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A Comparison of the Effect Size Estimators in Meta-Analysis

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Abstract. The objective of a meta-analysis is usually to estimate the overall treatment effect and make inferences about the difference between the efffects of the two treatments. This article presents several forms of effect size estimators and compares these effect size estimators and the variance of overall treatment effect estimator within each group. as outcome measures, standarized difference is considered. Four modes of effect size estimators are discussed. Effect size estimators by Glass, Hedges, The Maximum Likelihood, and Shrunken Estimators of Effect Size are employed in this study. Finally, with the help of a software the results of these four effect size estimators are discussed. Estimators are illustrated using a comparison of the effectiveness of amlodipine and placebo on work capacity.

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1. *INTRODUCTION*

Meta-analysis was defined by Glass (1976) to be statistical analysis of large collection of analysis results from individual studies for the purpose of interpreting the findings. Meta -analysis may be broadly defined as the quantitative review and synthesis of the results of related but independent studies [1-2]. Such analysis have become increasingly popular in medical research where information about treatment efficiency is available from a number of clinical trials with inconclusive or inconsistent results [3]. In terms of a selected set of outcome measure for each chosen outcome measures, this paper aims at comparing the effects of the two treatments. The goal is usually to estimate and make inferences about the difference between the effects of the two treatments. There are three forms of data which are commonly encountered: binary data, ordinal data, and normally distributed data. In meta-analysis of studies which are measured on a continuous scale, there are two situations. In the first situation, all of the eligible studies use the same measure of the effect. Accordingly, absolute mean difference is used. As for the second situation, if a different instrument and different scales as effect measure are used, then standardized mean differences are employed [4]. In this paper the responses can be considered to be approximately normally distributed. Therefore as outcome measures standardized mean differences, commonly used effect size estimators, are used [5].The aim of this paper is to estimate four forms of effect size estimators and compare these effect size estimators and the variance of overall treatment effect estimator within each group. Deriving an improved tool for the part of the meta-analysis process in which no covariates are available for explaining the heterogeneity [6-8] of the study results is targeted. In this study the results of commonly used methods for effect size estimator with normal mean case can be very conservative. In the next section continuos data type effect size estimators are presented; in the following section model types in meta-analysis are presented; and examples to compare estimators are presented in the fourth section. In the last section the conclusions of comparing effect size estimators are presented.

2. DATA TYPE

Outcomes have been categorized into one of three groups depending on the type of data from which they are derived: binary data, ordinal data, and normally distributed data

2.1. **Binary Data.** A binary variable takes one of two possible values commonly referred to as "success or failure", or patients "alive or dead", or "diseased/nondiseased". A binary outcome is recorded for each patient. The underlying model for the data recorded from one study is that patients in the experimental group succeed with probability p_E and patients in the control group succeed with probability p_C. When the response is a binary variable

knowledge of the individual patient, data adds nothing to the summary shown in Table1

Table1. data for a parallel group study with a binary outcome

	Experimental	Control	
Outcome	Group	Group	Total
Success (A live)	b	d	b+d
Failure (Dead)	a	c	a+c
Total	n_E	n_C	N

For binary data there are three measures of treatment difference which are Risk Difference, Relative Risk, and Odds Ratio. These measures of parameter estimator and standard error are given Table 2.

Table 2. Study summaries, E denotes experimental group, C denotes control group, $q_i = 1 - p_i$ n_{E_i} and n_{C_i} denote the total number of treated and control patients, respectively; and a, b, c, and d denote the number of observations in each of the cells defined by experimental and outcome

	Risk Difference	Relative Risk	Odds Ratio
Parameter	$RD = P_E - P_C$	$RR = P_E/P_C$	$OR = \frac{P_E(1-P_E)}{P_C(1-P_C)}$
Estimator	$d_i = \hat{p}_{E_i} - \hat{p}_{C_i}$	$y_i = \hat{p}_{E_i}/\hat{p}_{C_i}$	$w_i = \frac{\hat{p}_{E_i}\hat{q}_{C_i}}{\hat{q}_{E_i}\hat{p}_{C_i}}$
Standart Error	$s_{d_i} = \sqrt{\frac{P_{E_i}q_{E_i}}{n_{E_i}} + \frac{P_{C_i}q_{C_i}}{n_{C_i}}}$	$s_{\log(r_i)} = \sqrt{\left(\frac{q_{e_i}}{n_{e_i}p_{E_i}} + \frac{q_{C_i}}{n_{C_i}p_{C_i}}\right)}$	$s_{\log(w_i)} = \sqrt{\frac{1}{n_a} + \frac{1}{n_b} + \frac{1}{n_c} + \frac{1}{n_d}}$

2.2. **Ordinal Data.** Patients response one of m categories $C_1, C_2, ..., C_m$ which are ordered in terms of desirability: C_1 is the best C_m is the worst .The data can be presented in the form of an $m \times 2$ table as shown Table 3.

Table 3. Data for a parallel group study with an ordinal outcome

Number of patients			
in category	Experimental	Control	Total
C_1	n_{E_1}	n_{C_1}	n_1
.	.	.	.
.	.	.	.
.	.	.	.
C_m	n_{E_m}	n_{C_m}	n_m
Total	n_E	n_C	N

Two measures of treatment difference will be considered for ordinal data. The first is a log odds ratio based on the proportional odds model; the second model a log odds ratio based on the continuation ratio model.

2.3. **Normally Distributed Data.** A quantitative measurement on a continuous scale can often be treated as following a normal distribution. Data from subjects in the experimental group $Y_{E_1}, Y_{E_2}, ..., Y_{E_{n_E}}$ are modeled as being normally distributed with mean μ_E and common variance σ^2.Similarly the control groups $Y_{C_1}, Y_{C_2}, ..., Y_{C_{n_C}}$ are normally distributed with mean μ_C and common variance σ^2, using $N(\mu, \sigma^2)$ to denote normally distributed with mean μ and variances σ

$Y_{E_j} \sim N\ (\mu_E, \sigma^2)$; $j = 1, 2, ..., n_E$ (1)

$Y_{C_j} \sim N\ (\mu_C, \sigma^2)$; $j = 1, 2, ..., n_C$ (2)

the combination of the results of k controlled clinical trials or epidemiological trials is given in Table 4 where in each study or trial an experimental observation

is shown.

Table 4. Normally distributed data from the i.th study

i. study	Treatment			Overall
		E*	C*	
Model	mean	μ_{E_i}	μ_{C_i}	μ_i
	standart deviation	σ_{E_i}	σ_{C_i}	σ_i
Data	number	n_{E_i}	n_{C_i}	N_i
	mean	$\bar{Y}_{E_j}$	$\bar{Y}_{C_j}$	$\bar{Y}_j$
	standart deviation	s_{E_i}	s_{C_i}	s_i

* Experimental

** Control

An experimental group (E) is compared to a control group (C), as notations those given by Whitehead and Whitehead are used [9].

Then absolute mean difference is defined as

$$\theta_i = \mu_E - \mu_C \tag{3}$$

the effect size is defined as

$$\delta = \frac{\mu_E - \mu_C}{\sigma} \tag{4}$$

Effect sizes are natural parameters for use in the synthesis of experimental results. The hypothesis that there is no overall treatment effect for absolute mean differences is tested. $Y = \mu_E - \mu_C$, for effect size $\delta = \frac{\mu_E - \mu_C}{\sigma}$ the hypotheses below are considered respectively.

$H_0 : \theta = 0$ and $H_0 : \delta = 0$

2.3.1. *Absolute difference between means.* All of eligible studies use the same measure of effect. For instance, all of the studies may measure the effect of the intervention on blood pressure or serum cholesterol level. In this situation as measure absolute mean difference is used. As outcome measures, absolute difference between means which can be normally distributed responses is formulated as

$$\theta_i = \bar{Y}_{E_i} - \bar{Y}_{C_i} \sim N\ (\theta, \frac{\sigma_{ei}^2}{n_{E_i}} + \frac{\sigma_{\mathrm{Ci}}^2}{n_{C_i}} + \tau^2) \qquad i = 1, 2, ..., k \tag{5}$$

if it is assumed that the variance of both experimental groups and control groups has equal variances, the variance of Y_i can be written as $\sigma_i^2(1/n_E + 1/n_C) + \tau^2$.

where τ^2 is the variance of unexplained heterogeneity between the studies and positive. If $\tau^2 = 0$ then this model is called fixed effects model of meta-analysis; otherwise the model is called random-effects model of meta-analysis [3, 5, 12].

Effect Size Estimator. In certain situations there are several studies measuring outcomes which are similar but not exactly the same, or the studies measure the same end point but under different circumstances. One solution to this problem is to create an outcome measure which does not depend on the scale of measurements. The method of creating such a measure has been termed as the method of "effect size". In meta-analysis of studies where effect size is measured on a continuous scale, all of eligible studies use different instruments and thus different scales. In these situations effect size as measure can be used.

2.3.2. *An Estimator of Effect Size Based on Standardized Mean Difference.* Effect sizes are natural parameters for use in the synthesis of experimental results. There are several alternative point estimators of the effect size δ. These estimators are based on the sample standardized mean difference but differ by multicative constants that depend on the sample sizes involved. The standardized difference $\delta = \frac{\mu_E - \mu_C}{\sigma}$ is estimated in each study by

$$g = \frac{\mu_E - \mu_C}{s} \tag{6}$$

where s is the pooled sample standard deviation,

$$s = \sqrt{\frac{(n_E-1)(s_E)^2+(n_C-1)(s_C)^2}{n_E+n_C-2}} \tag{7}$$

g represents the gain (or loss) as the fraction of the variability of the measurements [13].

Unbiased Estimator of Effect Size. Unbiased estimator is described by Hedges and Olkin. It is given by,

$$g^* = \left(1 - \frac{3}{4N-9}\right) g \tag{8}$$

where $N = n_E + n_C$, g^* is an approximation of the unbiased estimator of the standardized difference between means (g) is proposed by hedges (Hedges). As estimates of the variance of the optimal estimator for standardized difference $g^*$is proposed by Hedges and Olkin [10].

$$var\,(g^*) = \frac{\tilde{n}}{N} + \frac{(\hat{g}^*)^2}{2N} \tag{9}$$

where $\tilde{n} = n_E n_C$, weight is given by

$$\hat{w}_i = \left(\frac{\tilde{n}}{N} + \frac{1}{2}\frac{(g_i^*)^2}{N} + \hat{\tau}^2\right)^{-1} \tag{10}$$

where $\hat{\tau}^2$ denotes an estimator of the between-study variance.

The Maximum Likelihood Estimator of the Effect Size. Maximum likelihood estimates have the advantage of being consistent and asymptotically efficient. The maximum likelihood estimator of $\hat{g}$ of δ is given by

$$\hat{g} = \frac{\mu_E - \mu_C}{s}\sqrt{\frac{N}{N-2}} = \sqrt{\frac{N}{N-2}}g \tag{11}$$

and the variance of the maximum likelihood estimator is given by

$$var(\hat{g}) = \frac{N}{N-2}\left[\frac{n^*}{(n^*-2)\tilde{n}}\right] + \delta^2\left[\frac{n^*}{(n^*-2)} - \frac{1}{J(n^*)^2}\right] \tag{12}$$

where $J(m)$ is a constant .The constant $J(m)$ is less than unity and approaches unity when m is large. Values for $J(m)$ is given in Table 5 and an approximate calculation is made using

$$J(m) = 1 - \frac{3}{4m-1} \tag{13}$$

Table 5.Exact values of the bias correction factor

m	$J(m)$	m	$J(m)$	m	$J(m)$	m	$J(m)$
2	0,5642	15	0,9490	28	0,9729	41	0,9816
3	0,7236	16	0,9523	29	0,9739	42	0,9820
4	0,7979	17	0,9551	30	0,9748	43	0,9824
5	0,8408	18	0,9577	31	0,9756	44	0,9828
6	0,8686	19	0,9599	32	0,9764	45	0,9832
7	0,8882	20	0,9619	33	0,9771	46	0,9836
8	0,9027	21	0,9638	34	0,9778	47	0,9839
9	0,9139	22	0,9655	35	0,9784	48	0,9843
10	0,9228	23	0,9670	36	0,9790	49	0,9846
11	0,9300	24	0,9684	37	0,9796	50	0,9849
12	0,9359	25	0,9699	38	0,9801		
13	0,9410	26	0,9708	39	0,9806		
14	0,9453	27	0,9719	40	0,9811		

Shrunken Estimators of the Effect Size. The minimum variance unbiased estimator need not be the minimum mean squared error estimator. The principle in these shrunken estimates is that the increase in the bias term of the mean-squared is more than compensated for the by reduction of the variance term of mean-squared error [10]. The minimum variance unbiased estimator of δ is dominated by shrunken estimator which is denoted by $\tilde{g}$ and is given by

$$\tilde{g} = \frac{N-4}{N-2}\frac{g}{J(N-2)^2} \tag{14}$$

and the variance of the shrunken estimator is given by

$$var(\tilde{g}) = (\frac{N-4}{N-2})^2\frac{1}{J(N-2)^2}\frac{n^*}{(n^*-2)\tilde{n}} + \delta^2\frac{n^*}{(n^*-2)} - \frac{1}{J(n^*)^2} \tag{15}$$

where $n^* = n_E + n_C - 2$

3. MODEL TYPES

There are two models in meta-analysis. The first model is fixed effects model which assumes that studies being modeled are homogeneous; namely there are no differences in underlying study populations. The second model is random effects model that assumes that the studies estimate different effect sizes and take into account the extra variation implied in making this assumption. These underlying effects are assumed to vary at random typically; the distribution of such effects is assumed to be normally distributed. This model includes two sources of variation: the between and within study variance [11].

3.1. **FIXED EFFECTS MODEL.**

Fixed effects models for meta-analysis according to which the modeled studies are homogeneous. There are no differences in underlying study populations and in patient-selection criteria that might affect response to therapy, and the therapies are applied in the same way [12].

let θ_i be a sufficient statistics for the effect of interest. For large individual-study sample sizes, the response and the individual study approximately have normal distribution. Thus, the general fixed effects model is given by

$$\hat{\theta}_i = \theta + \varepsilon_i \tag{16}$$

where θ represents the mean of study effects, and ε_i is error terms and realizations of normally distributed random variables with expected value 0 and variance denoted by σ_i^2, which might be shortened as follows:

$$\hat{\theta}_i \sim N\ (\theta, \sigma_i^2) \tag{17}$$

where σ_i is the standard deviation of the response. Let w_i be the estimated inverse variance of $\hat{\theta}_i$, that is $w_i = 1/var(\hat{\theta}_i)$

[12-13].

$$\hat{\theta}^*_{FEM} = \frac{\sum \hat{\theta}_i w_i}{\sum w_i} \tag{18}$$

where $\hat{\theta}$ is estimator of the overall fixed effect θ. Standard error is given by

$$se(\ \hat{\theta}^*_{FEM}) = \sqrt{\frac{1}{\sum w_i}} \tag{19}$$

and an approximate 95 % confidence interval (CI) for θ is given by

$$\hat{\theta}^*_{FEM} \pm 1,96 \sqrt{\frac{1}{\sum w_i^*}} \tag{20}$$

3.2. **RANDOM-EFFECTS MODEL.** The random-effects model have been advocated as a more conservative model. The random effects formulations avoid homogeneity assumption. This approach assumes that based on the studies different effect sizes are estimated and the extra variation implied in making this assumption is taken into account. More specifically, this model includes two sources of variation: the between and within study variance. In the random effects model it is assumed that treatment difference parameters in k studies $(\theta_1, ..., \theta_k)$ are sample of independent observations from $N\ (\theta, \tau^2)$. The general random effects model is given by

$$\hat{\theta}_i = \theta + \nu_i + \varepsilon_i \tag{21}$$

for $i = 1, 2, ..., k$, where the ν_i are normally distributed random effects with mean 0 and variance τ^2.

$$\hat{\theta}_i \sim N\ (\theta, \sigma^2 + \tau^2) \tag{22}$$

where τ^2 is unknown and must be estimated from the data and $\tau^2 > \mathbf{0}$ is the true-between study variances. The parameters τ^2 is also named as the variance of a random interaction of response by centres or studies. There are several estimators for between study variance τ^2. Some of these estimators are iterative, whereas other estimators are non iterative. As non iterative estimator, Der-simonian Laird is a widely used estimator, and heterogeneity variances estimation is estimated by this estimator. Therefore estimate of treatment difference parameters may be written

$$\hat{\theta}_i \sim N\ \left(\theta, w_i^{-1} + \hat{\tau}^2\right) \tag{23}$$

where $w_i = 1/\sigma^2, \hat{\tau}^2$ is an estimate of τ^2. By setting

$$w_i^* = \left(w_i^{-1} + \hat{\tau}^2\right)^{-1} \tag{24}$$

$$\hat{\theta}_i \sim N\ (\theta, (w_i^*)^{-1}) \tag{25}$$

$$\hat{\theta}^*_{REM} = \frac{\sum \hat{\theta}_i w_i^*}{\sum w_i^*} \tag{26}$$

where $\hat{\theta}^*$ is asymptotically unbiased for θ, with variance approximately equal to $(w_i^*)^{-1}$. The term $(w_i^*)^{-1}$as if it were the true variance of $\hat{\theta}_i$ provides the test statistics

$$U^* = \frac{\sum (\hat{\theta}_i w_i^*)^2}{\sum w_i^*} \tag{27}$$

U^* has a chi-squared distribution with one degree of freedom under null hypothesis of no treatment difference $(\theta = 0)$. The estimated component of variance due to inter-study variation in effect size $\hat{\tau}^2$ is calculated as

$$\hat{\tau}^2 = 0 \quad \text{if} \quad Q \leq k-1$$

and

$$\hat{\tau}^2 = (Q - (k-1))/\ U \quad \text{if} \quad Q \geq k-1$$

where Q is the heterogeneity test statistics defined as

$$Q = \sum w_i \left(\hat{\theta}_i - \hat{\theta}\right)^2 \tag{28}$$

the standard error is given by

$$se(\ \hat{\theta}^*_{REM}) = \sqrt{\frac{1}{\sum w_i^*}} \tag{29}$$

and an approximate 95 % confidence interval is given by

$$\hat{\theta}^*_{REM} \pm 1,96 \sqrt{\frac{1}{\sum w_i^*}} \tag{30}$$

4. AN ILLUSTRATIVE EXAMPLE

In section 3 methods to illustrate as an example are given. The data set given by Li et al is used [14] which is presented in Table 6. There are eight randomized clinical trials. The effectiveness of a drug called amlodipine was tested in the treatment of angina against placebo. The response variable was the change in work capacity for each patient given as the ratio of the exercise time after the patient received the intervention (drug or placebo) before patient received intervention. The logarithm of the changes are assumed to be normally distributed.

Table 6. Results of eight randomized controlled clinical trials comparing the effectiveness of amlodipine and a placebo on work capacity

study number	Amlopidipine 10 mg(E)			Placebo (C)		
	n_{E_i}	$\bar{Y}_{E_i}$	$(s_{E_i})^2$	n_{C_i}	$\bar{Y}_{C_i}$	$(s_{C_i})^2$
1	46	0,2316	0,2254	48	-0,0027	0,0007
2	30	0,2811	0,1441	26	0,027	0,1139
3	75	0,1894	0,1981	72	0,0443	0,4972
4	12	0,093	0,1389	12	0,2277	0,0488
5	32	0,1622	0,0961	34	0,0056	0,0955
6	31	0,1837	0,1246	31	0,0943	0,1734
7	27	0,6612	0,706	27	-0,0057	0,9891
8	46	0,1366	0,1211	47	-0,057	0,1291

Regarding the results in Table 6 we can see difference of treatment and control group variance. For example in study 1 we have $(s_{E_1})^2 = 0,2254$ and $(s_{C_1})^2 = 0,0007$ similarly in study 3 $(s_{E_3})^2 = 0,1894$ and$(s_{C_3})^2 = 0,4972$. The results of in Table 3 there are difference treatment and control group. Clearly we can say study 1, study3, study 5 there are difference between groups so that equal variances assumptions is not convenient for this study.

Table 7.Estimates of effect size estimators and the corresponding variance estimates in eight randomized controlled clinical trials [14]

Study	Glass		Hedges		Max.Lik		Shrunken	
number	(g)	$Var\,(g)$	(g^*)	$Var\,(g^*)$	$(\hat{g})$	$Var\,(\hat{g})$	$(\tilde{g})$	$Var\,(\tilde{g})$
1	0,7044	0,066361	0,6988	0,04521	0,7199	0,046977	0,7479	0,044
2	0,7044	0,111951	0,6946	0,07623	0,7305	0,081391	0,7800	0,072778
3	0,2472	0,029099	0,2459	0,02743	0,2506	0,028103	0,2568	0,026961
4	−0,4397	0,204315	−0,4245	0,17070	−0,4797	0,201203	−0,5654	0,152427
5	0,5059	0,078191	0,5000	0,62600	0,5218	0,066155	0,5514	0,060203
6	0,2316	0,068481	0,2287	0,06495	0,2393	0,068891	0,2539	0,062299
7	0,7244	0,117915	0,7141	0,07893	0,7522	0,084497	0,8053	0,075229
8	0,4022	0,050866	0,3990	0,04387	0,4110	0,045618	0,4273	0,042696

The estimated outcome measures effect size estimator by Glass, unbiased effect size estimator (Hedges), the maximum likelihood estimator, and shrunken estimators and corresponding variance estimation in each study are given Table 7. In study 4 it is observed that the results of estimate are negative. Accordingly outcome measures yield results in favour of placebo where in all other studies the drug is better one. The four effect size estimators were ordered except for sign so that it can be easily seen that

$$g^* < g < \hat{g} < \tilde{g}.$$

it follows that variances are ordered in the same way

$$Var(\tilde{g}) < Var(g^*) < Var(\hat{g}) < Var(g)$$

The estimated outcome measures effect size estimator by Glass, based on standardized unbiased effect size estimator, the maximum likelihood estimator, and Shrunken estimators both fixed and random effects estimate, heterogeneity

variance or corresponding to variance of between study and confidence interval in each study are given in Table 8.

The results of estimate outcome measures and confidence interval for fixed and random effects are nearly similar. Considering the confidence intervals, fixed effect confidence interval is narrower than the random effects confidence interval for all methods. For example, for Glass estimator has $(0, 26996 ; 0, 60484)$ for fixed-effects model $(0, 24689 ; 0, 63662)$ for random-effects model. For unbiased effect size estimator (Hedges) it has confidence interval $(0, 26103 ; 0, 58699)$ for fixed effects model, for other model has $(0, 22672 ; 0, 624592)$.Similarly the confidence intervals are examined for maximum likelihood estimator with confidence interval $(0, 26996 ; 0, 60484)$ for fixed-effects model $(0, 24689 ; 0, 63662)$ for random-effects model.

For Shrunken Estimator with confidence interval, the confidence intervals are $(0, 00668 ; 0, 081787)$ for fixed-effects model and $(0, 23949 ; 0, 56729)$ for random-effects model. As the four modes of effect size estimator have about the same confidence interval it can be inferred that almost all these estimators have the same confidence interval. First confidence interval of fixed effect is narrower than that of random effects interval for effects size estimator. Secondly if the performance of effect size estimators are compared with one another, shrunken estimator has the narrowest interval. Compared to the other effect size estimators, this difference is almost negligible.The test for heterogeneity value of $Q = 9, 7611 ; k - 1 = 7$ is found.

Table 8. Results of four effect size estimators for fixed and random effects model with confidence interval

	Fixed		Random		
	Effects	95%CI	Effects	95% CI	
	Estimate	(Fixed effects)	Estimate	(Random effects)	$\hat{\tau}^2$
Effect size estimator					
by Glass	$0, 03969$	$(0, 21712; 0, 57678)$	$0, 4026$	$(0, 20742; 0, 59761)$	$0, 00934$
Standardized					
difference (Hedges)	$0, 4240$	$(0, 26103; 0, 58699)$	$0, 4256$	$(0, 22672; 0, 6246)$	$0, 02326$
Maximum					
Likelihood Estimator	$0, 4371$	$(0, 26996; 0, 60424)$	$0, 4418$	$(0, 24689; 0, 63662)$	$0, 01759$
Shrunken Estimator	$0, 416$	$(0, 25571; 0, 576326)$	$0, 4034$	$(0, 23949; 0, 56729)$	$0, 01753$

In Table 8 a tabulation of four estimators for fixed and random effects model is given. Heterogeneity variance estimation of τ^2 by using the Der-Simonian Laird estimator by Glass is $\hat{\tau}^2 = 0, 00934$;by using unbiased effect size estimator by Hedges $\hat{\tau}^2 = 0, 02326$;by using maximum likelihood effect size estimator with $\hat{\tau}^2 = 0, 01759$,Shrunken effect size estimator with $\hat{\tau}^2 = 0, 01753$. In this table maximum likelihood and Shrunken effect size estimator have nearly the same $\hat{\tau}^2$ value while the smallest value of estimator of heterogeneity variance value is found with Glass. In order to evaluate these results, some criteria should be taken into consideration. In this paper two criteria for the evaluation of the performance of effect size estimator are defined: confidence interval and heterogeneity variance estimation. If the first criterion is used, Glass estimator is preferred; whereas if the second criterion is used, Shrunken with the smallest variance of effect size estimator is preferred.

5. CONCLUSION

In this paper the four effect size estimators are used in both fixed and random-effects meta-analysis models. Like Glass and Hedges, two commonly used effect size estimators in literature, it is also easy to apply Maximum likelihood and Shrunken effect size estimator. The effect size estimators were applied to one example of meta-analytical data to obtain inference for an overall effect and confidence interval and to estimate heterogeneity parameter $\hat{\tau}^2$. For the data set, the estimates of $\hat{\tau}^2$ for the Maximum likelihood and Shrunken were comparable with the results of existing commonly used effect size estimators like Glass and Hedges. Thus the meta-analytical inference for the effectiveness of amlodipine and placebo on work capacity was made by using the effect size estimators Glass, Hedges, Maximum Likelihood and Shrunken methods. To estimate τ^2 Der-Simonian Laird estimator is used since it does not need iterative and also is easy to calculate. The effectiveness of amlodipine and placebo on work capacity studies were carried out in order to assess the performance of the four effect size estimators based on the Der-Simonian Laird estimator. The study showed that Glass estimator has the smallest heterogeneity variance in all effect size estimators and that the Shrunken estimator of effect size has approximately the nearly same heterogeneity variance estimation than that of Maximum likelihood estimator. Shrunken estimator performs well in terms of variance and the smallest confidence interval. the difference in performance of the estimators are small for 16 or more degrees freedom . The differences are appreciable only for small degrees of freedom which is unrealistics in the most applications. Many applications involve sample sizes at least 10 subjects per group and these cases the differences among are neglible.[10,15].

In summary, Glass estimator is used in many applications. Especially, with small sample sizes (at least 10 subjects per group) it has better empirical properties than the effect size estimators. However, the Hedges estimator may be preferred over the other effect size estimators when sample size is small. The difference among these four estimators is negligible.

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