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Analysis of Warm Standby Systems Subject to Common-Cause Failures with Time Varying Failure and Repair Rates

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Abstract

This paper presents reliability and availability analysis of a k-out-of-(M+S): G warm standby system with time varying failure and repair rates in presence of common-cause failure. The system composed of two categories of components. One category of the components is of type 1 has *M* components and the other is of type 2 has *S* components. Type 1 have a lower failure rate. Markov method is used to obtain the system reliability and availability. A numerical example is given to illustrate the theoretical results obtained. Selective plots are shown to demonstrate the effects of common-cause failures, number of repairmen and number of components of type 1 on system reliability and availability.

Keywords: k-out-of-(M+S):G system, time varying failure and repair rates, *r* repair facilities , generalized transition probability, availability, reliability

1. Introduction

Standby systems often find applications in various industrial and other setups. The system reliability and availability are increased considerably by the use of this form of redundancy. However, the type of redundancy application is dictated by the circumstances under consideration and various other factors. The standby redundancy represents a situation with one unit operating and a number of on standby. In general, there are three types in standby, i.e. cold, hot and warm standby. Cold standby implies that the inactive components have a zero failure rate and cannot fail while in standby state. Hot standby implies that an inactive component has the same failure rate as when it is in operation. Warm standby is an intermediate case and it implies that an inactive component has a failure rate between that for the cold and hot standby. It is also called dormant failure in some papers.

In usual reliability and availability analysis of standby systems, the occurrence of common-cause failures is overlooked and only the general failures are considered. In recent years, it has been realized that in order to predict realistic reliability and availability of standby systems, the occurrence of common-cause failures must be considered. A common-cause failure is defined as any instance where multiple units or components fail due to a single cause.

Here we study a general k-out-of-n: G warm standby system with time varying failure and repair rates subject to common-cause failure. Many articles concerning the reliability and availability of k-out-of-n warm standby systems have been published. Among them, 1-out-of-2 warm standby systems have been studied in detail by considering different conditions, such as those cases with or by considering general distributions [1], different types of repair facilities [2], correlated failures and repairs [3].

An algorithm and its computer program for calculating fail-safe and fail-danger probabilities of k-out-of-n: G system with non identical components is presented [4]. Goel et al. [5] analyzed a 1-out-of-3 warm standby system with two types of spare units a warm and a cold standby unit, and inspection. A general closed-form equation was developed for system reliability of a k-out-of-n warm standby system where components in k-out-of-n: G standby systems were assumed to be statistically identical. In addition, as a general case, system availability of m-out-of-n: G warm standby system with identical components was studied [6].

Zhang [7] dealt with a repairable standby system consisting of (N+1) units and a single repair facility, in which unit 1 has preemptive priority both in getting operation and in getting repaired. Chryssaphinou et al. [8] considered a 1-out-of-(m+1) warm standby system with non-identical units. Zhang and Horigome [9] derived the availability of a specific warm standby system with two kinds of components where priority-standby for one kind of components applies. The reliability of a k-out-of-(m+w) warm standby system with *m* operating units, *w* warm standby and *R* repairmen in which the balking and reneging of units are considered is studied [10]. Zhang et al [11] studied systems with two categories of components named k-out-of-(M+N): G warm standby system. It is assumed that the failure and repair rates of each component in the system are constants.

The results mentioned above are limited to 1-out-of-n: G warm standby systems or a general m-out-of-n: G warm standby one with identical components or a specific system with two kinds of units or a general systems in which the failure and repair rates are assumed to be constant.

In the current paper we study the availability and reliability of a general k-out-of-(M+S): G warm standby repairable system with time varying failure and repair rates in presence of common-cause failures. Such a system is composed of two categories of components, one of which is named type 1 and the other type 2. Type 1 has *M* components and type 2 has *S* components. Here, type 1 components have priority of operation and repair over type 2, which means that type 1 components are desired to perform system function whenever available for use. Type 2 components operate if the number of type 1 in operation is not sufficient for the system to function normally. By using Markov model, the system state transition process can be clearly illustrated, and furthermore, the solutions of system availability and reliability are obtained based on this. Some special models are studied to illustrate the solutions of the system availability and reliability and the effect of common-cause failure on them. The system under consideration fails when there is either a common-cause failure or when there are only (k-1) good components.

Notations

The following symbols are used in the current paper:

$\lambda_1(t)$, $\lambda_2(t)$	time varying failure rates of [type 1, type 2] components in active mode
$\lambda'_1(t)$, $\lambda'_2(t)$	time varying failure rates of [type 1, type 2] components in inactive (standby) mode
λ_c	constant common-cause failure rate
$\mu_1(t)$, $\mu_2(t)$	time varying repair rates of [type 1, type 2] components
r	number of repairmen
(i, j)	represent the state of the system, where *i* and *j* represent the number of failed components of type1 and type2 respectively
$P_{(i,j)}(t)$	probability that the system is in state (i, j) at time *t*
$P'_{(i,j)}(t)$	derivative of $P_{(i,j)}(t)$
A(t), R(t)	availability, reliability of system at time *t*

2. Assumptions

In order to describe such a kind of systems clearly, the following assumptions are needed:

1. Common-cause failure and other failures are statistically independent.
2. Common-cause failure rates are constant.
3. Common-cause failures can only occur in a system with more than one good unit.
4. Common-cause failure rates are the same for the partially or fully operating system.
5. Type 1 components are identical, and each of them has time varying operative failure rate and standby failure rate.
6. Type 2 components are also identical, and each of them has time varying operative failure rate and standby failure rate, which different from type 1 components.
7. Repair rates for types 1 and 2 components are different and time varying.
8. The states of all components are statistically independent.
9. There are *r* repair facilities applied to the system.
10. When one component fails, it is instantaneously replaced by one of the standby components if there is one.
11. When system fails, no failure will occur for other working components.

3. The general model

The Markov state transition diagram is used in analyzing reliability and availability of the system. Here, we use the homogenous Markov model to analyze system state transition process. The system state transition diagram of a repairable standby redundant system with two categories of components and priority-standby for one kind can be expressed as one with two dimensions [11].

Where the system state is represented by (i, j) where *i*, *j* denotes the number of types 1 and 2 components in failure. Because type 1 components have priority in operation and repair, the state transition diagram can be obtained as shown in Figure (1).

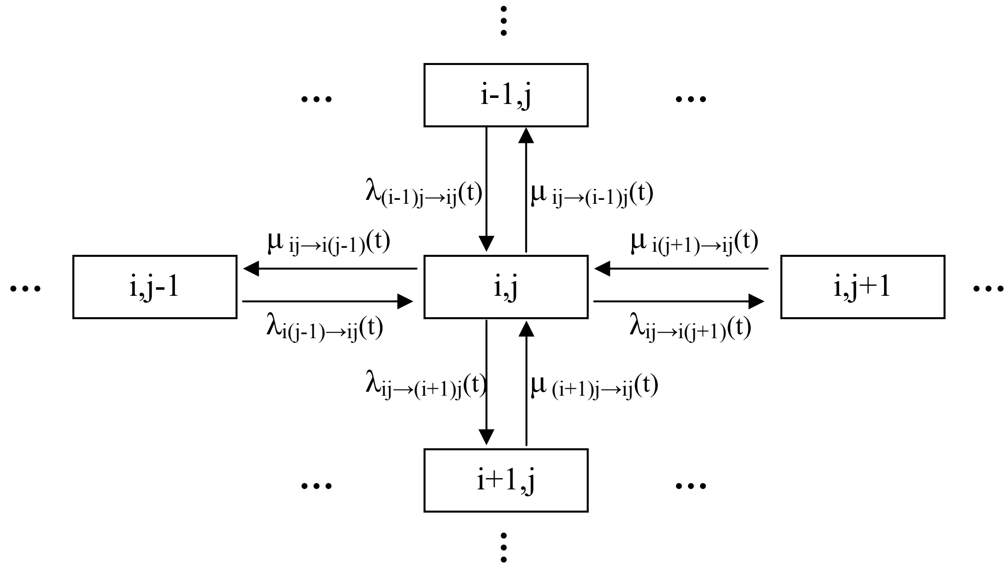

Figure (1) the state transition diagram.

The state transition rates which are related to the state (i, j) shown in Figure (1) are given as follows:

$$\lambda_{gj\to ij}(t)=\begin{cases}\lambda_c & if\ R\succ 1, i=M\\ k\lambda_1(t)+(M-k-i+1)\lambda_1^{/}(t), & if\quad M-k-i+1\ge 0,\\ (M-i+1)\lambda_1(t) & otherwise\end{cases} \tag{1}$$

$$\lambda_{ih\to ij}(t)=\begin{cases}\lambda_c & if\quad Q\succ 1, j=S\\ (S-j+1)\lambda_2^{/}(t), & if\quad M-k-i\ge 0,\\ (k-(M-i))\lambda_2(t)+(M+S-(k+i+j-1))\lambda_2^{/}(t) & otherwise\end{cases} \tag{2}$$

where

R is the number of good components of type 1
Q is the number of good components of type 2
g < i, $0\le g\le M$, and h < j, $0\le h\le S$.

$$\mu_{ij\to(i-1)j}(t)=\mu_1(t).\min(i,r) \tag{3}$$

$$\mu_{ij\to i(j-1)}(t)=\mu_2(t).\min(j,r-i) \tag{4}$$

In addition, $\mu_{ij\to i(j-1)}(t)=0$ when $r\le i$, $\lambda_{(i-1)j\to ij}(t)=0$, when i=0, $\lambda_{i(j-1)\to ij}(t)=0$, when j=0, and $\lambda_{(i-1)j\to ij}(t)=\lambda_{i(j-1)\to ij}(t)=\mu_{ij\to i(j-1)}(t)=\mu_{ij\to(i-1)j}(t)=0$, when (i+j) > M+S-k+1 .

4. System availability and reliability

Based on Figure (1), we have the following differential equation:

$$P^{/}(t)=P(t)\Lambda \tag{5}$$

where

$$P(t)=(P_{(0,0)}(t),P_{(0,1)}(t),\ldots,P_{(0,S)}(t),P_{(1,0)}(t),\ldots,P_{(M,(S-1))}(t),P_{(M,S)}(t))$$

and Λ is the transition matrix rate of the system, which can be obtained for a specific system.
Assume that the system is in state (0, 0) at time zero, the initial condition for equation (5) is:

$$P_{(0,0)}(0)=1, P_{(i,j)}(0)=0 \quad for\,(i+j)\geq 1 \tag{6}$$

Then the system availability is given by

$$A(t)=\sum_{i+j\leq M+S-k} P_{(i,j)}(t) \tag{7}$$

Based on the above analysis, if all failure states of the system are regarded as absorbing states, we can solve equation (5) to obtain $\widetilde{P}_{(i,j)}(t)$ under the following conditions

$\mu_{ij\to(i-1)j}(t)=\mu_{ij\to i(j-1)}(t)=0$ when (i+j) ≥ M+S-k+1.

Then the system reliability is obtained as

$$R(t)=\sum_{i+j\leq M+S-k} \widetilde{P}_{(i,j)}(t) \tag{8}$$

5. Special case models

Availability and reliability of 3-out-of-(4+2): G system

The system state transition diagram for the model is shown in Figure (2) with two repair facilities applied.

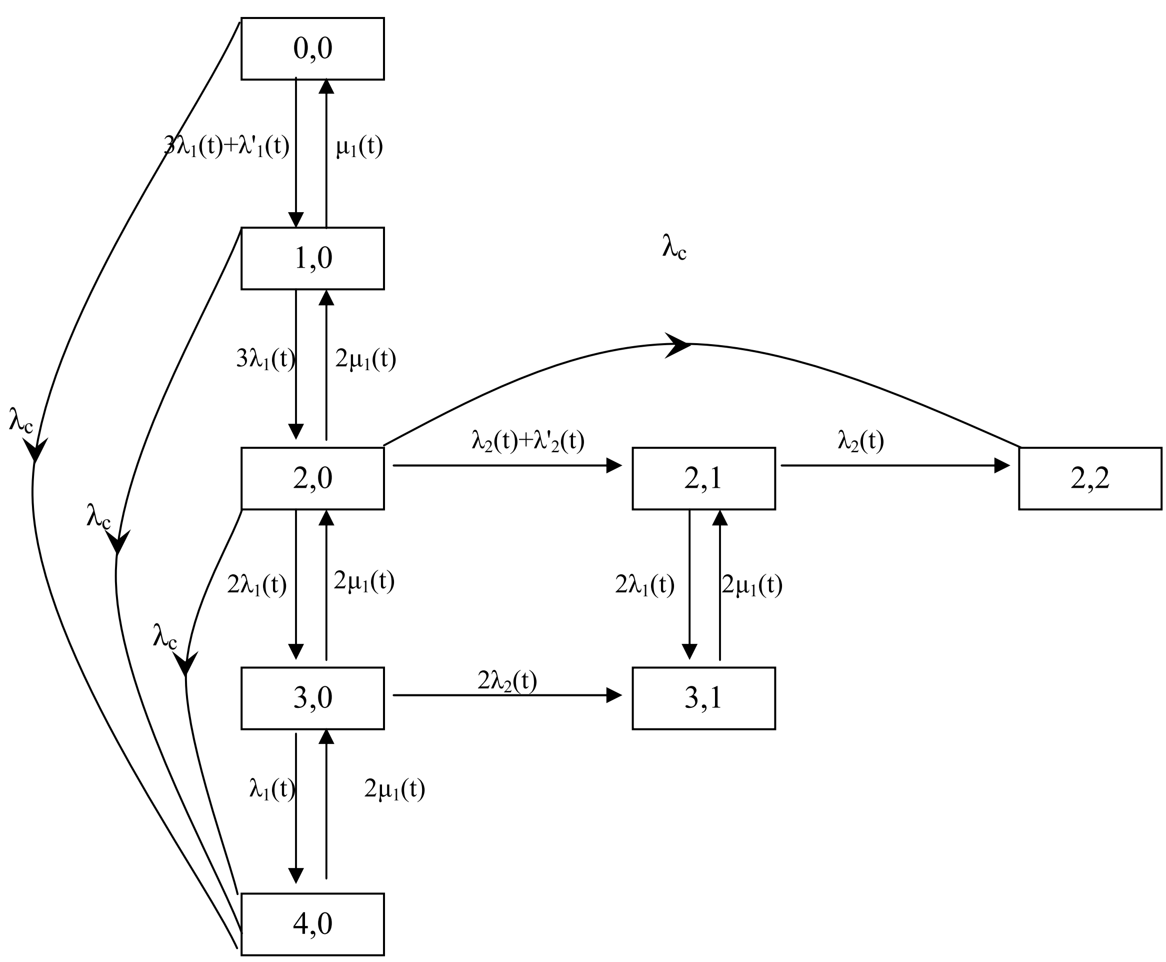

Figure (2) state transition diagram of 3-out-of-(4+2): G system

Assuming that the repair rates follow Weibull distribution and the failure rates follow Gamma distribution and with the help of equation (5):

$$P(t) = (P_{(0,0)}(t), P_{(1,0)}(t), P_{(2,0)}(t), P_{(2,1)}(t), P_{(2,2)}(t), P_{(3,0)}(t), P_{(3,1)}(t), P_{(4,0)}(t))$$

and Λ is written as

Λ=

$$\begin{pmatrix} -E(t) & 3\lambda_1(t)+\lambda_1^{/}(t) & 0 & 0 & 0 & 0 & 0 & \lambda_c \\ \mu_1(t) & -B(t) & 3\lambda_1(t) & 0 & 0 & 0 & 0 & \lambda_c \\ 0 & 2\mu_1(t) & -C(t) & \lambda_2(t)+\lambda_2^{/}(t) & \lambda_c & 2\lambda_1(t) & 0 & \lambda_c \\ 0 & 0 & 0 & -(2\lambda_1(t)+\lambda_2(t)) & \lambda_2(t) & 0 & 2\lambda_1(t) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\mu_1(t) & 0 & 0 & -D(t) & 2\lambda_2(t) & \lambda_1(t) \\ 0 & 0 & 0 & 2\mu_1(t) & 0 & 0 & -2\mu_1(t) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu_1(t) & 0 & -2\mu_1(t) \end{pmatrix} \quad (9)$$

where

$E(t) = (3\lambda_1(t) + \lambda'_1(t) + \lambda_c)$, $C(t) = (2\lambda_1(t) + \lambda_2(t) + \lambda'_2(t) + 2\mu_1(t) + 2\lambda_c)$

$B(t) = (3\lambda_1(t) + \lambda_c + \mu_1(t))$, $D(t) = (\lambda_1(t) + 2\lambda_2(t) + 2\mu_1(t))$.

$$\lambda_1(t) = [\frac{t^{\beta_1 - 1}}{\alpha_1^{\beta_1}\Gamma(\beta_1)} e^{-t/\alpha_1}]/[\sum_{k=0}^{\beta_1 - 1}\frac{(t/\alpha_1)^k e^{-t/\alpha_1}}{k!}],$$

$$\lambda_2(t) = [\frac{t^{\beta_2 - 1}}{\alpha_2^{\beta_2}\Gamma(\beta_2)} e^{-t/\alpha_2}]/[\sum_{k=0}^{\beta_2 - 1}\frac{(t/\alpha_2)^k e^{-t/\alpha_2}}{k!}],$$

$$\lambda_1^{/}(t) = [\frac{t^{\beta_3 - 1}}{\alpha_3^{\beta_3}\Gamma(\beta_3)} e^{-t/\alpha_3}]/[\sum_{k=0}^{\beta_3 - 1}\frac{(t/\alpha_3)^k e^{-t/\alpha_3}}{k!}].$$

$$\lambda_2^{/}(t) = [\frac{t^{\beta_4 - 1}}{\alpha_4^{\beta_4}\Gamma(\beta_4)} e^{-t/\alpha_4}]/[\sum_{k=0}^{\beta_4 - 1}\frac{(t/\alpha_4)^k e^{-t/\alpha_4}}{k!}]$$

$$\mu_1(t) = \alpha_5^{\beta_5}\beta_5\, t^{\beta_5 - 1}$$

with the initial conditions

$$[P_{(0,0)}(0), P_{(1,0)}(0), P_{(2,0)}(0), P_{(2,1)}(0), P_{(2,2)}(0), P_{(3,0)}(0), P_{(3,1)}(0), P_{(4,0)}(0)] = [1,0,0,0,0,0,0,0]$$

The system availability is given by

$$A(t) = P_{(0,0)}(t) + P_{(1,0)}(t) + P_{(2,0)}(t) + P_{(2,1)}(t) + P_{(3,0)}(t) \quad (10)$$

If all failure states of the system are regarded as absorbing states.
Then the system reliability is obtained as

$$R(t) = \widetilde{P}_{(0,0)}(t) + \widetilde{P}_{(1,0)}(t) + \widetilde{P}_{(2,0)}(t) + \widetilde{P}_{(2,1)}(t) + \widetilde{P}_{(3,0)}(t) \quad (11)$$

For

$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 1$, $\beta_1=4$, $\beta_2=2$, $\beta_3=5$, $\beta_4=3$, $\beta_5=1.2$ and $\lambda c=0.25$

The system availability and reliability are illustrated in Figure (3) with numerical solutions based on Runge-Kutta method.

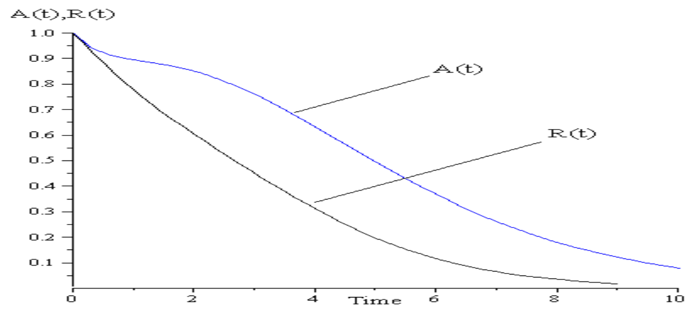

Figure (3) system reliability and availability versus time

Now we will study the effect of the number of repairmen on system reliability using equation (9) for r=1 as ($2\mu_1(t) \rightarrow \mu_1(t)$), and r=2.

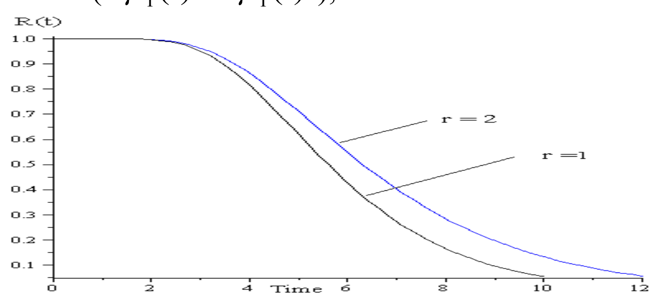

Figure (4) system reliability for different values of r

From figure (4) we deduced that the system reliability increases with corresponding increasing in number of system repairmen.

From the current model and with other two values for *M*; for example M=2, M=3 Assume that the failure and repair rates of the system in these cases follow Weibull distribution. Using the previous values of the parameters we find that the system reliability increases with corresponding increasing in numbers of components of type 1 as shown in the following figure.

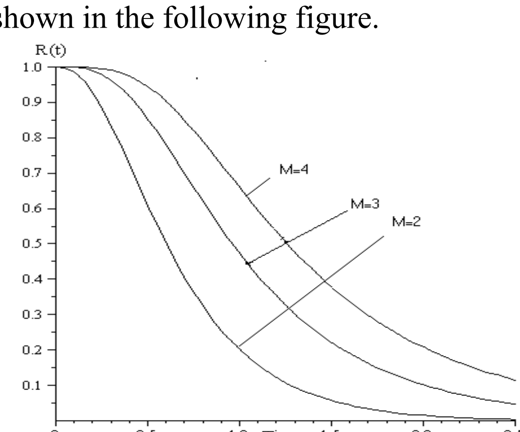

Figure (5) system reliability for different values of M

6. Conclusion

A warm standby system with two types of components subjected to common-cause failure is studied. The system transition diagram for this model is expressed as one with two dimensions. Markov method is used to obtain the system reliability and availability with numerical solution. In the future we hope that we can deal with warm standby systems with three types of components.

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