Solving Fully Fuzzy Linear Programming Problem by the Ranking Function

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Abstract

The modeling and solving the optimization problem is one of the most important daily problem.By notation the nature of data in practice which are imprecise, fully fuzzy linear programming problem (FFLP) is a power full tool to modeling the practical optimization problem. In This paper after introducing FFLP, a new method to solve it is proposed. a linear ranking function for defuzzifying the FFLP is used , Equivalency between two problems is proved by some theorems.

Keywords: fuzzy number, triangular fuzzy number, ranking, extension

1 Introduction

Linear programming is one of the most frequently applied operation research techniques. Although it has been investigated and expanded for more than six decades by many researchers and from the various point of views, it is still useful to develop new approaches in order to better fit the real world problems within the framework of linear programming.

In conventional approach, parameters of linear programming models must be well defined and precise. However, in real world environment, this is not a realistic assumption. Usually, the value of many parameters of a linear programming model is estimated by experts. Clearly, it can not be assumed the knowledge of experts is so precise. Since Bellman and Zadeh [3] proposed the concept of decision making in fuzzy environments, a number of researchers have exhibited their interest to solve the fuzzy linear programming problems.

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Existing methods can be divided into two groups, depending on the fuzziness of decision parameters and decision variables.

In the first group, the researchers assumed the decision parameters are fuzzy numbers while the decision variables are crisp ones, see [5, 6, 15, 17, 18, 21, 24, 29, 30, 31]. This means that in an uncertain environment, a crisp decision is made to meet some decision criteria.

Tanaka *et al.* [33] can be considered as the pioneers for the second group of fuzzy linear programming problems with fuzzy decision variables and crisp decision parameters. Tanaka *et al.* [33] initially proposed a possibilistic linear programming formulation and applied linear programming (LP) technique to obtain the largest possibility distribution of the decision variables. Three kinds of possibility distributions of fuzzy decision variables in possibilistic linear programming problems were introduced by Tanaka *et al.* [32]. These possibility distributions reflect the inherent fuzziness in fuzzy decision problems. Maleki *et al.* [23] proposed a method for solving the fuzzy linear programming problems with fuzzy variables in which all parameters are fuzzy numbers except the technological coefficients.

Recently, Buckley and Feuring [7] introduced a general class of fuzzy linear programming called fully fuzzified linear programming (FFLP) problems, where all decision parameters and variables are fuzzy numbers. In fact, fully fuzzified linear programming problem is a generalized version of both classes mentioned above. As pointed out by Buckley and Feuring [7], searching for the optimal solutions of FFLP problems is a very difficult task. They employed a directed search technique of evolutionary algorithm type.

To best of our knowledge, in all the methods developed for fuzzy decision making problems Zadeh's extension principle [36] (based on triangular norm T_M) is applied for fuzzy arithmetic operations on fuzzy numbers. Here, we solve full fuzzy linear programming (FFLP) by the same multiplication and special type of distance. In this study, we propose a new method for solving FFLP problems, by applying the concept of comparison of fuzzy numbers. For comparison of fuzzy numbers, there are many methods [1, 4, 9, 10, 12, 13, 20, 21, 28]. There are also some methods using crisp relations to rank fuzzy numbers [9, 13, 20, 24] by mapping every fuzzy number into a point on the real line $\Re$. Wang and Kerre [34] presented a comprehensive survey of the existing methods. The paper is organized as follows: In Section 2 we present the basic definitions of fuzzy arithmetics. In Section 3 the FFLP is introduced and solved using the above mentioned multiplication and a special distance and the properties of the presented linear ranking function with related theorems, are introduced.

2   Preliminaries

We review the fundamental notation of fuzzy set theory initialed by Bellman and Zadeh[1]. Below, we give definitions and notations taken from Zimmerman[3].

Definition 2.1. *If X is a collection of objected generically by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs:*
    $\tilde{A} = \{(x, \mu_A(x)) | x \in X)$
    $\mu_A(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth)of that maps to the membership space. When this space contains only the two points 0 and 1, $\tilde{A}$ is nonfuzzy and $\mu_A(x)$ is identical to the characteristic function of a nonfuzzy set. The family of all fuzzy sets in X is denoted by $F(X)$.

Definition 2.2. *The support of a fuzzy set $\tilde{A}$ is the crisp set of all $x \in X$ such that $\mu_A(x) \geq 0$.*

Definition 2.3. *The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α-level set:*
$\tilde{A}_\alpha = \{x \in X | \mu_A(x) \geq \alpha\}$

Definition 2.4. *A fuzzy set $\tilde{A}$ is convex if:*
    $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$. , $x_1, x_1 \in X$ and $\lambda \in [0, 1]$
    alternatively , a fuzzy set is convex if all α-level sets are convex.

Definition 2.5. *A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set $\tilde{A}$ of the real line $\Re$, such that:*

- *It exists exactly one $x_0 \in \Re$ with $\mu_{\tilde{A}}(x_0) = 1$ (x_0 is called the mean value of $\tilde{A}$) that we show with $F(\Re)$*

- *$\mu_{\tilde{A}}(x)$ is piecewise continuous.*
  parametric form of a fuzzy number has been introduced and presented by $\tilde{A} = (\underline{A}(r), \overline{A}(r))$,where $\underline{A}(r)$ and $\overline{A}(r)$,$0 \leq r \leq 1$, satisfying the following requirements:

- *1.$\underline{A}(r)$ is monotonically increasing left continuous function.*

- *2.$\overline{A}(r)$is monotonically decreasing left continuous function.*

- *3.$\underline{A}(r) \leq \overline{A}(r)$, $0 \leq r \leq 1$.*

- $4.\underline{A}(r) = \overline{A}(r) = 0,\ r \leq 0, r \geq 1$

Definition 2.6. *A fuzzy set $\tilde{A}$, is called triangular fuzzy number with peak (or center) a , left width α and right width β if its membership function has the following form:*

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - (a - x)/\alpha, \ a - \alpha \leq x \leq a \\ 1 - (x - a)/\beta, \ a \leq x < a + \beta \\ 0, \quad otherwise \end{cases} \tag{2.1}$$

And the set of all triangular fuzzy numbers is denoted by $FT(\Re)$, Where in parametric form is:
$\tilde{A} = (\alpha(r - 1) + a, \beta(1 - r) + a)$

Definition 2.7. *A fuzzy number $\tilde{A}$ is said to be a L_R type fuzzy number iff:*

$$\mu_{\tilde{A}}(x) = \begin{cases} L((a - x)/\alpha), \ x \leq m, \ \alpha > 0 \\ R((x - a)/\beta), \ x \geq m, \ \beta > 0 \\ 0, \quad otherwise \end{cases} \tag{2.2}$$

L is for left and R for right reference .a is the mean value of $\tilde{A}$.

α and β are called left and right spreads, respectively. Symbolically, we write;
$\tilde{A} = (a, \alpha, \beta)$
if $L(x)$ and $R(x)$ be linear functions then the corresponding LR number is said to be a triangular fuzzy number.

Theorem 2.1. *Let $\tilde{A}$, $\tilde{B}$ are two fuzzy numbers of LR type:*

$\tilde{A} = (a, \alpha, \beta)_{LR}$, $\tilde{B} = (b, \lambda, \delta)_{LR}$
Then:

- $(a, \alpha, \beta)_{LR} \oplus (b, \lambda, \delta)_{LR} = (a + b, \alpha + \lambda, \beta + \delta)_{LR}$

- $-(b, \lambda, \delta) = (-b, \delta, \lambda)_{LR}$

- $(a, \alpha, \beta)_{LR} - (b, \lambda, \delta)_{LR} = (a - b, \alpha + \delta, \beta + \lambda)_{LR}$

Proof:[2].

Theorem 2.2. *Let $\tilde{A}$, $\tilde{B}$ are two fuzzy numbers of LR type:*
$\tilde{A} = (a, \alpha, \beta)_{LR}$, $\tilde{B} = (b, \lambda, \delta)_{LR}$
Then: If $\tilde{A}$, $\tilde{B}$ are positive:

$$(a, \alpha, \beta)_{LR} \otimes (b, \lambda, \delta)_{LR} = (ab, a\lambda + b\alpha, a\delta + b\beta)$$

If $\tilde{B}$ positive, $\tilde{A}$ negative:

$$(a, \alpha, \beta)_{LR} \otimes (b, \lambda, \delta)_{LR} = (ab, b\alpha - a\lambda, b\beta - a\delta)$$

If $\tilde{A}$, $\tilde{B}$ negative:

$$(a, \alpha, \beta)_{LR} \otimes (b, \lambda, \delta)_{LR} = (ab, -a\delta - b\beta, -b\alpha - a\lambda,)$$

Proof:[2].

Definition 2.8. *Scalar multiplication of fuzzy number:*

$$\lambda \otimes \tilde{A} = \lambda \otimes (a, \alpha, \beta)_{LR} = \begin{cases} (\lambda a, \lambda\alpha, \lambda\beta)_{LR}, \ \lambda > 0, \\ (\lambda a, -\lambda\beta, -\lambda\alpha)_{LR}, \ \lambda < 0, \end{cases} \qquad (2.3)$$

Definition 2.9. *A matrix $\tilde{A} = (a_{ij})_{m*n}$ is called a fuzzy matrix if each element of $\tilde{A}$ is a fuzzy number [2].*
$\tilde{A}$ Will be positive(negative) and denoted by $\tilde{A} \succeq 0$ ($\tilde{A} \preceq 0$). If each element of $\tilde{A}$ be positive (negative).
Up to rest of this, we use $\tilde{a} = (a, a', a'')$ for fuzzy number that a is the core, a' is the left margin and a" is the right margin.

Definition 2.10. *Let $\tilde{A} = (\tilde{a_{ij}})$ and $\tilde{B} = (\tilde{b_{ij}})$ be two $m*n$ and $n*p$ fuzzy matrices, we define $\tilde{A} \otimes \tilde{B} = \tilde{C} = [\tilde{c_{ij}}]$ which is the $m*p$ matrix where:*
$\tilde{c_{ij}} = \sum_{k=1,...,n} \tilde{a_{ik}} \otimes \tilde{b_{kj}}$

3   Ranking function

In fact, an efficient approach for ordering the elements is to define a ranking function $D : F(\Re) \longrightarrow \Re$ which maps for each fuzzy number into the real line, where a natural order exists. We define orders on by:
$\tilde{A} \succeq \tilde{B}$ If and only if $D\tilde{(A)} \geq D\tilde{(B)}$

$\tilde{A} \preceq \tilde{B}$ If and only if $D(\tilde{A}) \leq D(\tilde{B})$

$\tilde{A} = \tilde{B}$ If and only if $D(\tilde{A}) = D(\tilde{B})$

Where $\tilde{A}, \tilde{B}$ are in $F(\Re)$. Also we write $\tilde{A} \succeq \tilde{B}$ if and only if $-\tilde{A} \preceq -\tilde{B}$. The following lemma is now immediate.

Lemma 3.1. *Let D be any linear ranking function then:*

(i) $\tilde{A} \succeq \tilde{B}$ iff $\tilde{A} - \tilde{B} \succeq 0$ iff $-\tilde{B} \succ -\tilde{A}$

(ii) $\tilde{A} \succeq \tilde{B}$ and $\tilde{C} \succeq \tilde{D}$ then $\tilde{A} \oplus \tilde{C} \succeq \tilde{B} \oplus \tilde{D}$

Proof[6].

We restrict our attention to linear ranking function ,that is a ranking function D such that :

For any $\tilde{A}$, $\tilde{B}$ belonging to $F(\Re)$ and any $k \in \Re$ [6].

Here we introduce a linear ranking function that is similar to the ranking function [4]. For any arbitrary fuzzy number $\tilde{A} = (\underline{A}(r), \overline{A}(r))$, we use ranking function as follows:

$D(\tilde{A}) = 1/2 \int_{[0,1]} (\underline{A}(r)) + \int_{[0,1]} (\overline{A}(r))$

For triangular fuzzy number this reduces to:

$D(\tilde{A}) = A + 1/4(A'' - A')$

Then, for triangular fuzzy number $\tilde{A}$ and $\tilde{B}$, we have:

$\tilde{A} \succeq \tilde{B}$ if and only if $A + 1/4(A'' - A') \geq B + 1/4(B'' - B')$

4   Full fuzzy number linear programming problems(FFLP)

$$
\begin{cases}
min \ \tilde{z} = \tilde{c_1} \otimes \tilde{x_1} \oplus \ldots \tilde{c_n} \otimes \tilde{x_n} \\
\quad s.t. \\
\tilde{a_{11}} \otimes \tilde{x_1} \oplus \ldots \tilde{a_{1n}} \oplus \tilde{x_n} \succeq \tilde{b_1} \\
\quad \vdots \\
\tilde{a_{m1}} \otimes \tilde{x_1} \oplus \ldots \tilde{a_{mn}} \otimes \tilde{x_n} \succeq \tilde{b_m} \\
\\
\tilde{x_1} \succeq 0, \tilde{x_2} \succeq 0, \ldots, \tilde{x_n} \succeq 0
\end{cases}
\tag{4.4}
$$

The matrix form of the above equation is:

$$
\begin{cases}
min \ \tilde{z} = \tilde{c} \otimes \tilde{x} \\
\quad s.t. \\
\tilde{A} \otimes \tilde{x} \succeq \tilde{b} \\
\tilde{x} \succeq 0
\end{cases}
\tag{4.5}
$$

The coefficient matrix $\tilde{A} = [\tilde{a_{ij}}]_{m*n}$, $1 \leq i, j \leq n$ is $m * n$ fuzzy matrix where $\forall\ i, j$, $\tilde{a_{ij}} \succ 0$ or $\tilde{a_{ij}} \prec 0$ and $\tilde{x}_i$, $\tilde{b}_j \in F(\Re)$. If matrix $\tilde{A}$ denoted by $\tilde{A} = (A, A', A'')$ that $A = [a_{ij}]$, $A' = [a'_{ij}]$, $A'' = [a''_{ij}]$, $\tilde{x} = (x, x', x'')$, $\tilde{b} = (b, b', b'')$ Then we have:

$$\begin{cases} min\ \tilde{z} = (c, c', c'') \otimes (x, x', x'') \\ s.t. \\ (A, A', A'') \otimes (x, x', x'') \succeq (b, b', b'') \\ \\ (x, x', x'') \succeq 0 \end{cases} \qquad (4.6)$$

Regarding theorem (2.1) if S, T are two matrices defined as follows:
$S = \{\tilde{a_{ij}} | \tilde{a_{ij}} \succeq 0\}$, $T = \{\tilde{a_{ij}} | \tilde{a_{ij}} \prec 0\}$
And I^+, I^- are two indexes where:
$I^+ = \{i | \tilde{c}_i \succeq 0\}$, $I^- = \{i | \tilde{c}_i \prec 0\}$
And we have two vectors:
$X = \{\tilde{x}_i | \tilde{a_{ij}} \succeq 0\}$, $Y = \{\tilde{x}_i | \tilde{a_{ij}} \prec 0\}$
With these definitions we rewrite (4.1) as:

$$\begin{cases} min\ \ \tilde{Z} = \sum_{i \in I^+} \tilde{c}_i \otimes \tilde{x}_i \oplus \sum_{i \in I^-} \tilde{c}_i \otimes \tilde{x}_i \\ s.t. \\ \tilde{S} \otimes \tilde{X} \oplus \tilde{T} \otimes \tilde{Y} \succeq \tilde{b} \\ \\ \tilde{X}, \tilde{Y} \succeq 0 \end{cases} \qquad (4.7)$$

And

$$\begin{cases} min\ \ \tilde{Z} = \sum_{i \in I^+}(c_i, c'_i, c''_i) \otimes\ \ (x_i, x'_i, x''_i)\ \ \oplus \sum_{i \in I^-}(c_i, c'_i, c''_i)\ \ \otimes (x_i, x'_i, x''_i) \\ s.t. \\ (S, S', S'') \otimes (X, X', X'') \oplus\ \ (T, T', T'') \otimes (Y, Y', Y'') \succeq (b, b', b'') \\ \\ (X, X', X''), (Y, Y', Y'') \succeq 0 \end{cases}$$
$$(4.8)$$

Now with theorem (2.2) we have:

$$
\begin{cases}
min \quad \tilde{Z} = \sum_{i \in I^+} (c_i x_i, c_i x_i' + c_i' x_i, c_i x_i'' + c_i'' x_i) \oplus \sum_{i \in I^-} (c_i x_i, c_i' x_i - c_i x_i', c_i'' x_i - c_i x_i'') \\
\quad s.t. \\
(SX, SX' + XS', SX'' + XS'') \oplus (TY, YT' - TY'', YT'' - TY') \; \succeq \; (b, b', b'') \\
\\
(X, X', X''), (Y, Y', Y'') \succeq 0
\end{cases}
$$

$$(4.9)$$

Definition 4.1. *Any which satisfies the set of constraints of FFLP is called a feasible solution. Let S be the set of all feasible solutions of FFLP. We say that x^* is optimal feasible solution for FFLP if for all $x \in S$:*
$$\tilde{c} \otimes \tilde{x}^* \succeq \tilde{c} \otimes \tilde{x}$$

Definition 4.2. *In the equation (3-1) if $\tilde{A}, \tilde{b}, \tilde{c} \in FT(\Re)$ then FFLP is transformed into the triangular fully fuzzy linear programming (TFFLP).*

Lemma 4.1. *If $\tilde{x} \in FT(\Re)$ was positive then $D(\tilde{x}) \geq 0$.*
Proof: It is obvious from the equation (3.5).

Definition 4.3. *we define the LP As follow:*

$$
\begin{cases}
min \; z = C_1 X_1 + C_2 X_2 \\
\quad s.t. \\
AX \geq b
\end{cases}
$$

$$(4.10)$$

$$\text{Where: } A = \begin{bmatrix} S_1 & T_1 \\ e & 0 \\ 0 & e \end{bmatrix}, \; \text{Where: } X = \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix}, \; b = \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} S + 1/4S'' - 1/4S', & -1/4S, & 1/4S \end{bmatrix},$$
$$T_1 = \begin{bmatrix} T + 1/4T'' - 1/4T', & -1/4T, & 1/4T \end{bmatrix},$$
$$e = \begin{bmatrix} 1, & -1/4, & 1/4 \end{bmatrix}, \; b_1 = b + 1/4(b'' - b')$$

$$C_1 = [C - 1/4C' + 1/4C'', -1/4C, 1/4C] \; if \; \tilde{C} \succeq 0 \; in \; the \; equation$$
$$C_2 = [C - 1/4C' + 1/4C'', 1/4C, -1/4C] \; if \; \tilde{C} \prec 0 \; in \; the \; equation$$

Theorem 4.1. *TFFLP and definition (4.5) are equivalent.*
Let S_1 and S_2 be the set of all feasible solutions of TFFLP and definition(4.5) respectively. Then $\tilde{X} \in S_1$ iff :

$$(SX, SX' + XS', SX'' + XS'') \oplus (TY, YT' - TY'', YT'' - TY') \succeq \tilde{b}$$

$(X, X', X''), (Y, Y', Y'') \succeq 0$
Where $a_{\widetilde{i,j}}$, $\tilde{b}_i$, $\tilde{c}_i \in FT(\Re)$ *and iff :*

$$(SX + TY, S'X + SX' + YT' - TY'', SX'' + XS'' + YT'' - TY') \succeq \tilde{b}$$

$(X, X', X''), (Y, Y', Y'') \succeq 0$
Iff:

$$SX + TY + 1/4(SX'' + XS'' + YT'' - TY' - S'X - SX' - YT' + TY'') \geq$$
$b + 1/4(b'' - b')$

$X + 1/4(X'' - X') \geq 0$
$Y + 1/4(Y'' - Y') \geq 0$

Iff: $X, X', X'', Y, Y', Y'' \in S_2$
Hence $S_1 \subset S_2$ *and* $S_2 \subset S_1$.
Now we prove that any optimal solution in LP definition(4.5) will make an optimal solution in TFFLP. If X^* is optimal solution then for all $X \in S_2$ we have:

$$CX^* \leq CX$$

But if any $X = (x, x', x'')$ it is assumed as fuzzy number, according to the definition of C_1, C_2 we have:

$$D(\tilde{C} \otimes \tilde{X}^*) \leq D(\tilde{C} \otimes \tilde{X})$$

Then with (3.4):

$$\tilde{C} \otimes \tilde{X}^* \preceq \tilde{C} \otimes \tilde{X}$$

And this means that $\tilde{X}^* = (x^*, x'^*, x''^*)$ is an optimal solution for S_1.

Remark If z^* be optimal solution of equation(4.10) then we have $z^* = D(\tilde{z}^*)$ that $\tilde{z}^* = \tilde{C} \otimes \tilde{X}^*$.

5 Examples

Example 5.1.

$$\begin{cases} min \ (1,1,1) \otimes \tilde{x}_1 \oplus (2,1,2) \otimes \tilde{x}_2 \\ \quad s.t. \\ (4,1,0) \otimes \tilde{x}_1 \oplus (-3,2,1) \otimes \tilde{x}_2 \succeq (2,1,2) \\ (-3,1,2) \otimes \tilde{x}_1 \oplus (2,1,1) \otimes \tilde{x}_2 \succeq (1,0,1) \\ \tilde{x}_1 \succeq, \tilde{x}_2 \succeq 0 \end{cases} \qquad (5.11)$$

This LP transform to:

$$\begin{cases} min \ x_1 - 0.25x_1' + 0.25x_1'' + 2.25x_2 - 0.5x_2' + 0.5x_2'' \\ \quad s.t. \\ 3.75x_1 - 0.25x_1' + 0.25x_1'' - 3.25x_2 + 0.75x_2' - 0.75x_2'' \geq 2.25 \\ -2.75x_1 + 0.75x_1' - 0.75x_1'' + 2x_2 - 0.5x_2' + 0.5x_2'' \geq 1.25 \\ x_1 - 0.25x_1' + 0.25x_1'' \geq 0 \\ x_2 - 0.25x_2' + 0.25x_2'' \geq 0 \\ x_1 - x_1' \qquad\qquad\quad \geq 0 \\ x_2 - x_2' \qquad\qquad\quad \geq 0 \end{cases} \qquad (5.12)$$

The solation of this problem are:
$$\tilde{X}_1 = (8.25, 8.25, 0) \quad , \quad \tilde{X}_2 = (0, 0, 35.5)$$

Example 5.2.

$$max \quad \tilde{C}\tilde{X}$$
$$s.t. \quad \tilde{A}\tilde{X} \preceq \tilde{b}$$
$$\tilde{0} \leq \tilde{X}$$

Which $\tilde{C} = \begin{bmatrix} (-1,1,0) \\ (2,1,1) \\ (1,0,1) \end{bmatrix}, \tilde{A} = \begin{bmatrix} (12,1,5) & (7,5,2) & (2,1,1) \\ (-5,2,1) & (4,2,2) & (10,1,5) \end{bmatrix}, \tilde{b} = \begin{bmatrix} (5,3,5) \\ (7,2,5) \end{bmatrix},$

Also the problem could be written as the following form:

$$\begin{cases} max \ (-1,1,0) \otimes \tilde{x}_1 \oplus (2,1,1) \otimes \tilde{x}_2 \oplus (1,0,1) \otimes \tilde{x}_3 \\ \quad s.t. \\ (12,1,5) \otimes \tilde{x}_1 \oplus (7,5,2) \otimes \tilde{x}_2 \oplus (2,1,1) \otimes \tilde{x}_3 \preceq (5,3,5) \\ (-5,2,1) \otimes \tilde{x}_1 \oplus (4,2,2) \oplus \tilde{x}_2 (10,1,5) \otimes \tilde{x}_3 \preceq (7,2,5) \\ \tilde{x}_1 \succeq 0, \tilde{x}_2 \succeq 0, \tilde{x}_3 \succeq 0, \end{cases} \qquad (5.13)$$

This LP transform to:

$$
\begin{cases}
max & -1.25x_1 + 0.25x_1' - 0.25x_1'' + 2x_2 - 0.5x_2' + 0.5x_2'' + 1.25x_3 - 0.25x_3' + 0.25x_3'' \\
& s.t. \\
& 13x_1 - 3x_1' + 3x_1'' + 6.25x_2 - 1.75x_2' + 1.75x_2'' + 2x_2 - 0.5x_2' + 0.5x_2'' \leq 5.5 \\
& -5.25x_1 + 1.25x_1' - 1.25x_1'' + 4x_2 - x_2' + x_2'' + 11x_3 - 2.5x_3' + 2.5x_3'' \leq 7.75 \\
& x_1 - 0.25x_1' + 0.25x_1'' \geq 0 \\
& x_2 - 0.25x_2' + 0.25x_2'' \geq 0 \\
& x_3 - 0.25x_3' + 0.25x_3'' \geq 0 \\
& x_1 - x_1' \qquad\qquad \geq 0 \\
& x_2 - x_2' \qquad\qquad \geq 0 \\
& x_3 - x_3' \qquad\qquad \geq 0
\end{cases}
$$

The solation of this problem are:

$$\tilde{X}_1 = (0, 0, 3.9474) \quad , \quad \tilde{X}_2 = (4.2281, 4.2281, 0) \quad , \quad \tilde{X}_3 = (0, 0, 0)$$

6 Conclusion

In this work we concentrate on a full fuzzy linear programming problem, We solve this problem by using a kind of defuzzification method. In our approach the Core of the nearest symmetric triangular fuzzy number is applied for an approximation of fuzzy numbers in objective function and coefficient matrix in the constraints.

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