.16	302.83	84.83	253.16
28	69914.33	4925.33	4303.66	91	24	12	13219.16	447.16	488.83	212.16	71.16	48.83
29	38993.33	325.33	212.66	32.5	9.5	10	1494	19	2 2	107.16	45.16	109.83
30	62304.5	1440.5	2479.5	30.83	15.83	5	12275.33	393.33	345.66	262.5	75.5	143.5

Table 4: The triangular Fuzzy outputs for 30 branches of insurance organization.

	M	α	β	$\tau(FCE)$
1	0.329	0.10923	0.401879	0.32
2	0.409891	0.194388	0.583153	0.40
3	0.101049	0.784549	0.356754	0.99
4	0.460762	0.732926	0.154034	0.31
5	0.690704	0.408566	0.156735	0.68
6	0.432239	0.119957	0.161581	0.43
7	0.496686	0.173127	0.105414	0.49
8	0.9312	0.288673	0.202303	0.92
9	1.01037	0.120585	0.52927	0.99
10	0.592834883	0.034355672	0.019400101	0.58
11	1.005896546	0.053961775	0.03023638	0.99
12	0.216379563	0.007262093	0.003085393	0.21
13	0.531368859	0.026882343	0.015680375	0.52
14	0.630700563	0.009956417	0.008569832	0.63
15	0.73374437	0.043078981	0.018009086	0.72
16	0.825338325	0.069075007	0.039122374	0.81
17	0.397703387	0.034370109	0.015933034	0.39
18	1.00143333	0.065951032	0.060209516	0.99
19	0.337097242	0.023674717	0.007517068	0.33
20	0.999111749	0.040628035	0.044177881	0.99
21	0.567347972	0.029774873	0.018264348	0.56
22	0.893509191	0.026972827	0.022160194	0.89
23	0.709780253	0.083252926	0.024014336	0.69
24	0.991219878	0.048508612	0.083320729	0.99
25	0.855896241	0.032745542	0.022247776	0.85
26	0.357184927	0.008379029	0.005743403	0.35
27	0.665941657	0.044349254	0.015261436	0.65
28	0.732594475	0.265182296	0.211218982	0.71
29	0.361352138	0.008082337	0.007220011	0.36
30	0.531988664	0.009164672	0.007300458	0.53

Table 5: Fuzzy Cost Efficiency and Defuzzy of it by Algorithm (1)

	M	α	β	$\tau(FCE)$
1	0.267792149	0.008900787	0.24782517	0.32
2	0.331343966	0.0157138	0.321069793	0.40
3	0.834619795	0.064800118	0.733322367	1.00
4	0.374705863	0.596038623	0.469889354	0.34
5	0.552831097	0.032701106	0.567454377	0.68
6	0.339162943	0.009412573	0.385330083	0.43
7	0.408598682	0.014242313	0.362427243	0.49
8	0.750753805	0.023273404	0.738248877	0.92
9	0.792443939	0.094576582	0.93869962	1.00
10	0.471723085	0.027337062	0.499883961	0.58
11	0.857942182	0.046024696	0.617606588	1.00
12	0.176127647	0.005911165	0.163519117	0.21
13	0.417049343	0.021098834	0.469584989	0.52
14	0.512006963	0.008082686	0.481731447	0.63
15	0.604382507	0.035483996	0.532281392	0.72
16	0.67048561	0.056114925	0.651192999	0.81
17	0.319192168	0.027585055	0.326832542	0.39
18	0.806801053	0.053133205	0.82703669	1.00
19	0.266773848	0.018735826	0.287242427	0.33
20	0.812183508	0.033026756	0.783625391	0.99
21	0.457528034	0.024011435	0.45400887	0.56
22	0.718092848	0.021677442	0.719475068	0.89
23	0.58642773	0.068784422	0.51325097	0.69
24	0.819540399	0.040106911	0.755607501	0.99
25	0.699857659	0.026775697	0.642346123	0.85
26	0.284891353	0.006683129	0.293755289	0.35
27	0.532945764	0.035492219	0.544197131	0.66
28	0.607974361	0.22007269	0.673769485	0.72
29	0.296632244	0.006634752	0.264806422	0.36
30	0.436163213	0.007513868	0.389287269	0.53

Table 5: Fuzzy Cost Efficiency and Defuzzy of it by Algorithm (2)

7 Conclusion

In this work we apply linear ranking function proposed by Mahdavi-Amiri et al. [14] for solving minimal cost model with data set of fuzzy numbers. Then, cost efficiency measurement with certain price obtained similar proposed algorithms. Hence, by using ranking function fuzzy number linear programming problem (FNLPP) and linear programming problem (Lpp) are equivalent.

But we saw that algorithm (2) is better than algorithm (1), because the results of algorithm (2) are nearest to real.

References

[1] Bellman R. E., L. A. Zadeh, Decision making in a fuzzy environment, *Manag. Sci.* 17(1970) 141-164.

[2] Bezdek J. C., Fuzzy models-What are they, and Why? IEEE Trans. *Fuzzy Sys.* 1(1)(1993) 1-9.

[3] Camanho A. S, Dyson R. G. Cost efficiency measurment with price uncertainly: A DEA application to bank banch assessments. *European Journal of Operational Research.* 161(2006) 432-446.

[4] Charnes, A. Cooper, W. W., and Rhodes, E. Measuring the efficiency of decision making unitis, *European Journal of Operational Research* 2(1987), 429-444.

[5] Debreu, G., The coefficient of resource utilisation Econometrica 19(1951), 273-267.

[6] Dubois D., Dubois, H. Prade, Fuzzy sets and systems, Theorey and Applications, Academic Press, New York, 1980.

[7] Fare, R. Grosskopf, S., Lovell, C. A. K., The measurement of efficiency of production. *Kluwer Academic Publishers.* (1985).

[8] Farrell, M. J., The measurement of productive efficiency Journal of the Royal Statistical Society. *Series A.* 120(1957), 253-281.

[9] Fortemps P., M. Roubens, Ranking and defuzzification methods based on area compensation, *Fuzzy Sets Sys.* 82(1996)319-330.

[10] Fuller R., H. J. Zimmermann, Fuzzy reasoning for solving fuzzy mathematical programming problems, *fuzzy Sets Sys.* 60(1993) 121-133.

[11] Garcia-Aguado C., J. L. Verdegay, On the sensitivity of membership functions for fuzzy linear programming problems, *Fuzzy Sets Sys.* 56(1993)47-49.

[12] Hosseinzadeh F., G. R. Jahanshahloo,M. Alimardani, A New Approach for Efficiency measures by Fuzzy linear programming and Application in Insurance Organization, *Applied Mathematical Sciences*, Vol. 1, 2007,no. 13-16.

[13] Jahanshahloo G. R.,Hosseinzadeh F.,H. Nikoomaram, M. Alimardani, Using a certain Linear Ranking Funcion to Measure the Malmquist Productivity Index withFuzzy Data and Application in Insurance Organization, *Applied Mathematical Sciences*, Vol. 1, 2007,no. 13-16.

[14] Koopmans. Activity analysis of production and Allocation. *John Wiley, New York* (1957).

[15] Mahdavi-Amiri N., S. H. Nasseri, Duality in fuzzy number linear programming by use of a certain linear ranking function, *Applied Mathematics and Computation* 180(2006) 206-216.

[16] Maleki H. R., Ranking functions and their applications to fuzzy linear programming, *Far East J. Math. Sci. (FJMS)* 4(2002) 283-301.

[17] Maleki H. R., M. Tata, M. Mashinchi, Linear programing with fuzzy variables, *Fuzzy Sets Sys.* 109(2000) 21-33.

[18] Tanaka H. C., Fuzzy set theory and its practical applications.

[19] Zadeh L. A., *Fuzzy sets, Inform, Conhol* 8:338-353 (1965).

[20] Zimmermann H. J., Fuzzy mathematical programming, *Compact. Oper.Res.*q 10(4) (1993)291-298.

Received: August 8, 2007