

# Sensitivity and Stability Analysis in DEA on Interval Data by Using MOLP Methods

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## Abstract

In this paper, we suppose a method for analyzing sensitivity and stability of all the decision making units, while inputs and outputs are interval data. Therefore, for estimating radius of stability of a DMU; firstly, we classify the decision making units then we obtain the radius of stability for each classification. For analyzing the sensitivity and estimating the radius of stability analogous of each DMU, a MOLP is defined. Therefore, the interactive methods are used for finding the efficient solution in which the comment of Decision Maker is important. At the end numerical example has been solved by using the weighted-sums of the target function and also the interactive method (STEM) in MOLP problems.

**Keywords:** Data envelopment analysis, Sensitivity and radius stability analysis, MOLP, Interval data

## 1 Introduction

Data envelopment analysis (DEA), introduced by Charnes et al. [1] (CCR) and extended by Banker et.al. [2] (BCC), is a useful method to evaluate relative efficiency of multiple-inputs and outputs units based on observed data. Its

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goal is to classify the decision making units(DMU)into two classes:efficient or inefficient ones. How ever, uncertainty such as a measurement error should be incorporated in observed data. This indicated the necessity to assess the sensitivity of classification in DEA. In the recent years, many DEA researchers have studied the sensitivity of efficiency and inefficiency classification with respect to perturbations in data [3], [4], [5], [6]. The original DEA models assume that inputs and outputs are measured by exact values on a ratio scale.

Recently, Cooper et al. [7]addressed the problem of imprecise in DEA in its general form. Imprecise data means that some data are known only to the extent that the true values lie within prescribe bounds while other data are known only to satisfy certain ordinal relations. Jahanshahloo et.al. [8] discussion a method for analyzing sensitivity and stability of all the decision making units, while inputs and outputs are interval data. They consider proportional output and input changes. In this paper we can easily consider non-proportional changes. we discuss a technique for assessing the sensitivity of efficiency classification in DEA with interval data.

In this paper, a modified CCR model is suggested to sensitive the DMUs with interval data. We develop several linear programming formulation for investigating radius of stability for all DMUs with interval data. The possible data perturbation for preserving the DMUs classification are computed from the optimal values.

Interval DEA models are extended to interval data Nagano et al.[9]. Then interval DEA for interval data can be extended to fuzzy data as well, since the level sets of fuzzy data are interval data. Therefore, fuzzy efficiency for fuzzy data can be obtained by interval DEA through the resolution identity Guo et al. [10].

The current article proceeds as follows: In Section 2, we review DEA models for dealing with interval data. Then, on the basis of these models, in Section 3, we propose some models for determining radius of stability for DMUs. In Section 4, the sensitivity analysis and radius of stability via STEM algorithm introduced. In Section 5, the sensitivity and stability analysis methods to several data sets are introduced. A conclusion section summarize our main results.

## 2 Preliminary Notes

Consider  $n$  DMUs with  $m$  inputs and  $s$  outputs. The input and output vectors of  $DMU_j$  ( $j = 1, \dots, n$ ) are  $X_j = (x_{1j}, \dots, x_{mj})^t$ ,  $Y_j = (y_{1j}, \dots, y_{sj})^t$ , respectively, where  $X_j \geq 0$ ,  $X_j \neq 0$ ,  $Y_j \geq 0$ ,  $Y_j \neq 0$ . Unlike the classic DEA model, we assume further that the levels of inputs and outputs are not known exactly, the true input and output data known to lie within bounded interval .i.e.  $x_{ij} \in [x_{ij}^L, x_{ij}^U]$  and  $y_{rj} \in [y_{rj}^L, y_{rj}^U]$  with upper and lower bounds of the

intervals given as constants and assumed strictly positive. In this case, the efficiency can be an interval. The upper bound of interval efficiency for  $DMU_o$  is obtained from the best viewpoints and the lower bound is obtained from the worst viewpoint. The following model provides such an upper bound for  $DMU_o$ :

$$\begin{aligned} \theta_o^U = \min \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1, j \neq o}^n \lambda_j x_{ij}^U + \lambda_o x_{io}^L \leq \theta x_{io}^L, \quad i = 1, \dots, m \\ & \sum_{j=1, j \neq o}^n \lambda_j y_{rj}^L + \lambda_o y_{ro}^U \geq y_{ro}^U, \quad r = 1, \dots, s \\ & \lambda_j \geq 0, \quad j = 1, \dots, n. \end{aligned} \quad (1)$$

We denote by  $\theta_o^U$  the efficiency score attained by  $DMU_o$  in model(1). the model below provides a lower bound of The efficiency score for  $DMU_o$ :

$$\begin{aligned} \theta_o^L = \min \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1, j \neq o}^n \lambda_j x_{ij}^L + \lambda_o x_{io}^U \leq \theta x_{io}^U, \quad i = 1, \dots, m \\ & \sum_{j=1, j \neq o}^n \lambda_j y_{rj}^U + \lambda_o y_{ro}^L \geq y_{ro}^L, \quad r = 1, \dots, s \\ & \lambda_j \geq 0, \quad j = 1, \dots, n. \end{aligned} \quad (2)$$

The efficiency  $\theta_o^L$  attained by  $DMU_o$  in model (2) serves as a lower bound of its possible efficiency scores. While considering (1) and (2), it is evident that  $\theta_o^L \leq \theta_o^U$ .

On the basis of the above efficiency score intervals,  $DMUs$  can be classified in three subsets as follows:

$E^{++} = \{j \in J \mid \theta_j^L = 1\}$ ,  $E^+ = \{j \in J \mid \theta_j^L < 1 \text{ and } \theta_j^U = 1\}$  and  $E^- = \{j \in J \mid \theta_j^U < 1\}$ .

### 3 Sensitivity analysis in DEA with interval data

Suppose that  $DMUs$  are evaluated by model (1) and model (2) are classified in  $E^{++}$ ,  $E^+$  and  $E^-$ . Having identified efficient and inefficient  $DMUs$  in a DEA analysis, one may want to know how sensitive these identification are to possible variation in the data. We determine "radius of stability" within which data variations will not alter a  $DMU$ 's classification from efficient to

inefficient status or vice versa. We consider the radius of stability of  $DMU_o$  in three cases as follow.

### 3.1 Radius of stability for $DMU$ in $E^{++}$

In this case, we assume that  $DMU_o$  is in  $E^{++}$ , that is,  $\theta_o^L = 1$ . It is obvious that  $DMU_o$  remains in  $E^{++}$  if its outputs increase or its inputs decrease. Our aim is to find the scalars  $\alpha_i, (i = 1, \dots, m)$ ,  $\beta_r, (r = 1, \dots, s)$ ,  $\delta_i, (i = 1, \dots, m)$  and  $\varphi_r, (r = 1, \dots, s)$  such that if we decrease  $r$ th upper bound of output of  $DMU_o$  by  $\beta_r$  and increase  $i$ th lower bound of input of  $DMU_o$  by  $\alpha_i$  then  $\theta_o^U = 1$ , also if we decrease  $r$ th lower bound of output of  $DMU_o$  by  $\varphi_r$  and increase  $i$ th upper bound of input of  $DMU_o$  by  $\delta_i$  then  $\theta_o^L = 1$ , i.e.  $DMU_o$  remains in  $E^{++}$ .

It has been assumed that  $\beta_r, \alpha_i, \varphi_r$  and  $\delta_i$  are scalar and non negative. Here we consider the following cases:

(1) It is obvious if  $x_{io}^L, (i = 1, \dots, m)$  are replace with  $x_{io}^L - \alpha_i, (i = 1, \dots, m)$  and  $y_{ro}^U, (r = 1, \dots, s)$  are replace with  $y_{ro}^U + \beta_r, (r = 1, \dots, s)$  then  $\theta_o^U = 1$  and if  $x_{io}^U, (i = 1, \dots, m)$  are replace with  $x_{io}^U - \delta_i, (i = 1, \dots, m)$ , and  $y_{ro}^L, (r = 1, \dots, s)$  are replace with  $y_{ro}^L + \varphi_r, (r = 1, \dots, s)$  then  $\theta_o^L = 1$ ; consequently,  $DMU_o \in E^{++}$ .

(2) If  $x_{io}^L, (i = 1, \dots, m)$  are replace with  $x_{io}^L + \alpha_i, (i = 1, \dots, m)$  and  $y_{ro}^U, (r = 1, \dots, s)$  are replace with  $y_{ro}^U - \beta_r, (r = 1, \dots, s)$  then it is possible for  $DMU_o$  not to be in the  $E^{++}$ . We are concerned with finding the largest value for  $\beta_r$  and  $\alpha_i$  such that  $DMU_o \in E^{++}$ . For this purpose, the following model is proposed:

$$\begin{aligned}
 & \min \quad \{\alpha_1, \dots, \alpha_m, \beta_1, \dots, \beta_s\} \\
 & s.t. \quad \sum_{j=1}^n \lambda_j x_{ij}^U \leq x_{io}^L + \alpha_i, \quad i = 1, \dots, m \\
 & \quad \quad \sum_{j=1}^n \lambda_j y_{rj}^L \geq y_{ro}^U - \beta_r, \quad r = 1, \dots, s \\
 & \quad \quad 0 \leq \alpha_i \leq x_{io}^U - x_{io}^L, \quad i = 1, \dots, m \\
 & \quad \quad 0 \leq \beta_r \leq y_{ro}^U - y_{ro}^L, \quad r = 1, \dots, s \\
 & \quad \quad \lambda_j \geq 0, \quad j = 1, \dots, n.
 \end{aligned} \tag{3}$$

Model (3) is a MOLP. We solve model (3) through the method of Weighted-

sums. We have:

$$\begin{aligned}
 \min \quad & \sum_{i=1}^m W_i \alpha_i + \sum_{r=1}^s W'_r \beta_r \\
 \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij}^U \leq x_{io}^L + \alpha_i, \quad i = 1, \dots, m \\
 & \sum_{j=1}^n \lambda_j y_{rj}^L \geq y_{ro}^U - \beta_r, \quad r = 1, \dots, s \\
 & 0 \leq \alpha_i \leq x_{io}^U - x_{io}^L, \quad i = 1, \dots, m \\
 & 0 \leq \beta_r \leq y_{ro}^U - y_{ro}^L, \quad r = 1, \dots, s \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n
 \end{aligned} \tag{4}$$

(3) If  $x_{io}^U$ , ( $i = 1, \dots, m$ ) are replace with  $x_{io}^U + \delta_i$ , ( $i = 1, \dots, m$ ) and  $y_{ro}^L$ , ( $r = 1, \dots, s$ ) are replace with  $y_{ro}^L - \varphi_r$ , ( $r = 1, \dots, s$ ) then it is possible for  $DMU_o$  not to be in the  $E^{++}$ . We are concerned with finding the largest value for  $\varphi_r$  and  $\delta_i$  such that  $DMU_o \in E^{++}$ . For this purpose, the following model is proposed:

$$\begin{aligned}
 \min \quad & \{\delta_1, \dots, \delta_m, \varphi_1, \dots, \varphi_s\} \\
 \text{s.t.} \quad & \sum_{j=1, j \neq o}^n \lambda_j x_{ij}^L \leq x_{io}^U + \delta_i, \quad i = 1, \dots, m \\
 & \sum_{j=1, j \neq o}^n \lambda_j y_{rj}^U \geq y_{ro}^L - \varphi_r, \quad r = 1, \dots, s \\
 & \delta_i \geq 0, \quad i = 1, \dots, m \\
 & \varphi_r \geq 0, \quad r = 1, \dots, s \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n.
 \end{aligned} \tag{5}$$

We solve model (5) through the method of Weighted-sums. So we have:

$$\begin{aligned}
 \min \quad & \sum_{i=1}^m W_i \delta_i + \sum_{r=1}^s W'_r \varphi_r \\
 \text{s.t.} \quad & \sum_{j=1, j \neq o}^n \lambda_j x_{ij}^L \leq x_{io}^U + \delta_i, \quad i = 1, \dots, m \\
 & \sum_{j=1, j \neq o}^n \lambda_j y_{rj}^U \geq y_{ro}^L - \varphi_r, \quad r = 1, \dots, s \\
 & \delta_i \geq 0, \quad i = 1, \dots, m \\
 & \varphi_r \geq 0, \quad r = 1, \dots, s \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n.
 \end{aligned} \tag{6}$$

**Theorem 3.1.** *If  $DMU_o \in E^{++}$  and  $(\lambda_1^*, \dots, \lambda_n^*, \alpha_1^*, \dots, \alpha_m^*, \beta_1^*, \dots, \beta_s^*)$  is an optimal Pareto solution of (3) then  $\overline{DMU}_o$  with inputs  $[x_{io}^L + \alpha_i^*, x_{io}^U]$  ( $i = 1, \dots, m$ ) and outputs  $[y_{ro}^L, y_{ro}^U - \beta_r^*]$ , ( $r = 1, \dots, s$ ) is in  $E^{++}$ , i.e.  $\theta_o^U = 1$ .*

**Proof.** Suppose  $(\hat{\lambda}, \theta_o^U)$  is optimal value of model(1) when evaluating  $\overline{DMU}_o$  so we have

$$\hat{\lambda}_o(x_{io}^L + \alpha_i^*) + \sum_{j=1, j \neq o}^n \hat{\lambda}_j x_{ij}^U \leq \theta_o^L(x_{io}^L + \alpha_i^*), \quad i = 1, \dots, m$$

$$\hat{\lambda}_o(y_{ro}^U - \beta_r^*) + \sum_{j=1, j \neq o}^n \hat{\lambda}_j y_{rj}^L \geq y_{ro}^U - \beta_r^*, \quad r = 1, \dots, s.$$

Suppose  $\theta_o^U < 1$  then  $\overline{DMU}_o$  is inefficient so  $\hat{\lambda}_o = 0$  we have:

$$\sum_{j=1, j \neq o}^n \hat{\lambda}_j x_{ij}^U < x_{io}^L + \alpha_i^*, \quad i = 1, \dots, m$$

$$\sum_{j=1, j \neq o}^n \hat{\lambda}_j y_{rj}^L \geq y_{ro}^U - \beta_r^*, \quad r = 1, \dots, s. \tag{7}$$

Exist  $\varepsilon_i > 0$ , ( $i = 1, \dots, m$ ) such that :

$$\sum_{j=1, j \neq o}^n \hat{\lambda}_j x_{ij}^U \leq x_{io}^L + \alpha_i^* - \varepsilon_i, \quad i = 1, \dots, m$$

$$\sum_{j=1, j \neq o}^n \hat{\lambda}_j y_{rj}^L \geq y_{ro}^U - \beta_r^*, \quad r = 1, \dots, s.$$

Let  $\tilde{\alpha}_i = \alpha_i^* - \varepsilon_i$  obviously,  $(\hat{\lambda}_1, \dots, \hat{\lambda}_n, \tilde{\alpha}_1, \dots, \tilde{\alpha}_m, \beta_1^*, \dots, \beta_s^*)$  is a feasible solution of model (3) we have  $\tilde{\alpha}_i < \alpha_i^*$  for all i and  $\beta_r^* = \beta_r^*$  for all r, which is a contradiction.

**Theorem 3.2.** *If  $DMU_o \in E^{++}$  and  $(\lambda_1^*, \dots, \lambda_n^*, \delta_1^*, \dots, \delta_m^*, \varphi_1^*, \dots, \varphi_s^*)$  is an optimal Pareto solution of (5) then  $\overline{DMU}_o$  with inputs  $[x_{io}^L, x_{io}^U + \delta_i^*]$ , ( $i = 1, \dots, m$ ) and outputs  $[y_{ro}^L - \varphi_r^*, y_{ro}^U]$ , ( $r = 1, \dots, s$ ) is in  $E^{++}$ , i.e.  $\theta_o^L = 1$ .*

**Proof.** Suppose  $(\hat{\lambda}, \theta_o^L)$  is an optimal value of model (2) when evaluating  $\overline{DMU}_o$  so we have

$$\begin{aligned} \hat{\lambda}_o(x_{io}^U + \delta_i^*) + \sum_{j=1, j \neq o}^n \hat{\lambda}_j x_{ij}^L &\leq \theta_o^L(x_{io}^U + \delta_i^*), \quad i = 1, \dots, m \\ \hat{\lambda}_o(y_{ro}^L - \varphi_r^*) + \sum_{j=1, j \neq o}^n \hat{\lambda}_j y_{rj}^U &\geq y_{ro}^L - \varphi_r^*, \quad r = 1, \dots, s. \end{aligned} \quad (8)$$

Suppose  $\theta_o^L < 1$  then  $\overline{DMU}_o$  is inefficient so  $\hat{\lambda}_o = 0$  we have:

$$\begin{aligned} \sum_{j=1, j \neq o}^n \hat{\lambda}_j x_{ij}^L &< x_{io}^U + \delta_i^*, \quad i = 1, \dots, m \\ \sum_{j=1, j \neq o}^n \hat{\lambda}_j y_{rj}^U &\geq y_{ro}^L - \varphi_r^*, \quad r = 1, \dots, s. \end{aligned} \quad (9)$$

Exist  $\varepsilon_i > 0$ , ( $i = 1, \dots, m$ ) such that :

$$\begin{aligned} \sum_{j=1, j \neq o}^n \hat{\lambda}_j x_{ij}^L &\leq x_{io}^U + \delta_i^* - \varepsilon_i, \quad i = 1, \dots, m \\ \sum_{j=1, j \neq o}^n \hat{\lambda}_j y_{rj}^U &\geq y_{ro}^L - \varphi_r^*, \quad r = 1, \dots, s. \end{aligned}$$

Let  $\tilde{\delta}_i = \delta_i^* - \varepsilon_i$  obviously,  $(\hat{\lambda}_1, \dots, \hat{\lambda}_n, \tilde{\delta}_1, \dots, \tilde{\delta}_m, \varphi_1^*, \dots, \varphi_s^*)$  is a feasible solution of model (5) we have  $\tilde{\delta}_i < \delta_i^*$  for all  $i$  and  $\varphi_r^* = \varphi_r^*$  for all  $r$ , which is a contradiction.

**Theorem 3.3.** If  $DMU_o \in E^{++}$  and  $(\lambda_1^*, \dots, \lambda_n^*, \alpha_1^*, \dots, \alpha_m^*, \beta_1^*, \dots, \beta_s^*)$  is an optimal Pareto solution of (3) then for any  $\alpha_i$ , ( $i = 1, \dots, m$ ) and  $\beta_r$ , ( $r = 1, \dots, s$ ), where  $\alpha_i \in [0, \alpha_i^*]$  ( $i = 1, \dots, m$ ) and  $\beta_r \in [0, \beta_r^*]$ , ( $r = 1, \dots, s$ ) if  $x_{io} \in [x_{io}^L + \alpha_i, x_{io}^U]$ , ( $i = 1, \dots, m$ ) and  $y_{ro} \in [y_{ro}^L, y_{ro}^U - \beta_r]$  ( $r = 1, \dots, s$ ) then  $DMU_o \in E^{++}$  i.e.  $\theta_o^U = 1$ .

**Proof.** The proof is analogous with that of theorem (3.1) and is omitted .

**Theorem 3.4.** If  $DMU_o \in E^{++}$  and  $(\lambda_1^*, \dots, \lambda_n^*, \delta_1^*, \dots, \delta_m^*, \varphi_1^*, \dots, \varphi_s^*)$  is an optimal Pareto solution of (5) then for any  $\delta_i$ , ( $i = 1, \dots, m$ ) and  $\varphi_r$ , ( $r = 1, \dots, s$ ), where  $\delta_i \in [0, \delta_i^*]$ , ( $i = 1, \dots, m$ ) and  $\varphi_r \in [0, \varphi_r^*]$ , ( $r = 1, \dots, s$ ) if

$x_{io} \in [x_{io}^L, x_{io}^U + \delta_i], (i = 1, \dots, m)$  and  $y_{ro} \in [y_{ro}^L - \varphi_i, y_{ro}^U] (r = 1, \dots, s)$  then  $DMU_o \in E^{++}$  i.e.  $\theta_o^L = 1$ .

**Proof.** The proof is analogous with that of theorem (3.2) and is omitted.

### 3.2 Radius of stability for DMU in $E^+$

In this case, we assume that  $DMU_o$  is in  $E^+$ , that is  $\theta_o^L < 1$  and  $\theta_o^U = 1$ . Our aim is to find the scalers  $\alpha_i, (i = 1, \dots, m), \beta_r, (r = 1, \dots, s), \delta_i, (i = 1, \dots, m)$  and  $\varphi_r, (r = 1, \dots, s)$  such that if we decrease rth upper bound of output of  $DMU_o$  by  $\beta_r$  and increase ith lower bound of input of  $DMU_o$  by  $\alpha_i$  then  $\theta_o^U = 1$ , also if we increase rth lower bound of output of  $DMU_o$  by  $\varphi_r$  and decrease ith upper bound of input of  $DMU_o$  by  $\delta_i$  then  $\theta_o^L < 1$ , i.e.  $DMU_o$  remains in  $E^+$ .

It has been assumed that  $\beta_r, \alpha_i, \varphi_r$  and  $\delta_i$  are scaler and non negative. Here we consider the following cases:

(1) It is obvious if  $x_{io}^L, (i = 1, \dots, m)$  are replace with  $x_{io}^L - \alpha_i, (i = 1, \dots, m)$  and  $y_{ro}^U, (r = 1, \dots, s)$  are replace with  $y_{ro}^U + \beta_r (r = 1, \dots, s)$  then  $\theta_o^U = 1$  and if  $X_o^U, Y_o^L$  are fixed, then  $\theta_o^L < 1$ ; consequently,  $DMU_o \in E^+$ .

(2) If  $x_{io}^U, (i = 1, \dots, m)$  are replace with  $x_{io}^U + \delta_i, (i = 1, \dots, m)$  and  $y_{ro}^L, (r = 1, \dots, s)$  are replace with  $y_{ro}^L - \varphi_r (r = 1, \dots, s)$  then  $\theta_o^L < 1$  and if  $X_o^L, Y_o^U$  are fixed, then  $\theta_o^U = 1$ ; consequently,  $DMU_o \in E^+$ .

(3) If  $x_{io}^L, (i = 1, \dots, m)$  are replace with  $x_{io}^L + \alpha_i (i = 1, \dots, m)$  and  $y_{ro}^U, (r = 1, \dots, s)$  are replace with  $y_{ro}^U - \beta_r (r = 1, \dots, s)$  then it is possible for  $DMU_o$  not to be in the  $E^+$ . We are concerned with finding the largest value for  $\beta_r$  and  $\alpha_i$  such that  $DMU_o \in E^+$ . For this purpose, the following model is proposed:

$$\begin{aligned}
 & \min \quad \{\alpha_1, \dots, \alpha_m, \beta_1, \dots, \beta_s\} \\
 & \text{s.t.} \quad \sum_{j=1}^n \lambda_j x_{ij}^U \leq x_{io}^L + \alpha_i, \quad i = 1, \dots, m \\
 & \quad \quad \sum_{j=1}^n \lambda_j y_{rj}^L \geq y_{ro}^U - \beta_r, \quad r = 1, \dots, s \\
 & \quad \quad 0 \leq \alpha_i \leq x_{io}^U - x_{io}^L, \quad i = 1, \dots, m \\
 & \quad \quad 0 \leq \beta_r \leq y_{ro}^U - y_{ro}^L, \quad r = 1, \dots, s \\
 & \quad \quad \lambda_j \geq 0, \quad j = 1, \dots, n
 \end{aligned} \tag{10}$$



We solve model (13) through the method of Weighted-sums. We have:

$$\begin{aligned}
 \min \quad & \sum_{i=1}^m W_i \alpha_i + \sum_{r=1}^s W'_r \beta_r \\
 \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij}^U \leq x_{io}^L + \alpha_i, \quad i = 1, \dots, m \\
 & \sum_{j=1}^n \lambda_j y_{rj}^L \geq y_{ro}^U - \beta_r, \quad r = 1, \dots, s \\
 & 0 \leq \alpha_i \leq x_{io}^U - x_{io}^L, \quad i = 1, \dots, m \\
 & 0 \leq \beta_r \leq y_{ro}^U - y_{ro}^L, \quad r = 1, \dots, s \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n
 \end{aligned} \tag{11}$$

(4) If  $x_{io}^U$ , ( $i = 1, \dots, m$ ) are replace with  $x_{io}^U - \delta_i$  ( $i = 1, \dots, m$ ) and  $y_{ro}^U$ , ( $r = 1, \dots, s$ ) are replace with  $y_{ro}^L + \varphi_r$  ( $r = 1, \dots, s$ ) then it is possible for  $DMU_o$  not to be in the  $E^+$ . We are concerned with finding the largest value for  $\varphi_r$  and  $\delta_i$  such that  $DMU_o \in E^+$ . For this purpose, the following model is proposed:

$$\begin{aligned}
 \max \quad & \{\delta_1, \dots, \delta_m, \varphi_1, \dots, \varphi_s\} \\
 \text{s.t.} \quad & \sum_{j=1, j \neq o}^n \lambda_j x_{ij}^L \leq x_{io}^U - \delta_i, \quad i = 1, \dots, m \\
 & \sum_{j=1, j \neq o}^n \lambda_j y_{rj}^U \geq y_{ro}^L + \varphi_r, \quad r = 1, \dots, s \\
 & 0 \leq \delta_i \leq x_{io}^U - x_{io}^L, \quad i = 1, \dots, m \\
 & 0 \leq \varphi_r \leq y_{ro}^U - y_{ro}^L, \quad r = 1, \dots, s \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n.
 \end{aligned} \tag{12}$$

We solve model (15) through the method of Weighted-sums. We have:

$$\begin{aligned}
 \max \quad & \sum_{i=1}^m W_i \delta_i + \sum_{r=1}^s W'_r \varphi_r \\
 \text{s.t.} \quad & \sum_{j=1, j \neq o}^n \lambda_j x_{ij}^L \leq x_{io}^U - \delta_i, \quad i = 1, \dots, m \\
 & \sum_{j=1, j \neq o}^n \lambda_j y_{rj}^U \geq y_{ro}^L + \varphi_r, \quad r = 1, \dots, s \\
 & 0 \leq \delta_i \leq x_{io}^U - x_{io}^L, \quad i = 1, \dots, m \\
 & 0 \leq \varphi_r \leq y_{ro}^U - y_{ro}^L, \quad r = 1, \dots, s \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n
 \end{aligned} \tag{13}$$

**Theorem 3.5.** *If  $DMU_o \in E^+$  and  $(\lambda_1^*, \dots, \lambda_n^*, \alpha_1^*, \dots, \alpha_m^*, \beta_1^*, \dots, \beta_s^*)$  is an optimal Pareto solution of (13) then  $\overline{DMU}_o$  with inputs  $[x_{io}^L + \alpha_i^*, x_{io}^U]$ , ( $i = 1, \dots, m$ ) and outputs  $[y_{ro}^L, y_{ro}^U - \beta_r^*]$ , ( $r = 1, \dots, s$ ) is in  $E^+$ , i.e.  $\theta_o^U = 1$ .*

**Proof.** *The proof is analogous with that of theorem (3.1) and is omitted.*

**Theorem 3.6.** *If  $DMU_o \in E^+$  and  $(\lambda_1^*, \dots, \lambda_n^*, \delta_1^*, \dots, \delta_m^*, \varphi_1^*, \dots, \varphi_s^*)$  is an optimal Pareto solution of (15) then  $\overline{DMU}_o$  with inputs  $[x_{io}^L, x_{io}^U - \delta_i], (\delta_i < \delta_i^* (i = 1, \dots, m))$  and outputs  $[y_{ro}^L + \varphi_r, y_{ro}^U], (\varphi_r < \varphi_r^* (r = 1, \dots, s))$  is in  $E^+$ , i.e.  $\theta_o^L < 1$ .*

**Proof.**  $(\lambda_1^*, \dots, \lambda_n^*, \delta_1^*, \dots, \delta_m^*, \varphi_1^*, \dots, \varphi_s^*)$  is optimal value of objective function(15) so we have

$$\sum_{j=1, j \neq o}^n \lambda_j^* x_{ij}^L \leq x_{io}^U - \delta_i^* < x_{io}^U - \delta_i, \quad i = 1, \dots, m$$

$$\sum_{j=1, j \neq o}^n \lambda_j^* y_{rj}^U \geq y_{ro}^L + \varphi_r^* > y_{ro}^L + \varphi_r, \quad r = 1, \dots, s.$$

So exist  $\hat{\theta}$  where  $0 < \hat{\theta} < 1$  such that

$$\sum_{j=1, j \neq o}^n \lambda_j^* x_{ij}^L \leq \hat{\theta}(x_{io}^U - \delta_i), \quad i = 1, \dots, m$$

$$\sum_{j=1, j \neq o}^n \lambda_j^* y_{rj}^U > y_{ro}^L + \varphi_r, \quad r = 1, \dots, s.$$

So  $(\hat{\theta}, \lambda^*)$  where  $\lambda_o^* = 0$  is a feasible solution of model (2). Obviously  $\theta_o^L \leq \hat{\theta} < 1$ .

**Theorem 3.7.** *If  $DMU_o \in E^+$  and  $(\lambda_1^*, \dots, \lambda_n^*, \alpha_1^*, \dots, \alpha_m^*, \beta_1^*, \dots, \beta_s^*)$  is an optimal Pareto solution of (13) then for any  $\alpha_i, (i = 1, \dots, m)$  and  $\beta_r, (r = 1, \dots, s)$ , where  $\alpha_i \in [0, \alpha_i^*] (i = 1, \dots, m)$  and  $\beta_r \in [0, \beta_r^*], (r = 1, \dots, s)$  if  $x_{io} \in [x_{io}^L + \alpha_i, x_{io}^U], (i = 1, \dots, m)$  and  $y_{ro} \in [y_{ro}^L, y_{ro}^U - \beta_r] (r = 1, \dots, s)$  then  $DMU_o \in E^+$  i.e.  $\theta_o^U = 1$ .*

**Proof.** The proof is analogous with that of theorem (3.5) and is omitted.

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### 3.3 Radius of stability for DMU in $E^-$

In this case, we assume that  $DMU_o$  is in  $E^-$ . Our aim is to find the scalers  $\alpha_i, (i = 1, \dots, m), \beta_r, (r = 1, \dots, s), \delta_i, (i = 1, \dots, m)$  and  $\varphi_r, (r = 1, \dots, s)$  such that if we increase rth upper bound of output of  $DMU_o$  by  $\beta_r$  and decrease

ith lower bound of input of  $DMU_o$  by  $\alpha_i$  then  $\theta_o^U < 1$ , also if we increase rth lower bound of output of  $DMU_o$  by  $\varphi_r$  and decrease ith upper bound of input of  $DMU_o$  by  $\delta_i$  then  $\theta_o^L < 1$ , i.e.  $DMU_o$  remains in  $E^-$ .

It has been assumed that  $\beta_r$ ,  $\alpha_i$ ,  $\varphi_r$  and  $\delta_i$  are scalar and non negative. Here we consider the following cases:

(1) It is obvious if  $x_{io}^L$ , ( $i = 1, \dots, m$ ) are replace with  $x_{io}^L + \alpha_i$ , ( $i = 1, \dots, m$ ) and  $y_{ro}^U$ , ( $r = 1, \dots, s$ ) are replace with  $y_{ro}^U - \beta_r$ , ( $r = 1, \dots, s$ ) then  $\theta_o^U < 1$  and if  $x_{io}^U$ , ( $i = 1, \dots, m$ ) are replace with  $x_{io}^U + \delta_i$  ( $i = 1, \dots, m$ ) and  $y_{ro}^L$ , ( $r = 1, \dots, s$ ) are replace with  $y_{ro}^L - \varphi_r$  ( $r = 1, \dots, s$ ) then  $\theta_o^L < 1$  consequently  $DMU_o$  is in  $E^-$ .

(2) If  $x_{io}^L$ , ( $i = 1, \dots, m$ ) are replace with  $x_{io}^L - \alpha_i$ , ( $i = 1, \dots, m$ ) and  $y_{ro}^U$ , ( $r = 1, \dots, s$ ) are replace with  $y_{ro}^U + \beta_r$ , ( $r = 1, \dots, s$ ) then it is possible for  $DMU_o$  not to be in the  $E^-$ . We are concerned with finding the largest value for  $\beta_r$  and  $\alpha_i$  such that  $DMU_o \in E^-$ . For this purpose, the following model is proposed:

$$\begin{aligned}
 & \max \quad (\alpha_1, \dots, \alpha_m, \beta_1, \dots, \beta_s) \\
 & \text{s.t.} \quad \sum_{j=1, j \neq o}^n \lambda_j x_{ij}^U \leq x_{io}^L - \alpha_i, \quad i = 1, \dots, m \\
 & \quad \quad \sum_{j=1, j \neq o}^n \lambda_j y_{rj}^L \geq y_{ro}^U + \beta_r, \quad r = 1, \dots, s \\
 & \quad \quad \alpha_i \geq 0, \quad i = 1, \dots, m \\
 & \quad \quad \beta_r \geq 0, \quad r = 1, \dots, s \\
 & \quad \quad \lambda_j \geq 0, \quad j = 1, \dots, n
 \end{aligned} \tag{14}$$

We solve model (19) through the method of Weighted-sums. We have:

$$\begin{aligned}
 & \max \quad \sum_{i=1}^m W_i \alpha_i + \sum_{r=1}^s W'_r \beta_r \\
 & \text{s.t.} \quad \sum_{j=1, j \neq o}^n \lambda_j x_{ij}^U \leq x_{io}^L - \alpha_i, \quad i = 1, \dots, m \\
 & \quad \quad \sum_{j=1, j \neq o}^n \lambda_j y_{rj}^L \geq y_{ro}^U + \beta_r, \quad r = 1, \dots, s \\
 & \quad \quad \alpha_i \geq 0, \quad i = 1, \dots, m \\
 & \quad \quad \beta_r \geq 0, \quad r = 1, \dots, s \\
 & \quad \quad \lambda_j \geq 0, \quad j = 1, \dots, n
 \end{aligned} \tag{15}$$

**Theorem 3.8.** *If  $DMU_o \in E^-$  and  $(\lambda_1^*, \dots, \lambda_n^*, \alpha_1^*, \dots, \alpha_m^*, \beta_1^*, \dots, \beta_s^*)$  is an optimal Pareto solution of (19) For any  $\alpha_i$ , ( $i = 1, \dots, m$ ) and  $\beta_r$ , ( $r =$*

$1, \dots, s$ ), where  $\alpha_i \in [0, \alpha_i^*)$ , ( $i = 1, \dots, m$ ) and  $\beta_r \in [0, \beta_r^*)$ , ( $r = 1, \dots, s$ ) if  $x_{io} \in [x_{io}^L - \alpha_i, x_{io}^U]$ , ( $i = 1, \dots, m$ ) and  $y_{ro} \in [y_{ro}^L, y_{ro}^U + \beta_r]$ , ( $r = 1, \dots, s$ ) then  $DMU_o \in E^-$  i.e.  $\theta_o^U < 1$ .

**Proof.**  $(\lambda_1^*, \dots, \lambda_n^*, \alpha_1^*, \dots, \alpha_m^*, \beta_1^*, \dots, \beta_s^*)$  is optimal value of objective function (19) so we have

$$\sum_{j=1, j \neq o}^n \lambda_j^* x_{ij}^U \leq x_{io}^L - \alpha_i^* < x_{io}^L - \alpha_i, \quad i = 1, \dots, m$$

$$\sum_{j=1, j \neq o}^n \lambda_j^* y_{rj}^L \geq y_{ro}^U + \beta_r^* > y_{ro}^U + \beta_r, \quad r = 1, \dots, s.$$

So exist  $\hat{\theta}$  where  $0 < \hat{\theta} < 1$  such that

$$\sum_{j=1, j \neq o}^n \lambda_j^* x_{ij}^U \leq \hat{\theta}(x_{io}^L - \alpha_i), \quad i = 1, \dots, m$$

$$\sum_{j=1, j \neq o}^n \lambda_j^* y_{rj}^L > y_{ro}^U + \beta_r, \quad r = 1, \dots, s.$$

So  $(\hat{\theta}, \lambda^*)$  where  $\lambda_o^* = 0$  is a feasible solution of model (1). Obviously  $\theta_o^U \leq \hat{\theta} < 1$ .

## 4 Another method for solving Sensitivity analysis models(MOLP), via STEM algorithm

For finding efficient solution of MOLP, the interactive methods are used according to decision maker comment; therefore, the interactive methods are argued such as SETEM, Z.W,... in [11].

In this paper, we solve the MOLPs which were obtained for estimating the sensitivity analysis and radius of stability via STEM algorithm. so; we argue about STEM algorithm for model (3) briefly as the following:

step 1: By individually optimizing each objective function, construct a payoff table to obtain the ideal criterion vector  $z^* \in R^{m+s}$ . Let  $z_i^* = \alpha_i^*$ , ( $i = 1, \dots, m$ ) (ith Optimal value of ith objective function) ,  $z_{m+i}^* = \beta_i^*$ , ( $i = 1, \dots, s$ )(i+m)th Optimal value of (i+m)th objective function) ,  $z_{ij}^* = \alpha_j^*$ , ( $j = 1, \dots, m$ ) (jth Optimal value of ith objective function ( $i = 1, \dots, m$ ))and  $z_{m+ij}^* = \beta_j^*$ , ( $j = 1, \dots, s$ )(jth Optimal value of (i+m)th objective function( $i = m + 1, \dots, m + s$ )) .

a payoff table is of the form

|            |            |          |             |
|------------|------------|----------|-------------|
| $z_1^*$    | $z_{12}$   |          | $z_{1m+s}$  |
| $z_{21}$   | $z_2^*$    |          | $z_{2m+s}$  |
| $\vdots$   | $\vdots$   | $\ddots$ | $\vdots$    |
| $z_{m+s1}$ | $z_{m+s2}$ |          | $z_{m+s}^*$ |

Table 1. The payoff table.

Where the rows are the criterion vectors resulting from individually optimizing each of the objective. The  $z_i^*$  entries along the main diagonal form the  $z^*$  ideal criterion vector.

step 2: Let iteration counter  $h = 0$  . let  $m_i$  be the minimum value in the  $i$ th column of the payoff table. Calculate  $\pi_i$  values where

$$\pi_i = \begin{cases} \frac{z_i^* - m_i}{z_i^*} & z_i^* > 0 \\ 1 & z_i^* = 0. \end{cases}$$

step 3: Let  $S^1 = S$  ( $S$  is feasible region of model (3)) and index set  $J^* = \emptyset$ .

step4: Let  $h = h + 1$  ,calculate  $\eta_i^h$ :

$$\eta_i^h = \begin{cases} \frac{\pi_i}{m+s} & \text{otherwise} \\ \sum_{i=1} \pi_i & i \in J^*. \\ 0 & \end{cases}$$

step 5: Solve the weighted min max program

$$\begin{aligned} \min \quad & \gamma \\ \text{s.t.} \quad & \gamma \geq \eta_i^h(z_i^* - \alpha_i), \quad i = 1, \dots, m \\ & \gamma \geq \eta_r^h(z_{m+r}^* - \beta_r), \quad r = 1, \dots, s \\ & \alpha_i \in S^h \quad i = 1, \dots, m \\ & \beta_r \in S^h \quad r = 1, \dots, s \\ & \gamma \geq 0 \end{aligned} \tag{16}$$

for decision space solution  $(\alpha_1^h, \dots, \alpha_m^h, \beta_1^h, \dots, \beta_s^h)$ .  
 Step 6: Let  $z^h = (\alpha_1^h, \dots, \alpha_m^h, \beta_1^h, \dots, \beta_s^h)$ . compare  $z^h$  with  $z^*$ .  
 Step 7: If all components of  $z^h$  are satisfactory, stop with  $(z^h, \lambda_1, \dots, \lambda_n)$  as the final solution. otherwise go to step 8.  
 step 8: Specify the index set  $J^*$  of criterion values to be relaxed and specify the amounts  $(\Delta_j, j \in J^*)$  by which they are to be relaxed.  
 step 9: Form reduced feasible region

$$S^{h+1} = \{(\lambda_1, \dots, \lambda_n, \alpha_1, \dots, \alpha_m, \beta_1, \dots, \beta_s) \in S \mid \text{if } j \in J^* : z_j \geq z_j^h - \Delta_j; \text{ otherwise : } z_j \geq z_j^h\}$$

then go to step 4.

All MOLP models can also be solved through STEM method which we have explained it's process.

## 5 Numerical example

Consider the interval data setting of Table 2 contains eight DMUs with two inputs and one output and efficiency scores obtained by applying (1) and (2).

|         | Input      |            |            |            | Output  |         | Efficiency Score |              |                |
|---------|------------|------------|------------|------------|---------|---------|------------------|--------------|----------------|
| $DMU_j$ | $x_{1j}^L$ | $x_{1j}^U$ | $x_{2j}^L$ | $x_{2j}^U$ | $y_j^L$ | $y_j^U$ | $\theta_j^L$     | $\theta_j^U$ | Classification |
| 1       | 4          | 6          | 3          | 5          | 0.1     | 0.25    | 0.173            | 0.842        | $E^-$          |
| 2       | 7          | 9          | 0.5        | 2.5        | 0.1     | 0.5     | 0.0162           | 0.5714       | $E^-$          |
| 3       | 6          | 8          | 1          | 2          | 3.5     | 3.75    | 1                | 1            | $E^{++}$       |
| 4       | 4          | 4.25       | 2          | 2.5        | 3       | 3.25    | 0.8544           | 1            | $E^+$          |
| 5       | 2          | 2.5        | 4          | 4.75       | 2.5     | 3       | 1                | 1            | $E^{++}$       |
| 6       | 10         | 12         | 1          | 3          | 2       | 4       | 0.2481           | 1            | $E^+$          |
| 7       | 12         | 14         | 1          | 3          | 0.5     | 1       | 0.0548           | 0.5714       | $E^-$          |
| 8       | 10         | 12         | 1.5        | 3.5        | 0.25    | 0.75    | 0.03             | 0.2854       | $E^-$          |

Table 2. Data of numerical example.

Considering the data from Table 2 and using models in section 3, we calculate the radius of stability for all DMUs as the following:

**(i) Radius of stability  $E^{++}$**

For determining radius of stability of  $DMU_3$  and  $DMU_5$ , we apply (3) and (5)(because  $DMU_3$  and  $DMU_5$  are in  $E^{++}$ ) the data is as shown in Table 3.

|         | Weighted -sums method |              |           |              |              |             | STEM method  |              |           |              |              |             |
|---------|-----------------------|--------------|-----------|--------------|--------------|-------------|--------------|--------------|-----------|--------------|--------------|-------------|
| $DMU_j$ | $\alpha_1^*$          | $\alpha_2^*$ | $\beta^*$ | $\delta_1^*$ | $\delta_2^*$ | $\varphi^*$ | $\alpha_1^*$ | $\alpha_2^*$ | $\beta^*$ | $\delta_1^*$ | $\delta_2^*$ | $\varphi^*$ |
| 3       | 2                     | 1            | 0.25      | 0            | 0            | 0           | 2            | 1            | 0.25      | 0            | 0            | 0           |
| 5       | 0.5                   | 0.75         | 0.5       | 0            | 0            | 0.4688      | 0.5          | 0.75         | 0.5       | 0            | 0            | 0           |

Table 3. The value of model(3)and model(5).

If  $x_{ij}^L$ , for  $i = 1, 2$  and  $j = 3, 5$  are increased by  $\alpha_i^*$  ( $i = 1, 2$ ) and  $y_j^U$  for  $j = 3, 5$  are decreased by  $\beta^*$ , and apply (1), then  $\theta_j^U = 1(j = 3, 5)$  and  $DMU_j(j = 3, 5)$  remains in  $E^{++}$ . If  $x_{ij}^U$ , for  $i = 1, 2$  and  $j = 3, 5$  are increased by  $\delta_i^*$  ( $i = 1, 2$ ) and  $y_j^U$  for  $j = 3, 5$  are decreased by  $\varphi^*$ , and apply (2), then  $\theta_j^L = 1(j = 3, 5)$  and  $DMU_j(j = 3, 5)$  remains in  $E^{++}$ .

|         | Input   |         | output | Efficiency | Input   |         | output | Efficiency |
|---------|---------|---------|--------|------------|---------|---------|--------|------------|
| $DMU_j$ | $x_1^L$ | $x_2^L$ | $y^U$  | $\theta^U$ | $x_1^U$ | $x_2^U$ | $y^L$  | $\theta^L$ |
| 3       | 8       | 2       | 3.5    | 1          | 8       | 2       | 3.5    | 1          |
| 5       | 2.5     | 4.75    | 2.5    | 1          | 2.5     | 7.75    | 2      | 1          |

Table 4. the sensitivity analysis result.

Table 4 reports the sensitivity analysis result for  $DMU_5$  and  $DMU_3$ .

**(ii)Radius of stability  $E^+$ .**

For determining radius of stability of  $DMU_4$  and  $DMU_6$ , we apply (13) and (15)(because  $DMU_4$  and  $DMU_6$  are in  $E^+$ ) the data is as shown in Table 5 and 6.

|         | Weighted-sums method |              |           |              | STEM method  |           |  |
|---------|----------------------|--------------|-----------|--------------|--------------|-----------|--|
| $DMU_j$ | $\alpha_1^*$         | $\alpha_2^*$ | $\beta^*$ | $\alpha_1^*$ | $\alpha_2^*$ | $\beta^*$ |  |
| 4       | 0.25                 | 0.5          | 0.25      | 0.25         | 0.5          | 0.25      |  |
| 6       | 0                    | 1.43         | 0         | 0            | 0.1285       | 0         |  |

Table 5. The value of model(13).

|         | Weighted-sums method |              |             | STEM method  |              |             |
|---------|----------------------|--------------|-------------|--------------|--------------|-------------|
| $DMU_j$ | $\delta_1^*$         | $\delta_2^*$ | $\varphi^*$ | $\delta_1^*$ | $\delta_2^*$ | $\varphi^*$ |
| 4       | 0.25                 | 0.5          | 0.1364      | 0.1765       | 0.5          | 0.1765      |
| 6       | 2                    | 1.9333       | 2           | 1.9474       | 1.9474       | 1.9474      |

Table 6. The value of model(15).

If  $x_{ij}^L$ , for  $i = 1, 2$  and  $j = 4, 6$  are increased by  $\alpha_i^*$  ( $i = 1, 2$ ) and  $y_j^U$  for  $j = 4, 6$  are decreased by  $\beta^*$ , and apply (1), then  $\theta_j^U = 1(j = 3, 5)$  and  $DMU_j(j = 4, 6)$  remains in  $E^+$ . If  $x_{ij}^U$ , for  $i = 1, 2$  and  $j = 4, 6$  are decreased by  $\delta_i$  ( $\delta_i < \delta_i^*$ ) ( $i = 1, 2$ ) and  $y_j^U$  for  $j = 4, 6$  are increased by  $\varphi$  ( $\varphi < \varphi^*$ ), and apply (2), then  $\theta_j^L < 1(j = 4, 6)$  and  $DMU_j(j = 4, 6)$ remains in  $E^+$ .

|         | Input   |         | output | Efficiency | Input   |         | output | Efficiency |
|---------|---------|---------|--------|------------|---------|---------|--------|------------|
| $DMU_j$ | $x_1^L$ | $x_2^L$ | $y^U$  | $\theta^U$ | $x_1^U$ | $x_2^U$ | $y^L$  | $\theta^L$ |
| 4       | 4.25    | 2.5     | 3      | 1          | 4.1     | 2.1     | 3.13   | 0.98       |
| 6       | 10      | 2.43    | 4      | 1          | 10.1    | 1.1     | 3.9    | 0.95       |

Table 7. the sensitivity analysis result.

Table 7, reports the sensitivity analysis result for  $DMU_4$  and  $DMU_6$ . In this case we assume  $\delta_1 = 0.24, \delta_2 = 0.4$  and  $\varphi = 0.13$  for  $DMU_4$  and  $\delta_1 = 1.9, \delta_2 = 1.9$  and  $\varphi = 1.9$  for  $DMU_6$ .

**(iii) Radius of stability  $E^-$  :**

For determining radius of stability of  $DMU_j$  ( $j = 1, 2, 7, 8$ ), we apply (19) (because  $DMU_j$  ( $j = 1, 2, 7, 8$ ) are in  $E^-$ ) the data is as shown in Table 8.

|         | Weighted-sums method |              |           | STEM method  |              |           |
|---------|----------------------|--------------|-----------|--------------|--------------|-----------|
| $DMU_j$ | $\alpha_1^*$         | $\alpha_2^*$ | $\beta^*$ | $\alpha_1^*$ | $\alpha_2^*$ | $\beta^*$ |
| 1       | 3.6458               | 2.7917       | 0         | 1.511        | 0.2583       | 2         |
| 2       | 6.2917               | 0.0333       | 0         | 5.5          | 0.0528       | 0.2135    |
| 7       | 10.5833              | 0.1667       | 0         | 9.5          | 0.039        | 0.3603    |
| 8       | 8.9375               | 0.875        | 0         | 7.5          | 0.1307       | 0.9343    |

Table 8. the value of model (19).

If  $x_{ij}^L$ , for  $i = 1, 2$  and  $j = 1, 2, 7, 8$  are decreased by  $\alpha_i$  ( $\alpha_i < \alpha_i^*$ ) ( $i = 1, 2$ ) and  $y_j^U$  for  $j = 1, 2, 7, 8$  are increased by  $\beta$  ( $\beta < \beta^*$ ), and apply (1), then  $\theta_j^U < 1$  ( $j = 1, 2, 7, 8$ ) and  $DMU_j$  ( $j = 1, 2, 7, 8$ ) remains in  $E^-$ .

|         | Input   |         | output | Efficiency |
|---------|---------|---------|--------|------------|
| $DMU_j$ | $x_1^L$ | $x_2^L$ | $y^U$  | $\theta^U$ |
| 1       | 0.4     | 0.3     | 0.25   | 0.84       |
| 2       | 0.8     | 0.5     | 0.5    | 0.84       |
| 7       | 1.5     | 0.9     | 1      | 0.94       |
| 8       | 1.1     | 0.7     | 0.75   | 0.95       |

Table 9. the sensitivity analysis result.

Table 9, reports the sensitivity analysis result for  $DMU_j$  ( $j = 1, 2, 7, 8$ ). In this case we assume  $\alpha_1 = 3.6, \alpha_2 = 2.7$  and  $\beta = 0$  for  $DMU_1, \alpha_1 = 6.2, \alpha_2 = 0$  and  $\beta = 0$  for  $DMU_2, \alpha_1 = 10.5, \alpha_2 = 0.1, \beta = 0$ , for  $DMU_7$  and  $\alpha_1 = 8.9, \alpha_2 = 0.8$  and  $\beta = 0$  for  $DMU_8$ .

## 6 conclusion

In this paper, for estimating the sensitivity analysis of DMUs in all the inputs and outputs; while the inputs and outputs are interval data, aMOLP problem will be suggested. We classify the DMUs then we obtained the radius of stability for all the different classification. So, by using efficient solutions, the sensitivity analysis can be obtained for inputs and outputs. In this case that



we want to use comment of Decision Maker for finding an efficient solution of MOLP, the STEM method which is an interactive method is used. But MOLP problem have many methods for solving for this reason we can not find the unique region. such a work with fuzzy data may be done and the model may be extended.

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