Applied Mathematical Sciences, Vol. 3, 2009, no. 15, 701 - 713

Some Oscillation Theorems for Systems of Partial Differential Equations with Deviating Arguments and a Note on Impulsive Hyperbolic Equation

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Abstract

In this paper, we give some results of the oscillations criteria of the solution for some higher - order equations with deviating arguments , and note of the impulsive hyperbolic equations. We get some new conclusions, which generalize the results in [4] and [5].

Mathematics Subject classification: 35R10, 35B05

Keywords : Oscillation ;Delay Hyperbolic equation ; Impulses

1 Introduction and Lemma

Recently, the oscillation of solutions for higher-order partial differential equations with deviating arguments is widely usually discussed (see [2]-[4]etc), Aside from their intrinsic interest, oscillation of solutions is very important in the domain of physics(this things are interesting with some example). In this paper, we consider a more generalized higher –order equation

Now in the direction of [2], our conclusions extend and complete the previous results in [1]-[5] .

Let Ω be a bounded domain of R^N having sufficiently smooth boundary $\partial\Omega$,and n be an even positive integer number , $(x,t) \in \Omega\times[0,\infty) \triangleq$ G, Let

$P_3(x,t) ==\ a_i(t)\Delta u_i(x,t)+b_i(t)\Delta^2 u_i(x,t)+c_i(t)\Delta^3 u_i(x,t)$.We consider the oscillation of solutions of systems:

$$\frac{\partial^n u_i(x,t)}{\partial t^n}+\frac{\partial^{n-1}u_i(x,t)}{\partial t^{n-1}}=P_3(x,t)+P_3(x,t-\rho_k(t))$$

$$-\sum_{j=1}^{m} p_{ij}(x,t)u_j(x,t-\sigma(t)), i=1,2,\cdots,m \qquad (*)$$

(where $P_3(x,\ t-\ \rho_k(t)\ )=\sum_{k=1}^{s}[a_{ik}(t)\Delta+b_{ik}(t)\Delta^2+c_{ik}(t)\Delta^3\ ]\ u_i(x\ ,\ t-\ \rho_k(t))$, γ denotes the derivative in outward normal direction on $\partial\Omega$, and. $u_i(x,t)$ is defined to be real function , and $\Delta=\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^{\ 2}},\ \Delta^2=\Delta(\Delta),\Delta^3=\Delta(\Delta^2),\cdots$ .Let $g_i(x,t)$ be a non-negative continuous function in $\partial\Omega\times[0,\infty)$, and satisfying two conditions:

$$\frac{\partial u_i(x,t)}{\partial\gamma}+g_i(x,t)u_i(x,t)=0,\frac{\partial\Delta u_i(x.t)}{\partial\gamma}+g_i(x,t)u_i(x,t)=0,$$

$$\frac{\partial\Delta^2 u_i(x,t)}{\partial\gamma}+g_i(x,t)u_i(x,t)=0,\quad (x,t)\in\partial\Omega\times[0,\infty),\ (i=1,2,\cdots,m)\ (Q_1)$$

and

$$u_i(x,t)=0,\ \Delta u_i(x,t)=0,\Delta^2 u_i(x,t)=0,\ (x,t)\ \in\partial\Omega\times[0,\infty)\ (i=1,2,\cdots,m).\ (Q_2)$$

By the definition and some prescribe in [2] , we assume that it satisfies (H):

(G_1) $\sigma,\rho_k\in C([0,\infty),[0,\infty))$,and $\lim_{t\to\infty}(t-\sigma(t))=\infty,$

$\lim_{t\to\infty}(t-\rho_k(t))=\infty, k=1,2,\cdots,s$.

(G_2) $p_{ij}(x,t)\in C(\overline{G},R)$, $p_{ii}(x,t)>0$,$p_{ii}(t)=\min_{x\in\bar G} p_{ii}(x,t)$,$\bar p_{ij}(t)=\sup_{x\in\bar G}\left|p_{ij}(x,t)\right|$,

$$Q(t)=\min_{1\le i\le m}\left\{p_{ii}(t)-\sum_{j=1,j\ne i}^{m}\bar p_{ij}(t)\right\}\ge 0,\quad i=1,2,\cdots,m;\ j=1,2,\cdots,m .$$

(G_3) $a_i, a_{ik} \in C([0,\infty),[0,\infty)), k = 1,2,\cdots,s.$

We will give two theorems to extend some results which are similar to in [2].

Remark If we take $b_i(t), c_i(t) \equiv 0$ in (*),then we get theorem 1 in [2],and

From the last part of proof of theorem 1 in [2],we have the following two lemmas (lemma 1 and lemma 2).

Now we list following lemma (by $V_i(t) = \int_\Omega Z_i(x,t)dx$, and $V(t) = \sum_{i=1}^{m} V_i(t)$)

Lemma 1 If

$$\frac{d^n}{dt^n}\left(\int_\Omega Z_i(x,t)dx\right) + \frac{d^{n-1}}{dt^{n-1}}\left(\int_\Omega Z_i(x,t)dx\right) \le -p_{ii}(t)\int_\Omega Z(x,t-\sigma(t))dx +$$

$$\sum_{j=1, j\ne i}^{m} \overline{P_{ij}}(t)\int_\Omega Z_j(x,t-\sigma(t))dx, t \ge t_1, i = 1,2,\cdots,m. \qquad (1)$$

then

$$V^{(n)}(t) + V^{(n-1)}(t) + Q(t)V(t-\sigma(t)) \le 0, \quad t \ge t_1 .$$

Proof From $V_i(t) = \int_\Omega Z_i(x,t)dx$, and $V(t) = \sum_{i=1}^{m} V_i(t)$,and the last part of proof of theorem 1 in [2] ,it is easy to get it, So the proof of the lemma is omitted.

Lemma 2 If

$$V^{(n)}(t) + V^{(n-1)}(t) + Q(t)V(t-\sigma(t)) \le 0, \quad t \ge t_1 \qquad (2)$$

then $\int_{t_1}^{\infty} Q(t)dt < \infty$

Proof It is as same as the last part of proof of theorem 2 in [2]

In the following part, we will give out oscillation criteria of theorems for system (*)-(Q_1).

2 Several theorems

Theorem 1 If $\int_{t_0}^{\infty} Q(t)dt = \infty$, $t_0 > 0$,then all solutions of the system (*)-(Q_1) are oscillation in G .

Proof We suppose to the contrary there exists a non-oscillation solution

$u(x,t)$ = $\left(u_1(x,t), u_2(x,t), ..., u_m(x,t)\ \right)$ of the system (*)-(Q_1) for some $0 \le t_0 \le t$, $|u_i(x,t)| > 0$.Let $\delta_i = sign u_i(x,t), i = 1,2,\cdots,m$ and Z_i (x, t)= $\delta_i\ u_i(x,t)$.

Then we have Z_i (x, t)>0, where (x ,t) $\in \Omega\times[t_0,\infty)$.

From condition (G_1), we easily know that there exists $t_1 \ge t_0$ such that when $t \ge t_1$, we have $Z_i(x,t) > 0$, Z_i (x, t- $\rho_k(t)$) >0 , Z_i (x, t- $\sigma(t)$) >0. where (x, t) $\in \Omega\times[t_1,\infty)$, $i = 1,2,\cdots,m$; $k = 1,2,\cdots,s$.

Integrating both side of (*) for x over Ω, we have that

$$\frac{d^n}{dt^n}\left(\int_\Omega u(x,t)dx\right) + \frac{d^{n-1}}{dt^{n-1}}\left(\int_\Omega u(x,t)dx\right) = \int_\Omega P_3(x,t)dx + \int_\Omega P(x,t-\rho_k(t))dx$$

$$-\sum_{j=1}^{m} P_{ij}(x,t)\int_\Omega u_j(x,t-\sigma(t))dx \quad, t \ge t_1, \quad i = 1,2,\cdots,m.$$ That is

$$\frac{d^n}{dt^n}\left(\int_\Omega Z_i(x,t)dx\right) + \frac{d^{n-1}}{dt^{n-1}}\left(\int_\Omega Z_i(x,t)\right) = \int_\Omega P_3(x,t)dx + \int_\Omega P(x,t-\rho_k(t))dx$$

$$-\sum_{j=1}^{m} P_{ij}(x,t)\int_\Omega u(x,t-\sigma(t))dx, \ t \ge t_1, \quad i = 1,2,\cdots,m. \tag{3}$$

Similar to the proof of theorem 1 , by Green identity and boundary value conditions (Q_1),we have that

$$\int_\Omega \Delta^3 Z_i(x,t)dx = \int_{\partial\Omega} \frac{\partial\Delta^2 Z_i(x,t)}{\partial\gamma} ds = -\int_{\partial\Omega} g_i(x,t)\ Z_i(x,t)ds \le 0,$$ and

$$\int_\Omega \Delta^3 Z_i(x,t-\rho_k(t))dx = \int_{\partial\Omega} \frac{\partial\Delta^2 Z_i(x,t-\rho_k(t))}{\partial n} ds$$

$$= -\int_{\partial\Omega} g_i(x,t-\rho_k(t)) Z_i(x,t-\rho_k(t))ds \le 0.$$

$$\int_\Omega \Delta^2 Z_i(x,t)dx = \int_{\partial\Omega} \frac{\partial\Delta Z_i(x,t)}{\partial n} = -\int_{\partial\Omega} g_i(x,t) Z_i(x,t)ds \le 0,$$ and also that

$$\int_{\Omega}\Delta^2 Z_i(x,t-\rho_k(t))dx=\int_{\partial\Omega}\frac{\partial\Delta Z_i(x,t-\rho_k(t))}{\partial n}ds$$

$$=-\int_{\partial\Omega} g_i(x,t-\rho_k(t))Z_i(x,t-\rho_k(t))ds \le 0.$$

and $\int_{\Omega}\Delta Z_i(x,t)dx=\int_{\partial\Omega}\frac{\partial Z_i(x,t)}{\partial n}ds = -\int_{\partial\Omega} g_i\,(x,t)Z_i(x,t)ds \le 0$, $i=1,2,\cdots,m$.

$$\int_{\Omega}\Delta Z_i(x,t-\rho_k(t))dx=\int_{\partial\Omega}\frac{\partial Z_i(x,t-\rho_k(t))}{\partial n}ds$$

$$=-\int_{\partial\Omega} g_i(x,t-\rho_k(t))Z(x,t-\rho_k(t))ds \le 0.$$

Thus from above stating and combing conditions (G_2), (3) holds. Now by lemma 1 and lemma 2, we have $\int_{t_1}^{\infty}Q(t)dt<\infty$, which is contradictory to the condition of theorem .Then this theorem is proved.

Corollary If the differential inequality (2) has no eventually positive solution, then all solution of (*)-(Q_1) are oscillation in G (the same as corollary 2 in [2]).

It is well known that the first eigenvalue λ_0 of the problem

$$\Delta\varphi+\lambda\varphi=0 \text{ in } \Omega, \quad \varphi=0 \text{ on } \partial\Omega$$

is positive and the corresponding eigenfunction φ is positive in Ω.

Lemma 3 (see the proof of theorem 2 in [2]) Assume that

$$\frac{d^n}{dt^n}\left(\int_{\Omega} Z_i(x,t)\varphi(x)dx\right)+\frac{d^{(n-1)}}{d^{(n-1)}}\left(\int_{\Omega} Z_i(x,t)\varphi(x)dx\right)$$

$$\le -p_{ii}(t)\int_{\Omega} Z_i(x,t-\sigma(t))\varphi(x)dx+\sum_{j=1,j\ne i}^{m}\overline{P_{ij}}(t)\int_{\Omega} Z_i(x,t-\sigma(t))\varphi(x)dx,\ t\ge t_1,\ (3)'.$$

Then $V_i^{(n)}(t)+V_i^{(n-1)}(t)+Q(t)V(x,t-\sigma(t))\le 0$, $t\ge t_1$.

Theorem 2 If $\int_{t_0}^{\infty} Q(t)dt = \infty, t_0 > 0,$ then all solutions of the systems (*)-(Q_2) are oscillation in G.

Proof . Suppose to the contrary .Then there exists a non-oscillation solution : $u(x,t) = (u_1(x,t), u_2(x,t), \cdots, u_m(x,t))$ of system (*)- (Q_2) in the domain

$\Omega \times [t_0, +\infty)$ for some $t_0 > 0$ .For convenience and simplicity, we may take as

$0 \le t_0 \le t$, $|u_i(x,t)| > 0$, ($i = 1,2,\cdots,m$) and $Z_i(x, t) = \delta_i u_i(x,t)$,

and δ_i =sign $u_i(x,t)$ .Then we have $Z_i(x,t) > 0$. From (G_1) there exists $t_1 \ge t_0$,such

that when $t \ge t_1$, we have $Z_i(x, t)>0$, $Z_i(x, t-\rho_k(t))>0, i = 1,2,\cdots,m$.

$k = 1,2,\cdots,s$. $(x, t) \in \Omega \times [t_1, \infty)$.

Multiplying both sides of (*) by $\varphi(x)$, and integrating for x on Ω, we get

$$\frac{d^n}{dt^n}\left(\int_\Omega u_i(x,t)\varphi(x)dx\right) + \frac{d^{n-1}}{dt^{n-1}}\left(\int_\Omega u_i(x,t)\varphi(x)dx\right) = \int_\Omega P_3(x,t)\varphi(x)dx +$$

$$\int_\Omega P_3(x,t-\rho_k(t))\varphi(x)dx - \sum_{j=1}^{m}\int_\Omega P_{ij}(x,t)u_j(x,t-\sigma(t))\varphi(x)dx, \quad t \ge t_1, \; i = 1,2,\cdots,m$$

Therefore, we have that

$$\frac{d^n}{dt^n}\left(\int_\Omega Z_i(x,t)\varphi(x)dx\right) + \frac{d^{n-1}}{dt^{n-1}}\left(\int_\Omega Z_i(x,t)\varphi(x)dx\right)$$

$$= a_i(t)\int_\Omega \Delta Z_i(x,t)\varphi(x)dx + \cdots + c_i(t)\int_\Omega \Delta^3 Z_i(x,t)\varphi(x)dx$$

$$+\sum_{k=1}^{s} a_{ik}(t)\int_\Omega Z_j(x,t-\rho_k(t))\varphi(x)dx + \cdots + \sum_{k=1}^{s} c_{ik}(t)\int_\Omega \Delta^3 Z(x,t-\rho_k(t))\varphi(x)dx$$

$$-\sum_{j=1}^{m}\frac{\delta_i}{\delta_j}\int_\Omega \overline{P_{ij}}(x,t)Z_j(x,t-\sigma(t))\varphi(x)dx \quad ,t \ge t_{1.}$$

From Green identity and boundary value conditions (Q_2) we obtain that

$$\int_\Omega \Delta Z_i(x,t)\varphi(x)dx = -\lambda_0\int_\Omega Z_i(x,t)\varphi(x)dx \le 0, \cdots, \le 0,$$

$$\int_\Omega \Delta Z_j(x, t-\rho_k(t))\varphi(x)dx \le 0, \cdots,$$

and

$\int_\Omega \Delta^2 Z_j(x, t-\rho_k(t))\varphi(x)dx \le 0, \cdots,$ $\int_\Omega \Delta^3 Z_j(x, t-\rho_k(t))\varphi(x)dx \le 0,$

$t \ge t_1, i = 1,2,\cdots,m$.

Then (3)' holds .By lemma 3 and lemma 2, we have $\int_{t_1}^{\infty} Q(t)dt < \infty$, which is contradictory to the condition of the theorem. Then all solutions of (*),(Q_2) are oscillation in G. The proof of theorem 2 is therefore completed .

3 Some Note of Several Oscillation Criteria

We may extend the results of the impulsive hyperbolic equations for (2r+1) order case by using some definitions and some stating results in [3] .When r=0 or r=1 we will give out some results in [3]-[4] respectively , which are is also new things for this direction .

In this section, let Ω also be a bounded domain in R^n with a piecewise smooth boundary $\partial\Omega$, and $PC(R_+, R_+) = \{x(t): R_+ \to R_+, x(t)$ is piecewise continuous for $t \in R_+, t \ne t_k$, $x(t_k^+), x(t_k^-)$ exist and $x(t_k) = x(t_k^+), k = 1,2,\cdots\}$**,**

$\lim_{k\to\infty} t_k = \infty, 0 < t_1 < t_2 < \cdots < t_k <$ **, etc.**

We make it satisfy following conditions:

(H_1)

$a(t), a_1(t) \in PC(R_+, R_+),\ \lambda_i(t) \in PC^2(R_+, R_+),\ (i = 1,2,\cdots,m);\ \sigma(t),\ \rho_j(t) \in PC$ (R_+, R_+) **, and** $\lim_{t\to\infty}\sigma(t) = \lim_{t\to\infty}\rho_j(t) = \infty$, and $I: \Omega\times R_+\times R \to R,\ f \in PC(G, R)$.

(H_2) $c(x,t,\xi,\eta) \in PC(G\times R\times R, R)$, $c(x,t,\xi,\eta) \ge p(t)h(\xi)$ for all $(x,t,\xi,\eta) \in G\times R_+\times R_+$, $t \ne t_k$, where $p(t) \in PC(R_+, R_+)$ and that his continuous ,positive and convex function in R_+.

We assume that they are left continuous, at the moments of impulse ,the following relations $u(x,t_k^-)=u(x,t_k)$,and $u(x,t_k^+)=u(x,t_k)+I(x,t_k,u(x,t_k))$, are satisfied.

We consider the systems:

$$\frac{\partial^2}{\partial t^2}[u+\sum_{j=1}^{m}\lambda_i(t)u(x,t-\tau_i)]$$

$$=a(t)\Delta^{2r+1}u+\sum_{j=1}^{k}a_j(t)\Delta^{2r+1}u(x,\rho_j(t))-C(x,t,u(x,t),u(x,\sigma(t))+f(x,t),$$

$(x,t)\in\Omega\times(0,\infty)=G, t\neq t_k$.

$$u(x,t_k^+)-u(x,t_k^-)=I(x,t,u), t=t_k, k=1,2,\cdots \qquad \textbf{(4)}$$

with boundary condition**:**

$$\frac{\partial u}{\partial\gamma}=\psi,\ \frac{\partial\Delta u}{\partial\gamma}=\psi_{1,}\ \frac{\partial\Delta^2 u}{\partial\gamma}=\psi_2,\cdots,\frac{\partial\Delta^{2r}u}{\partial\gamma}=\psi_{2r}\ \text{ on }\ \partial\Omega\times R_+, t\neq t_k,\ (B)$$

Theorem 3 Assume that $(H_1)-(H_2)$ hold, and satisfy (A) for any function $u\in PC(\Omega\times R,,R_+)$ and constant $\alpha_k>0$ those

$$\int_\Omega I(x,t_k,u(x,t_k))dx\leq\alpha_k\int_\Omega u(x,t_k)dx, k=1,2,\cdots$$

hold. .

If $u(x,t)$ is a positive solution of the problem (4)-(B) in the domain $\Omega\times[t_0,\infty)$ for some $t_0>0$, then the impulsive differential inequalities of neutral type

$$[W(t)=\sum_{i=1}^{m}\lambda_i(t)W(t-\tau_i)]''+p(t)h(W(\sigma(t))\leq H(t),\quad (x,t)\in\Omega\times[t_0,\infty),\ t\neq t_k\ ,$$

$$W(t_k^+)\leq(1+\alpha_k)W(t_k), k=1,2,\cdots \qquad (5)$$

have an eventually positive solution

$$W(t)=\frac{1}{|\Omega|}\int_\Omega u(x,t)dx$$

where $H(t)=\frac{1}{|\Omega|}\{\int_{\Omega}[a(t)\psi_{2r}(x,t) + \sum_{j=1}^{k} a_j(t)\psi_{2r}(x,\rho_j(t))]ds + \int_{\Omega} f(x,t)dx\}$,

$t \neq t_k$,

Proof Let $u(x,t)$ be a positive solution of problem (4)-(B) in the domain $\Omega\times[t_0,+\infty)$ for some $t._0>0$.

For $t\neq t_k$, it follows from (H_1) that there exists a $t_1\geq t_0$ such that $u(x,t-\tau_i)>0,\ u(x,\rho_j(t))>0,\ u(x,\sigma(t))>0$ in $\Omega\times[t_1,\infty),\ i=1,2,\cdots,m,$ $j=1,2,\cdots,k.$

Thus , we obtain that

$$\frac{\partial^2}{\partial t^2}[u+\sum_{i=1}^{m}\lambda_i(t)\ u(x,t-\tau_i)]\leq a(t)\Delta^{2r+1}u+\sum_{j=1}^{k}a_j(t)\Delta^{2r+1}u(x,\rho_j(t))$$

$$-p(t)h(u(x,\sigma(t)))+f(x,t),\ (x,t)\in\Omega\times[t_1,\infty),\ t\neq t_k. \qquad (6)$$

From condition (B), Green identity and Jensen's inequality, it follows that

$\int_{\Omega}\Delta u dx=\int_{\partial\Omega}\frac{\partial u}{\partial\gamma}ds=\int_{\partial\Omega}\psi ds,\ \int_{\Omega}\Delta^2 u dx=\int_{\partial\Omega}\frac{\partial\Delta u}{\partial\gamma}ds=\int_{\partial\Omega}\psi_1 ds,\cdots,\int_{\Omega}\Delta^{2r+1}u dx=$

$\int_{\partial\Omega}\psi_{2r}ds$;

$\int_{\Omega}\Delta u(x,\rho_j(t))dx=\int_{\partial\Omega}\frac{\partial}{\partial\gamma}u(x,\rho_j(t))ds=\int_{\partial\Omega}\psi(x,\rho_j(t))ds$,and by similar calculating this integration we have that

$$\int_{\Omega}\Delta^{wr+1}u(x,\rho_j(t))dx=\int_{\partial\Omega}\psi_{2r}(x,\rho_j(t))ds.$$

Therefore integrating (6) for x over Ω, we obtain

$$\frac{d^2}{dt^2}[\int_{\Omega}udx+\sum_{i=1}^{m}\lambda_i(t)\int_{\Omega}u(x,t-\tau_i)dx]$$

$$\le a(t)\int_\Omega \Delta^{2r+1} u dx + \sum_{j=1}^{k} a_j(t)\int_\Omega \Delta^{2r+1} u(x,\rho_j(t))dx - p(t)\int_\Omega h(u(x,\sigma(t)))dx +$$

$$\int_\Omega f(x,t)dx$$

$$\le a(t)\int_{\partial\Omega} \psi_{2r} ds + \sum_{j=1}^{k} a_j(t)\int_{\partial\Omega} \psi_{2r}(x,\rho_j(t))ds - p(t)|\Omega| h(\frac{1}{|\Omega|}\int_\Omega u(x,\sigma(t))dx)$$

$$+\int_\Omega f(x,t)dx, \quad t \ne t_k, t \ge t_1.$$

where $|\Omega| = \int_\Omega dx$ .Set $W(t) = \frac{1}{|\Omega|}\int_\Omega u(x,t)dx$, Thus we have

$$\{W(t) + \sum_{j=1}^{m} \lambda_j(t) W(t-\tau_j)\}'' + p(t)h\{W(\sigma(t))\}$$

$$\le \frac{1}{|\Omega|}\{\int_{\partial\Omega} [a(t)\psi_{2r}(x,t) + \sum_{j=1}^{k} a_j(t)\psi_{2r}(x,\rho_j(t))\}]ds + \int_\Omega f(x,t)dx\}$$

$$= H(t), (x,t) \in \Omega \times [t_1,\infty), t \ne t_k, \quad (7)$$

For $t = t_k$, by (4) we have $(k = 1,2,\cdots)$

$$\int_\Omega (u(x,t_k^+) - u(x,t_k))\varphi(x)dx = \int_\Omega I(x,t_k,u(x,t_k))\varphi(x)dx \le \alpha_k \int_\Omega u(x,t_k)\varphi(x)dx,$$

So $\int_\Omega u(x,t_k^+)\varphi(x)dx \le (1+\alpha_k)\int_\Omega u(x,t)\varphi(x)dx, (k = 1,2,\cdots)$ (8)

Hence the inequalities (7)-(8) imply that the function $W(t)$ is a positive solution of the impulsive differential inequality of neutral type in (4) for $t \ge t_1$. Therefore this ends the proof .

Remark. When r=0 we get the theorem 2.3 in [5] ,and when r=1 that is a sixth-order case .

Theorem 4 Assume that same as theorem3 that $(H_1)-(H_2)$ and (A) hold, and that

(A)' $c(x,t,-\xi,-\eta) = -c(x,t,,\xi,\eta)$ for all $(x,t,\xi,\eta) \in G \times R \times R,\ t \ne t_k$,

$I(x,t_k,-u(x,t_k)) = -I(x,t_k,u(x,t_k)), \quad t = t_k, \quad (k = 1,2,\cdots)$, and both the impulsive differential inequalities of neutral type (4) and

$$[V(t)+\sum_{i=1}^{m}\lambda_i(t)V(t-\tau_i)]'' + (\lambda_0)^{2r+1}a(t)V(t) + (\lambda_0)^{2r+1}\sum_{j=1}^{k}a_j(t)V(\rho_j(t))$$

$$+ p(t)h(V(o(t))) \le -F(t), \quad t \ne t_k,$$

$$V(t_k^+) \le (1+\alpha_k)V(t_k), \quad k = 1,2,\cdots \qquad (8^*)$$

have no eventually positive solution. Then there every nonzero solution of the problem (4)-(B_1) is Oscillation in the domain $G = \Omega \times R_+$.

Proof The proof is similar to the theorem 2 in [4], so we omit it.

Remark When r=1 we get the theorems 1-2 of [3] ,and When r=0 we get the theorem 2.2 of [5].There is taking $r = 2,3,\cdots$, then now we have more results.

4 Some examples

Example 1 We consider that system (5)-(5)':

$$\frac{\partial^6 u_1(x,t)}{\partial t^6} + \frac{\partial^5 u_1(x,t)}{\partial t^5} = (\Delta^3 + \Delta^2 + 4\Delta)u_1(x,t) + \frac{1}{2}\Delta u_1(x,t-\frac{3\pi}{2})$$

$$-3u_1(x,t-3\pi) - (\frac{3}{2})u_2(x,t-3\pi) \qquad \textbf{(9)}$$

$$\frac{\partial^6 u_2(x,t)}{\partial t^6} + \frac{\partial^5 u_2(x,t)}{\partial t^5} = (\Delta^3 + \Delta^2 + 4\Delta)u_2(x,t) + \frac{1}{2}\Delta u_2(x,t-\frac{3\pi}{2})$$

$$-(-\frac{3}{2})u_1(x,t-\pi)\,\mathbf{-3}\,u_2(x,t-\pi) \qquad \textbf{(9)'}$$

where $(x,t) \in (0,\pi) \times [0,\infty)$. The boundary value condition：

$$\frac{\partial}{\partial x}u_i(0,t) = \frac{\partial}{\partial x}u_i(\pi,t) = 0,\ \frac{\partial^2}{\partial x^2}u_i(0,t) = \frac{\partial^2}{\partial x^2}u_i(\pi,t) = 0,\ \mathbf{t} \ge 0, i = 1,2.$$

Let $n = 6, N = 1, m = 2, s = 1. a_1(t) = 4, a_{11}(t) = \frac{1}{2}, \rho_1(t) = \frac{3\pi}{2}, \sigma(t) = \pi$,

$p_{11}(x,t)=3, p_{12}(x,t)=\frac{3}{2}, p_{21}(x,t)=-\frac{3}{2}, p_{22}(x,t)=3$; $a_2(t)$ =4, $a_{21}(t)$ = $\frac{1}{2}$, $\Omega=(0,\pi)$, and Q(t)=$\frac{3}{2}$.

It satisfy all condition of theorem 1, then all solution .of this system are oscillation on $(0, \pi)\times[0,\infty)$ (In fact, we have that $u_1(x,t)$ = $\cos x \sin t$, $u_2(x,t)=\cos x$ cost are oscillation solution of the system (9)-(9)').

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Received: August, 2008