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Hybrid Stochastic Petri Nets*

LU Guang-song[1], GE Yun-jian[2]

(1. *School of Management, Hefei University of Technology, Hefei* 230009, *China*;
2. *Hefei Institute of Intelligent Machines, Chinese Academy of Sciences, Hefei* 230031, *China*)

Abstract: Second order Fluid Stochastic Petri nets are a modeling formalism used for performance and dependability evaluation of computer and communication systems. Hybrid Stochastic Petri nets are an extension of second order Fluid Stochastic Petri nets, in which the fluid jump arcs as a modeling primitive are assigned the function that instantaneously empties the fluid place connected to it. The dynamic equations of the stochastic marking process are given, and in the derivation of the equations the discrete state transitions concurrent with fluid jumps are taken into account for the first time. Finally, the boundary conditions for the case in which fluid flow rates depend on fluid levels are presented, upon which the solution of the dynamic equations can be obtained directly by numerical methods.

Key words: Petri nets; Hybrid Stochastic Petri nets; Fluid Stochastic Petri nets; stochastic models

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0 Introduction

Stochastic Petri nets[1] are well suited for specifying and solving performance and dependability models of complex computer and communication systems. Fluid Stochastic Petri nets (FSPNs) are a generalization of General Stochastic Petri nets by introducing stochastic fluid flow models. FSPNs are a high level modeling formalism that enables a simple description of a complex hybrid system with both continuous and discrete components[2]. FSPNs can also be thought of as a graphical language to represent (non-Markovian) stochastic processes with rewards[3].

The FSPN formalism was inspired by the work of Mitra etc. on queuing systems with a continuous flow of customers[4], which was first proposed in [5], then extended in [6]. A noticeable extension called the flush-out arcs was introduced in [3]. Second order fluid

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Biography: LU Guang-song, male, born in 1972, Ph. D candidate. E-mail: lgs@ustc. edu

diffusion approximation was first used in FSPNs in [7] in order to allow non-deterministic modeling of continuous quantities, and in [8] the jump transitions were added.

It has been shown that the flush-out arcs considerably increase the modeling power of the FSPN formalism[2, 3]. The modeling primitive of the flush-out arcs hasn't been included in the second order FSPN formalism up to now. In [8], the function of the flush-out arcs that instantaneously empties a continuous place was realized by choosing a large deterministic jump height and the force-jump strategy for the fluid jump arcs. But in some cases, the fluid upper bounds on some continuous places may be infinite; this would bring about much inconvenience in analyzing the dynamic equations of the marking process. The modeling means also impairs the close-to-intuition graphical representation effect of Stochastic Petri nets. In [8～10] the discrete state transitions concurrent with fluid jumps were ignored in the derivation of the dynamic equations, so the dynamic equations failed to describe the marking process correctly. Neither has probability mass component been taken into account in the literature. However, probability mass accumulated will result in discontinuities in the cumulative distribution functions, and then direct numerical solution can not be obtained due to the Dirac delta functions in the probability density functions.

In the present paper the name of Hybrid Stochastic Petri net (HSPN) is used[11]. We let the fluid jump arcs used as a modeling primitive have the function of the flush-out arcs directly, to increase the representative power of FSPNs as a mathematical and graphical tool. Then the dynamic equations of the stochastic marking process are given, in the derivation of the equations the discrete state transitions concurrent with fluid jumps are taken into account for the first time. The third contribution of the paper is the complete boundary conditions for the case in which fluid flow rates depend on fluid levels are presented, and the problem from probability mass accumulation is solved. Examples of the formalism will not be given for the space limitation.

1 Definition and Notations

The definition of the HSPNs is derived from [2, 8] with common notations inherited from [1]. With N, R, and R^+ we denote natural, real, and non-negative real numbers respectively.

An HSPN is an 11-tuple HSPN$=(P,T,A,B,\lambda,g,H,\gamma,\vartheta,w,M_0)$, where

(1) $P = P_d \cup P_c$ is the set of places consists of a set of discrete places and a set of continuous (fluid) places. The complete marking is defined as $M = (m, \boldsymbol{x})$, with $\boldsymbol{x} = (x_k, c_k \in P_c)$ giving the content of the continuous places and $m = (\# p_i, p_i \in P_d)$ for the discrete places. We use S to denote the state space and S_d the discrete component of the state space.

(2) $T = T_g \cup T_e \cup T_i$ is the set of transitions. T_g is a set of transitions with arbitrary firing time distributions. T_e is a set of exponentially distributed transitions and T_i is a set

of immediate ones with a constant zero firing time.

(3) $A = A_h \cup A_d \cup A_j \cup A_c$ is the set of arcs. The subset A_h contains inhibitor arcs, which can be seen as a function $A_h : (P_d \times T) \to N$. A_d is the set of discrete arcs, and they can be seen as a function $A_d : (P_d \times (T_e \cup T_i) \cup (T_e \cup T_i) \times P_d) \times S_d \to N$. A_j is the set of jump arcs, which is a subset of $(P_c \times (T_e \cup T_i)) \cup ((T_e \cup T_i) \times P_c)$. A jump arc can connect a fluid place and an immediate transition under certain restrictions[9]. A_c is a subset of $(P_c \times (T_e \cup T_g)) \cup ((T_e \cup T_g) \times P_c)$, the elements of which are called continuous arcs.

(4) $B : P_c \to R^+ \cup \{\infty\}$ defines the fluid upper bound for each continuous place. Each continuous place has an implicit lower bound at level 0.

(5) $\lambda : T \times S \to R^+ \cup \{\infty\}$ is the firing rate function for each transition.

(6) $w : T_i \times S_d \to R^+$ is the weight function of immediate transitions.

(7) $g : T \times S_d \to \{\text{True}, \text{False}\}$ is the guards.

(8) $H : A_j \times S_d \to \{h(\cdot)\} \cup \{*\}$ is the jump height function for each jump arc, in which $h(\cdot)$ is the jump height probability density function. If an input jump arc of a transition is labeled with the symbol $*$, it plays the role of a flush-out arc in [3], and it will empty in zero time the existing fluid from a continuous place when the corresponding transition fires. The output jump arcs of transitions cannot be labeled with the symbol $*$.

(9) $\gamma : A_c \times S \to (R^+)^2$ is the flow rate function of fluid arcs that assigns a cardinality in the form of a normal distribution to each fluid arc, and the distribution is specified by its expectation and variance. To preserve the flow rates' independence they can only depend on the fluid level of the fluid place the arc is connected to.

(10) $\vartheta : T \to N$ defines the priority for each transition.

(11) $M_0 = (m_0, \boldsymbol{x}_0)$ denotes the initial state of the HSPNs.

2 Model Parameters

In this section we demonstrate the computation of all the parameters that are needed to define the dynamics of the discrete as well as the continuous part of HSPNs.

Since the enabling and firing rules of transitions don't depend on the continuous part of HSPNs, the vanishing markings can be removed by using any analysis technique for General Stochastic Petri nets. From now on, we will only consider tangible states and S_d will be used to denote the set of all the discrete tangible states.

The fluid parameters have to be specified for each discrete state and for every fluid place. We first regard the continuous flow. The continuous dynamics are defined by the fluid change rate $\mathrm{d}\boldsymbol{x}/\mathrm{d}t$ that describes the change in the fluid level over time. The change in the fluid level is determined by the difference between the inflow and the outflow, each of which is normally distributed. The parameters of the normal distribution are either specified by the user or calculated as diffusion approximation. The details have been given in [9], and will not be repeated here. The expectations and variances for each fluid place are collected into diagonal matrices (if there is none in a discrete state, the corresponding entry is equal to zero):

$$\boldsymbol{M}_k(x_k) = \mathrm{diag}(\mu_{k,i}(x_k), i \in S_d),\ \sum\nolimits_k^2(x_k) = \mathrm{diag}(\sigma_{k,i}^2(x_k), i \in S_d),$$

Where $\mu_{k,i}(x_k)$ and $\sigma_{k,i}^2(x_k)$ are the expectation and variance of the flow rate of the fluid place $c_k \in P_c$ in the discrete state $i \in S_d$.

We use $\hat{l} = 1, \cdots, \hat{L}$ to enumerate all the output fluid jump arcs (the elements of $(T_e \cup T_i) \times P_c)$). All the input fluid jump arcs (the elements of $(P_c \times (T_e \cup T_i))$ labeled with a probability density function are enumerated by $\check{l} = 1, \cdots, \check{L}$, and all the flush-out arcs by $\bar{l} = 1, \cdots, \bar{L}$.

The infinitesimal generator matrix $\boldsymbol{Q}(\boldsymbol{x})$ accounts for the transition rates among discrete tangible states, and can be obtained by using standard analysis methods. If we don't consider the effect of immediate transitions, the entry $q_{i,j}(\boldsymbol{x})$ $(i, j \in S_d)$ can be defined as

$$q_{i,j}(\boldsymbol{x}) = \sum_{t \in E(i) \,\big|\, i \xrightarrow{t} j} \lambda(t, (i, \boldsymbol{x})) \quad i \neq j, \qquad q_{i,i}(\boldsymbol{x}) = -\sum_{t \in E(i)} \lambda(t, (i, \boldsymbol{x})).$$

Here, $E(i)$ denotes the set of enabled transitions in the discrete marking i.

In this paper we assume that any discrete tangible state transition concurs with at most one fluid jump, and if a tangible state transition may concur with a fluid jump, it must always occur with the fluid jump simultaneously. For a fluid jump, the jump rate and the jump height have to be specified. The height is drawn from a probability density function, and the density function can be arbitrary, but the jump height must depend on the bounds of the fluid place. To make sure that the fluid jumps will not go across the boundaries the jump height probability distributions must be transformed. Two kinds of transformation have been considered in [9].

For notational convenience all the parameters are collected in matrices $\boldsymbol{H}_l(\cdot)$ for the jump heights (There aren't corresponding matrices for flush-out arcs.), and $\Lambda_l(\boldsymbol{x})$ for the jump rates. $\boldsymbol{H}_l(\cdot)$ is defined as $\boldsymbol{H}_l(\cdot) = \mathrm{diag}(H(l,i)(\cdot), i \in S_d)$. $\Lambda_l(\boldsymbol{x})$ is defined as

$$(\Lambda_l(\boldsymbol{x}))_{i,j} = \lambda_{li,j}(\boldsymbol{x}) = \begin{cases} q_{i,j}(\boldsymbol{x}) \text{ if } i \to j \text{ and } l \text{ jump concur} \\ 0 \text{ otherwise} \end{cases} \quad i \neq j.$$

Namely, $\lambda_{li,j}(\boldsymbol{x})$ is the transition rate from the discrete state i to the state j, if the transition concurs with the fluid jump indicated by the arc l (l jump for short). A fluid jump can also occur without a discrete state transition. $\lambda_{li,i}(\boldsymbol{x})$ is used to show this case, $\lambda_{li,i}(\boldsymbol{x}) = (\Lambda_l(\boldsymbol{x}))_{i,i} = \lambda(t, (i, \boldsymbol{x}))$, in which t is the transition that l is connected with, and $\lambda(t, (i, \boldsymbol{x}))$ is the firing rate of the transition t in the marking $M = (i, \boldsymbol{x})$ (The transition t must be an exponential transition[9]).

We use $\xi_l^k (c_k \in P_c)$ to indicate if the arc l is connected with the continuous place c_k. ξ_l^k is defined as $\xi_l^k = \begin{cases} 1 \text{ if } l \text{ is connected with } c_k \\ 0 \text{ otherwise} \end{cases}$.

The matrix $Q'(\boldsymbol{x})$ is used to show the rates of those discrete state transitions without a fluid jump concurrently and its entry $q'_{i,j}(\boldsymbol{x})$ is defined as

$$q'_{i,j}(\boldsymbol{x}) = \begin{cases} q_{i,j}(\boldsymbol{x}) \text{ if } i \rightarrow j \text{ occurs without a fluid jump} \\ 0 \text{ otherwise} \end{cases} \quad i \neq j,$$

$$q'_{i,j}(\boldsymbol{x}) = q_{i,i}(\boldsymbol{x}) - \sum_{l=1}^{L} \lambda_{li,i}(\boldsymbol{x}).$$

In order to write dynamic equations in a more compact form, we define a projection operator $\theta(\boldsymbol{x},k)$ as $\theta(\boldsymbol{x},k) = (x_1, x_2, \cdots, x_{k-1}, y, x_{k+1}, \cdots, x_{|P_c|})$.

3 Dynamic equations

The stochastic marking process under consideration is a Markov process in continuous time with mixed discrete and continuous state space, $M(\tau) = \{(m(\tau), \boldsymbol{x}(\tau)), \tau \geqslant 0\}$, where $m(\tau)$ is the discrete tangible marking at time τ, and $\boldsymbol{x}(\tau)$ is a random variable vector, representing the fluid levels in the fluid places at time τ. In this section, we give out the equations for the stochastic marking process describing the dynamic behaviour of the HSPN model as a function of the time.

Define the cumulative distribution function (CDF) of the process $M(\tau) = \{m(\tau), \boldsymbol{x}(\tau), \tau \geqslant 0\}$ as $P(\tau, \boldsymbol{x}, i) = \Pr\{m(\tau) = i, \boldsymbol{x}(\tau) \leqslant \boldsymbol{x}\}$, and let $\boldsymbol{P}(\tau, \boldsymbol{x}) = [P(\tau, \boldsymbol{x}, i), i \in S_d]$ be a row vector of the CDFs of all the discrete states. Similarly, at the points where the CDF $P(\tau, \boldsymbol{x}, i)$ is differentiable, the probability density function (pdf) is defined as $g(\tau, \boldsymbol{x}, i) = \partial P(\tau, \boldsymbol{x}, i)/\partial \boldsymbol{x}$, and $\boldsymbol{g}(\tau, \boldsymbol{x}) = [g(\tau, \boldsymbol{x}, i), i \in S_d]$ is a row vector of the pdfs. Probability mass may be accumulated at certain points, and that will result in discontinuities in the CDFs. At these points, the derivative is generalized to include the Dirac delta function.

Theorem 1 For each discrete state $j \in S_d$, the function $g(\tau, \boldsymbol{x}, j)$ is governed by

$$\begin{aligned} \frac{\partial g(\tau, \boldsymbol{x}, j)}{\partial \tau} = & -\sum_{k=1}^{|P_c|} \frac{\partial [g(\tau, \boldsymbol{x}, j) \mu_{k,j}(x_k)]}{\partial x_k} + \sum_{k=1}^{|P_c|} \frac{\partial^2 [g(\tau, \boldsymbol{x}, j) \sigma_{k,j}^2(x_k)]}{\partial x_k^2} + \\ & \sum_{i=1}^{|S_d|} g(\tau, \boldsymbol{x}, i) q'_{i,j}(\boldsymbol{x}) + \sum_{k=1}^{|P_c|} \sum_{\hat{l}=1}^{\hat{L}} \sum_{i=1}^{|S_d|} \int_0^{x_k} g(\tau, x_1, \cdots, x_{k-1}, y, x_{k+1}, \cdots, x_{|P_c|}, i) \cdot \xi_{\hat{l}}^k \cdot \\ & H(\hat{l}, i)(x_k - y) \cdot \lambda_{\hat{l}i,j}(x_1, \cdots, x_{k-1}, y, x_{k+1}, \cdots, x_{|P_c|}) \mathrm{d}y + \\ & \sum_{k=1}^{|P_c|} \sum_{\check{l}=1}^{|\check{L}|} \sum_{i=1}^{|S_d|} \int_{x_k}^{B(c_k)} g(\tau, x_1, \cdots, x_{k-1}, y, \cdots, x_{|P_c|}, i) \cdot \xi_{\check{l}}^k \cdot \\ & H(\check{l}, i)(y - x_k) \cdot \lambda_{\check{l}i,j}(x_1, \cdots, x_{k-1}, y, \cdots, x_{|P_c|}) \mathrm{d}y + \\ & \sum_{k=1}^{|P_c|} \sum_{\bar{l}=1}^{\bar{L}} \sum_{i=1}^{|S_d|} \delta(x_k) \int_0^{B(c_k)} g(\tau, x_1, \cdots, x_{k-1}, y, \cdots, x_{|P_c|}, i) \cdot \\ & \xi_{\bar{l}}^k \cdot \lambda_{\bar{l}i,j}(x_1, \cdots, x_{k-1}, y, \cdots, x_{|P_c|}) \mathrm{d}y \end{aligned} \tag{1}$$

In vector form, (1) can be written as

$$\frac{\partial}{\partial\tau}\boldsymbol{g}(\tau,\boldsymbol{x}) = -\sum_{k=1}^{|P_c|}\frac{\partial[\boldsymbol{g}(\tau,\boldsymbol{x})\boldsymbol{M}_k(x_k)]}{\partial x_k} + \sum_{k=1}^{|P_c|}\frac{\partial^2\left\lfloor \boldsymbol{g}(\tau,\boldsymbol{x})\sum_k^2(x_k)\right\rfloor}{\partial x_k^2} + \boldsymbol{g}(\tau,\boldsymbol{x})\boldsymbol{Q}'(\boldsymbol{x}) +$$
$$\sum_{k=1}^{|P_c|}\sum_{\hat{l}=1}^{\hat{L}}\int_0^{x_k}\boldsymbol{g}(\tau,\theta(\boldsymbol{x},k))\cdot\xi_{\hat{l}}^k\cdot\boldsymbol{H}_{\hat{l}}(x_k-y)\cdot\Lambda_{\hat{l}}(\theta(\boldsymbol{x},k))\mathrm{d}y +$$
$$\sum_{k=1}^{|P_c|}\sum_{\check{l}=1}^{|\check{L}|}\int_{x_k}^{B(c_k)}\boldsymbol{g}(\tau,\theta(\boldsymbol{x},k))\cdot\xi_{\check{l}}^k\cdot\boldsymbol{H}_{\check{l}}(y-x_k)\cdot\Lambda_{\check{l}}(\theta(\boldsymbol{x},k))\mathrm{d}y +$$
$$\sum_{k=1}^{|P_c|}\sum_{\bar{l}=1}^{\bar{L}}\delta(x_k)\int_0^{B(c_k)}\boldsymbol{g}(\tau,\theta(\boldsymbol{x},k))\cdot\xi_{\bar{l}}^k\cdot\Lambda_{\bar{l}}(\theta(\boldsymbol{x},k))\mathrm{d}y \quad (2)$$

Equations (1) and (2) are parabolic partial differential equations of convection-diffusion type. The boundary conditions for the equations will be addressed in later section.

Theorem 1 can be derived in a way proposed in [2, 9]. Although the governing equations are given by Theorem 1, they are not amenable to a direct numerical solution due to the delta functions in $\boldsymbol{g}(\tau,\boldsymbol{x})$. For this reason, the probability mass functions (pmfs) will be treated separately from the pdfs.

4 Initial and boundary conditions

4.1 Initial conditions

The initial conditions are defined by the models' configuration at time zero. Let $\boldsymbol{g}_0$ be the vector of initial discrete state probabilities and let $\boldsymbol{x}_0$ be the vector of initial fluid levels, then the initial conditions are

$$\boldsymbol{g}(0,\boldsymbol{x}) = \boldsymbol{g}_0\cdot\delta(\boldsymbol{x}-\boldsymbol{x}_0)$$

4.2 Boundary conditions

In this subsection, the problem of boundary conditions is discussed. The lower and upper boundaries are formed naturally because the fluid levels of continuous places cannot exceed their upper and lower bounds. The treatment at the boundaries for fluid jumps is included in the transformation of the jump height functions, so there aren't separate boundary conditions needed. For the fluid flow boundary conditions at the upper and lower bounds are required. In addition, if the fluid flow rates depend on fluid levels, intermediate boundaries may be formed where the flow rates change their values or directions.

We present the complete boundary conditions for the case in which HSPNs have only one fluid place and the case of two fluid places. Extension to more continuous places is straight forward and can be carried out along the same lines. The unifying form of the boundary conditions will not be given, because it involves more cumbersome notations, and moreover, the numerical methods developed for the solution of second order FSPNs are so far feasible only for the models with one or two fluid places[9, 12].

(1) The case of one fluid place

The set of boundary points, Z, is defined as

$$Z = \{b \mid \exists i \in S_d, \mu_{1,i}(b^+) \neq \mu_{1,i}(b^-), 0 < b < B(c_1); \text{or } b = 0, B(c_1)\},$$

where 0 and $B(c_1)$ are the lower and upper bounds of the single fluid place, respectively. This means that the boundaries are formed at those points where the fluid flow in any of the discrete states changes rates.

Let $g(\tau,x,i) = p(\tau,x,i) + \sum_{b\in Z}\delta(x-b)c(\tau,b,i)$, where $c(\tau,b,i)$ is the probability mass accumulated at point b under the discrete state i, $p(\tau,x,i)$ is the probability density function at point x where $P(\tau,x,i)$ is differentiable with respect to x.

The pmf $\boldsymbol{c}(\tau,b)$ is given by

$$\frac{\mathrm{d}}{\mathrm{d}\tau}c(\tau,b,j) + p(\tau,b^+,j)\mu_{1,j}(b^+) - p(\tau,b^-,j)\mu_{1,j}(b^-) = \frac{1}{2}\left[\frac{\partial}{\partial x}(p(\tau,x,j)\sigma_{1,j}^2(x))\right]_{x=b^+} - \frac{1}{2}\left[\frac{\partial}{\partial x}(p(\tau,x,j)\sigma_{1,j}^2(x))\right]_{x=b^-} + \sum_{i\in S_d}c(\tau,b,i)q'_{i,j}(b) \quad b \neq 0 \quad \forall j \in S_d \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}c(\tau,0,j) + p(\tau,0^+,j)\mu_{1,j}(0^+) = \frac{1}{2}\left[\frac{\partial}{\partial x}p(\tau,x,j)\sigma_{1,j}^2(x)\right]_{x=0^+} + \sum_{i\in S_d}c(\tau,b,i)q'_{i,j}(b) + \sum_{\bar{l}=1}^{\bar{L}}\sum_{i=1}^{|S_d|}\int_0^{B(c_1)}\rho(\tau,y,i)\cdot\xi_{\bar{l}}^1\cdot\lambda_{\bar{l}i,j}(y)\mathrm{d}y \quad \forall j \in S_d \tag{4}$$

Equations (3) and (4) can be derived in a way proposed in [11, 2].

(2) The case of two fluid places

By the same way, we define

$$Z_1 = \{b_1 \mid \exists i \in S_d, \mu_{1,i}(b_1^+) \neq \mu_{1,i}(b_1^-), 0 < b_1 < B(c_1); \text{or } b_1 = 0, B(c_1)\};$$

$$Z_2 = \{b_2 \mid \exists i \in S_d, \mu_{2,i}(b_2^+) \neq \mu_{2,i}(b_2^-), 0 < b_2 < B(c_2); \text{or } b_2 = 0, B(c_2)\}.$$

Let $g(\tau,\boldsymbol{x},i) = p(\tau,x_1,x_2,i) + \sum_{b_1\in Z_1}\delta(x_1-b_1)p_1(\tau,b_1,x_2,i) + \sum_{b_2\in Z_2}\delta(x_2-b_2)p_2(\tau,x_1,b_2,i) + \sum_{b_1\in Z_1}\sum_{b_2\in Z_2}\delta(x_1-b_1)\delta(x_2-b_2)c(\tau,b_1,b_2,i)$, where $p(\tau,x_1,x_2,i)$ is the joint probability density function at point (x_1,x_2), $c(\tau,b_1,b_2,i)$ is the probability mass accumulated at point (b_1,b_2) under the discrete state i, and $p_1(\tau,b_1,x_2,i)$ ($p_2(\tau,x_1,b_2,i)$) is the marginal probability density function of the CDF $P_1(\tau,x_1(\tau)=b_1,x_2(\tau)\leqslant x_2,i)$ ($P_2(\tau,x_1(\tau)\leqslant x_1,x_2(\tau)=b_2,i)$ at point $x_2(x_1)$ where the CDF is differentiable with respect to $x_2(x_1)$.

The function $\boldsymbol{p}_1(\tau,b_1,x_2)$ is governed by

$$\frac{\partial}{\partial\tau}p_1(\tau,b_1,x_2,j) + p(\tau,b_1^+,x_2,j)\mu_{1,j}(b_1^+) - p(\tau,b_1^-,x_2,j)\mu_{1,j}(b_1^-) = \frac{1}{2}\left[\frac{\partial}{\partial x_1}(p(\tau,x_1,x_2,j)\sigma_{1,j}^2(x_1))\right]_{x_1=b_1^+} - \frac{1}{2}\left[\frac{\partial}{\partial x_1}(p(\tau,x_1,x_2,j)\sigma_{1,j}^2(x_1))\right]_{x_1=b_1^-} +$$

$$\sum_{i\in S_d} p_1(\tau,b_1,x_2,i)q'_{i,j}(b_1,x_2) \quad b_1\neq 0 \quad \forall j\in S_d.$$

The pmf $\boldsymbol{c}(\tau,b_1,b_2)$ is governed by

$$\frac{\partial}{\partial\tau}c(\tau,b_1,b_2,j)+p_1(\tau,b_1,b_2^+,j)\mu_{2,j}(b_2^+)-p_1(\tau,b_1,b_2^-,j)\mu_{2,j}(b_2^-)+$$
$$p_2(\tau,b_1^+,b_2,j)\mu_{1,j}(b_1^+)-p_2(\tau,b_1^-,b_2,j)\mu_{1,j}(b_1^-)=$$
$$\frac{1}{2}\left[\frac{\partial}{\partial x_2}(p_1(\tau,b_1,x_2,j)\sigma_{2,j}^2(x_2))\right]_{x_2=b_2^+}-\frac{1}{2}\left[\frac{\partial}{\partial x_2}(p_1(\tau,b_1,x_2,j)\sigma_{2,j}^2(x_2))\right]_{x_2=b_2^-}+$$
$$\frac{1}{2}\left[\frac{\partial}{\partial x_1}(p_2(\tau,x_1,b_2,j)\sigma_{1,j}^2(x_1))\right]_{x_1=b_1^+}-\frac{1}{2}\left[\frac{\partial}{\partial x_1}(p_2(\tau,x_1,b_2,j)\sigma_{1,j}^2(x_1))\right]_{x_1=b_1^-}+$$
$$\sum_{i\in S_d}c(\tau,b_1,b_2,i)q'_{i,j}(b_1,b_2) \quad b_1\neq 0 \quad b_2\neq 0 \quad \forall j\in S_d.$$

If $b_1=0$ or $b_2=0$, the flush-out fluid jumps must be considered.

For example, the $\boldsymbol{p}_1(\tau,0,x_2)$ is given by

$$\frac{\partial}{\partial\tau}p_1(\tau,0,x_2,j)+p(\tau,0^+,x_2,j)\mu_{1,j}(0^+)=\frac{1}{2}\left[\frac{\partial}{\partial x_1}(p(\tau,x_1,x_2,j)\sigma_{1,j}^2(x_1))\right]_{x_1=0^+}+$$
$$\sum_{i\in S_d}p_1(\tau,0,x_2,i)q'_{i,j}(0,x_2)+\sum_{\bar{l}=1}^{\bar{L}}\sum_{i=1}^{|S_d|}\int_0^{B(c_1)}p(\tau,y,x_2,i)\cdot\xi_{\bar{l}}^1\cdot\lambda_{\bar{l}i,j}(y,x_2)\mathrm{d}y \quad \forall j\in S_d.$$

The functions $\boldsymbol{p}_2(\tau,x_1,b_2)$, $\boldsymbol{p}_2(\tau,x_1,0)$ and $\boldsymbol{c}(\tau,0,0)$ can be analyzed analogously.

Note that the fluid behavior at upper and lower boundaries is different from that at intermediate boundaries. These boundaries may exhibit very different properties under different discrete states, according to the fluid flow rates associated with them[12]. For this reason, the discrete states should be classified according to their mean flow rates around the boundaries. For the continuous place c_k in the discrete state i whose mean flow rate $\mu_{k,i}$ does not change direction at the boundaries, there is no probability mass accumulated, and the probability mass transferred from other states are converted to changes in probability densities, otherwise, probability mass is accumulated. The details will be discussed in a separated paper.

5 Conclusions

In the present paper, Hybrid Stochastic Petri nets are defined as an extension of second order FSPNs. The fluid jump arcs in the formalism as a modeling primitive have the function that empties in zero time the existing fluid from a continuous place when the corresponding transition fires. The underlying stochastic process is described by parabolic partial differential equations of convection-diffusion type, in the derivation of the equations the discrete state transitions concurrently with fluid jumps are taken into account for the first time. And the complete boundary conditions for the case in which fluid flow rates depend on fluid levels are presented, so the direct numerical solution of the dynamic equations is possible. Efficient solution techniques for the dynamic equations of HSPNs with more than two fluid places are an open research area.

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混合随机 Petri 网

卢光松[1],葛运建[2]

(1. 合肥工业大学管理学院,安徽合肥 230009; 2. 中国科学院合肥智能机械研究所,安徽合肥 230031)

摘要:二阶流体随机 Petri 网是一种用于计算机和通信系统性能与可靠性评价的建模机制.混合随机 Petri 网是对二阶流体随机 Petri 网的进一步拓展,其中,流体跳跃弧作为建模原语被赋予瞬时清空与之相联接的连续库的功能.给出了混合随机 Petri 网随机标识过程的动态方程,在该方程的推导中,首次将同时伴有流体跳跃发生的离散状态转移考虑在内.最后对流体流动速度随连续标识变化的情况下的边界条件进行了分析,使得可直接用数值方法对动态方程进行求解.

关键词: Petri 网;混合随机 Petri 网;流体随机 Petri 网;随机模型