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Combined Space-Time Manifold and the Quantum Geometry

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Abstract

In a previous article we defined "combined manifold" M3 as the graph of a diffeomorphism from one manifold M1 to another M2, akin to the idea of a diagonal map. In this paper, we derive the values for the previously undetermined two parameters: (1) the energy distribution between a particle in M1 and its accompanied electromagnetic wave in M2 for the combined entity - - [particle, wave], and (2) the gravitational constant G2 for M2, where there exist only electromagnetic waves and gravitational forces. Because of a large G2, an astronomical black hole B arose in M2, branching out M1 (the Big Bang), with a fraction of a wave energy in M2 transferred to M1 as a photon, which collectively were responsible for the subsequent formation of matter. Being within the Schwarzschild radius, B in M2 is a complex (sub) manifold, which furnishes exactly the geometry for the observed quantum mechanics; moreover, B provides an energy interpretation to probabilities. In summary, our M3 casts quantum mechanics in the framework of General Relativity.

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1 Introduction

Previously we introduced in this publication [8] (cf. also [9]) a construct analogous to a diagonal map, namely, a combined space-time 4-manifold $\mathcal{M}^{[3]}$

$$:= \left\{ \left(p^{[1]}, p^{[2]}\right) \in \mathcal{M}^{[1]} \times \mathcal{M}^{[2]} \mid h\left(p^{[1]}\right) = p^{[2]}, h = \text{any diffeomorphism} \right\}, \quad (1)$$

where manifold $\mathcal{M}^{[i]}$, $i = 1, 2$, is determined by Einstein Field Equations

$$R^{[i]}_{\mu\nu} - \frac{1}{2}R^{[i]}g^{[i]}_{\mu\nu} = -\frac{8\pi G^{[i]}}{c^2}T^{[i]}_{\mu\nu} \tag{2}$$

($R_{\mu\nu} \equiv$ the Ricci $(0, 2)$ curvature tensor, $R \equiv$ the Ricci scalar curvature tensor, $g_{\mu\nu} \equiv$ a semi-Riemannian metric, $\{\mu, \nu\} \subset \{1, 2, 3, 4\}$, $G^{[i]} \equiv$ the gravitational constant in $\mathcal{M}^{[i]}$, $c \equiv$ the speed of light in empty spaces, $T_{\mu\nu} \equiv$ the energy-momentum tensor), and we proposed the entity of a "combined particle j" of energy

$$E^{[3]}_j = E^{[1]}_j + E^{[2]}_j, \tag{3}$$

where $E^{[1]}_j$ and $E^{[2]}_j$ contribute respectively to $T^{[1]}$ of $\mathcal{M}^{[1]}$ and $T^{[2]}$ of $\mathcal{M}^{[2]}$, with $\left\{E^{[1]}_j \mid j \in \mathbb{N}\right\}$ engaging in all the fundamental forces within $\mathcal{M}^{[1]}$ and $\left\{E^{[2]}_j\right\}$ engaging only in gravitational forces within $\mathcal{M}^{[2]}$. We did not however specify the ratio $E^{[1]}_j/E^{[3]}_j \equiv 1 - \left(E^{[2]}_j/E^{[3]}_j\right)$, but here in this paper we shall settle this ratio in the following Section 2 and moreover equate $\left[E^{[1]}_j, E^{[2]}_j\right]$ to the [*particle energy*, *wave energy*] of j. We shall also resolve the other previously undetermined parameter $G^{[2]}$, even though we had provided the relation

$$G^{[3]} = \frac{G^{[1]}G^{[2]}}{G^{[1]} + G^{[2]}}, \tag{4}$$

by a consideration of the form invariance of the time-time component of g $\forall i = 1, 2, 3$,

$$g^{[i]}_{tt} = 1 - \frac{2G^{[i]}M^{[i]}}{rc^2}, \tag{5}$$

where $M^{[i]} \equiv$ the total mass in $\mathcal{M}^{[i]}$, and $r \equiv$ the radius of the space in $\mathcal{M}^{[i]}$.

Upon settling $\left(E^{[1]}_j/E^{[2]}_j\right)$ and $G^{[2]}$, we shall next establish two propositions: (1) The wave part of a combined particle is just the particle's wave function ψ, but we shall add an energy interpretation to ψ. (2) The probability current is just a Poynting vector in $\mathcal{M}^{[2]}$. After remarking on the significance of our analysis, we shall conclude with a summary in Section 3.

2 The Quantum Geometry

Hypothesis: $\mathcal{M}^{[2]}$ consists solely of electromagnetic waves as described by Maxwell Equations for free space; $\mathcal{M}^{[2]}$ predates $\mathcal{M}^{[1]}$.

Due to a large $G^{[2]}$, an astronomical black hole $\mathbf{B} \subset \mathcal{M}^{[2]}$ came into being (cf. e.g., [2, 12], for formation of space-time singularities in Einstein manifolds), and resulted in $\mathcal{M}^{[1]} \times \mathbf{B}$ (the Big Bang; cf. e.g., [5], for how a black hole may give rise to a macroscopic universe): photons then emerged in $\mathcal{M}^{[1]}$ with their associated remainder electromagnetic waves existing in $\mathbf{B}$. Any energy entity j in $\mathcal{M}^{[1]}$ is a particle resulting from a superposition of electromagnetic waves in $\mathbf{B}$ and

$$\begin{aligned} \text{the combined entity} \quad &\equiv \quad [particle, wave] \qquad (6) \\ \text{has energy } E_j^{[3]} \quad &= \quad E_j^{[1]} + E_j^{[2]} \qquad (7) \end{aligned}$$

(where the term "particle wave" was exactly used in Feynman [4]). Particles in $\mathcal{M}^{[1]}$ engage in electromagnetic, nuclear weak or strong forces via exchanging virtual particles. Both particles and waves engage in gravitational forces separately and respectively in $\mathcal{M}^{[1]}$ and $\mathcal{M}^{[2]}$ as introduced in Section 1.

Remark 1 *As established in the previous paper* [8]*, the gravitational motions in* $\mathcal{M}^{[3]}$ *are determined by*

$$g^{[3]} = \left(\frac{G^{[2]}}{G^{[1]} + G^{[2]}}\right) g^{[1]} + \left(\frac{G^{[1]}}{G^{[1]} + G^{[2]}}\right) g^{[2]}, \qquad (8)$$

or

$$\begin{aligned} m^{[3]}\mathbf{a}^{[3]} \quad = \quad & -[\left(\frac{G^{[2]}}{G^{[1]} + G^{[2]}}\right)\left(\frac{G^{[1]}M^{[1]}m^{[1]}}{\|\mathbf{r}\|^2}\right) \\ & + \left(\frac{G^{[1]}}{G^{[1]} + G^{[2]}}\right)\left(\frac{G^{[2]}M^{[2]}m^{[2]}}{\|\mathbf{r}\|^2}\right)] \cdot \frac{\mathbf{r}}{\|\mathbf{r}\|}, \qquad (9) \end{aligned}$$

as the Newtonian limit of the gravitational dynamics between masses $\left[m^{[1]}, m^{[2]}\right]$ *at* $\mathbf{r} \neq \mathbf{0}$ *and* $\left[M^{[1]}, M^{[2]}\right]$ *at* $\mathbf{0}$ *as expressed in terms of the acceleration* $\mathbf{a}^{[3]}$ *of* $m^{[3]}$ $\left(= m^{[1]} + m^{[2]}\right)$*, which also implies that*

$$\mathbf{a}^{[3]} = -\frac{G^{[3]}M^{[3]}}{\|\mathbf{r}\|^2}\left(\frac{M^{[1]}}{M^{[3]}} \cdot \frac{m^{[1]}}{m^{[3]}} + \frac{M^{[2]}}{M^{[3]}} \cdot \frac{m^{[2]}}{m^{[3]}}\right)\frac{\mathbf{r}}{\|\mathbf{r}\|}, \qquad (10)$$

so that a laboratory-measured mass $M^{[3]}_{measured}$ *necessarily observes*

$$\begin{aligned} \mathbf{a}^{[3]} \quad &= \quad -\frac{G^{[3]}M^{[3]}_{measured}}{\|\mathbf{r}\|^2}\frac{\mathbf{r}}{\|\mathbf{r}\|} \qquad (11) \\ &= \quad -\frac{G^{[3]}\left[M^{[3]}\left(\frac{M^{[1]2}+M^{[2]2}}{M^{[3]2}}\right)\right]}{\|\mathbf{r}\|^2}\frac{\mathbf{r}}{\|\mathbf{r}\|}; \qquad (12) \end{aligned}$$

i.e., denoting by $\eta \equiv \frac{M^{[1]}}{M^{[3]}} \equiv 1 - \frac{M^{[2]}}{M^{[3]}}$*, we have*

$$M^{[3]}\left(\eta^2 + (1-\eta)^2\right) = M^{[3]}_{measured}, \tag{13}$$

implying that

$$M^{[1]} = \frac{M^{[3]}_{measured} \cdot \eta}{\eta^2 + (1-\eta)^2} \quad and \tag{14}$$

$$M^{[2]} = \frac{M^{[3]}_{measured} \cdot (1-\eta)}{\eta^2 + (1-\eta)^2}. \tag{15}$$

Incidentally, we also defined in [8]

$$G^{[3]} = G^{[1]} \quad \forall M^{[2]} m^{[2]} = 0, \text{ and} \tag{16}$$

$$G^{[3]} = G^{[2]} \quad \forall M^{[1]} m^{[1]} = 0, \tag{17}$$

so that a dark matter $\left[0, M^{[2]}\right]$ *acts on* $\left[m^{[1]}, m^{[2]}\right]$ *by*

$$\mathbf{a}^{[3]} = -\frac{G^{[2]} M^{[2]}}{\|\mathbf{r}\|^2}\left(\frac{m^{[2]}}{m^{[3]}}\right)\frac{\mathbf{r}}{\|\mathbf{r}\|}. \tag{18}$$

Proposition 1 *Any* $[particle, wave]$ *of electric charge* $+1$ *or* -1 *with energy* $E^{[3]}$ *has*

$$E^{[1]} = \frac{3}{4}E^{[3]} \text{ and} \tag{19}$$

$$E^{[2]} = \frac{1}{4}E^{[3]}. \tag{20}$$

Proof. Following Feynman ([4], II-28-4), we cite the discrepancy in the electromagnetic mass of an electron as measured in a stationary state and as measured in a moving state with a constant velocity of $\|\mathbf{V}\| << c$:

$$m_{\mathbf{V}=\mathbf{0}} = \frac{3}{4} m_{\mathbf{V}\neq\mathbf{0}} = \frac{3}{4}\frac{q^2}{4\pi\epsilon_o r_o}, \tag{21}$$

where $q \equiv$ the charge of electron, $\epsilon_o \equiv$ the permittivity constant, and $r_o \equiv$ the classical electron radius $\approx 2.82 \times 10^{-15}$ meter. By Hypothesis, electromagnetic forces take place only in $\mathcal{M}^{[1]}$, but motions necessarily take place in $\mathcal{M}^{[3]}$; thus, we attribute $m_{\mathbf{V}=\mathbf{0}}$ to $\mathcal{M}^{[1]}$ and $m_{\mathbf{V}\neq\mathbf{0}}$ to $\mathcal{M}^{[3]}$; i.e., $E^{[3]} = m_{\mathbf{V}\neq\mathbf{0}}c^2 = \frac{3}{4}E^{[3]} + \frac{1}{4}E^{[3]} = E^{[1]} + E^{[2]}$. Since Feynman's calculation applies to any electromagnetic field, the result of $E^{[1]} = \frac{3}{4}E^{[3]}$ and $E^{[2]} = \frac{1}{4}E^{[3]}$ applies to any particle of electric charge. ■

Corollary 1 *Any* $[photon,\ electromagnetic\ wave]$ *has* $E^{[1]} = \frac{3}{4}E^{[3]}$ *and* $E^{[2]} = \frac{1}{4}E^{[3]}$.

Proof. Since an electron and a positron annihilate each other into photons, by energy conservation the energy ratio of $\frac{3}{4} : \frac{1}{4}$ is preserved in [photon, electromagnetic wave]. ■

Remark 2 *By Equations* (14), (15), *and* (13), *we have* $\eta = \frac{3}{4}$, *and thus*

$$\begin{aligned} E^{[1]} &= 1.2E^{[3]}_{measured}, &(22)\\ E^{[2]} &= 0.4E^{[3]}_{measured}, &(23)\\ and\ E^{[3]} &= 1.6E^{[3]}_{measured}. &(24)\end{aligned}$$

Proposition 2

$$\begin{aligned} G^{[2]} &= \frac{c^5 \cdot second^2}{1.6h} \quad (h \equiv the\ Planck\ constant) &(25)\\ &\equiv \frac{c^5}{1.6\ddot{h}},\ where\ we\ define &(26)\\ \ddot{h} &:= \frac{h}{second^2} \approx 6.6 \times 10^{-34} \times \left(\frac{joule}{second}\right). &(27)\end{aligned}$$

Proof. Consider two reference frames, $S^{[1]}$ and $S^{[2]}$: $S^{[1]}$ observes a free photon with wave length λ, and $S^{[2]}$ is positioned on the boundary of exactly 1 wave cycle of the photon; i.e., $S^{[2]}$ and the photon are of a mean distance of $\frac{\lambda}{2}$. Clearly to $S^{[1]}$ the unit of $\nu = \frac{c}{\lambda} \equiv \nu^{[1]}$ is $\frac{1\ \text{(cycle)}}{\text{second}}$; however, the unit of $\nu^{[2]}$ to $S^{[2]}$ is $\frac{1\ \text{(cycle)}}{i\cdot\ \text{second}}$ since: in $\mathbf{B}$, any energy entity $E^{[2]}$ has its distance r to the center of $\mathbf{B}$ less than the Schwarzschild radius, so that we have

$$g_{tt} = \left(\frac{\text{proper time } t_o^{[2]}}{\text{proper time } t_o^{[1]}}\right)^2 = 1 - \frac{2G^{[2]}E^{[2]}}{rc^4} < 0. \quad (28)$$

By analytic continuation from $\left(\mathcal{M}^{[2]} - \mathbf{B}\right)$ (where $g_{tt} > 0$) to $\mathbf{B}$ (where $g_{tt} \leq 0$), $t_o^{[2]}$ changes from (δ_1 second) to (0 second) and then ($\delta_2\ i$ second), with $\delta_1, \delta_2 > 0$ unit-free (cf. e.g., [1], for the inherent necessity of complex numbers in standard quantum theory, and [7], for analytic continuation of Lorentzian metrics). Thus,

$$\frac{\Delta t_o^{[2]}}{\Delta t_o^{[1]}} \equiv \frac{\nu^{[1]}}{\nu^{[2]}} \equiv \frac{\nu\ \text{(cycle)}}{\frac{1\ \text{(cycle)}}{i\ \text{second}}} = i\nu\ \text{second}\ ; \quad (29)$$

accordingly,

$$\left(\frac{t_o^{[2]}}{t_o^{[1]}}\right)^2 = -\nu^2 \cdot \text{second}^2 \quad (30)$$

$$= \quad 1 - \frac{2G^{[2]}\left(0.4E^{[3]}_{measured}\right)}{\frac{\lambda}{2}\cdot c^4} \tag{31}$$

(recall Equation (23))

$$\approx \quad 1 - \frac{1.6G^{[2]}h\nu}{\frac{c}{\nu}\cdot c^4} \text{ (where } E^{[3]}_{measured} = h\nu \ \forall \nu >> 1 \tag{32}$$

as derived by Planck, cf. [10] , 206)

$$\approx \quad -\frac{1.6G^{[2]}h\nu^2}{c^5}; \tag{33}$$

thus,

$$G^{[2]} = \frac{c^5 \cdot \text{second}^2}{1.6h} \tag{34}$$

$$(\quad = \frac{\left(3\times 10^8\right)^5 \times \left(\frac{\text{meter}}{\text{second}}\right)^5 \times \text{second}^2}{1.6\times 6.6\times 10^{-34}\times \frac{\text{kilogram}\ \times\ \text{meter}^2}{\text{second}^2} \times\ \text{second}} \tag{35}$$

$$\approx \quad 2.3\times 10^{75}\times \frac{\text{meter}^3}{\text{kilogram}\times\ \text{second}^2}). \tag{36}$$

■

Proposition 3 *The wave function ψ of a photon γ (in $\mathcal{M}^{[1]}$) of frequency ν $\left(\equiv \frac{\omega}{2\pi}\right)$ as measured by a local laboratory frame with a parameter domain U is such that $\forall (t,\mathbf{x}) \in U$, one has*

$$\psi(t,\mathbf{x}) = z_0 \cdot \|\mathbb{E}(t,\mathbf{x})\|_{\mathbb{C}^3}\ , \tag{37}$$

where $\mathbb{E}(t,\mathbf{x}) = \mathbb{E}_o \cdot e^{-i(\omega\ t - \mathbf{k}\cdot\mathbf{x}+\phi)} \in \mathbb{C}^3$ is the electric field (in $\mathbf{B} \subset \mathcal{M}^{[2]}$) associated with γ ($\mathbf{k} \equiv$ the wave vector, $\phi \in [0,2\pi)$, and the complex norm

$$\left\|\left(z_1, z_2, z_3\right)^T\right\|^2_{\mathbb{C}^3} := z_1^2 + z_2^2 + z_3^2 \in \mathbb{C}, \tag{38}$$

cf. e.g., [6], 221, and [11] for metrics on complex manifolds), and $z_0 \in \mathbb{C}$ is a constant.

Proof. By the property of ψ,

$$|\psi(t,\mathbf{x})|^2 \equiv \rho \text{ is the probability density of } \gamma \text{ at } (t,\mathbf{x}). \tag{39}$$

Adopt now the following Axiom:

$$\rho = \beta \cdot (0.4\hat{u}), \tag{40}$$

where $\beta > 0$ is a proportional constant of unit $\left(\frac{1}{\text{joule}}\right)$ and

$$\hat{u} \equiv \epsilon_o \cdot \left|\left\|\mathbb{E}\left(t,\mathbf{x}\right)\right\|_{\mathbb{C}^3}\right|^2 \tag{41}$$

is the measured electric field energy density with $(0.4\hat{u})$ = the energy density apportioned to the electromagnetic wave of γ in $\mathbf{B} \subset \mathcal{M}^{[2]}$ (recall Equation (23)). Then,

$$\begin{aligned} \psi\left(t,\mathbf{x}\right) &= \sqrt{0.4\beta\epsilon_o}e^{i\theta}\left\|\mathbb{E}\left(t,\mathbf{x}\right)\right\|_{\mathbb{C}^3} \quad (\text{for some } \theta \in [0, 2\pi)\,) && (42) \\ &\equiv z_0 \cdot \left\|\mathbb{E}\left(t,\mathbf{x}\right)\right\|_{\mathbb{C}^3}. && (43) \end{aligned}$$

■

Corollary 2 *The wave function ψ_p of any arbitrary particle p is*

$$\psi_p\left(t,\mathbf{x}\right) = z_p \cdot \left\|\mathbb{E}_p\left(t,\mathbf{x}\right)\right\|_{\mathbb{C}^3}, \tag{44}$$

where

$$\mathbb{E}_p\left(t,\mathbf{x}\right) = \sum_j \mathbb{E}_{\gamma_j}\left(t,\mathbf{x}\right). \tag{45}$$

Proof. By Hypothesis, p results from a superposition of electromagnetic waves, and we arrive at the conclusion. ■

Remark 3 *For the next Proposition 4, we note that all the energy entities refer to electromagnetic waves in $\mathbf{B} \subset \mathcal{M}^{[2]}$.*

Proposition 4 *The probability current density of a particle*

$$\begin{aligned} \mathbf{j}\left(t,\mathbf{x}\right) \quad &:= \left(\frac{\hbar}{2\hat{m}i}\right)\left(\bar{\psi}\left(t,\mathbf{x}\right)\cdot\nabla\psi\left(t,\mathbf{x}\right) - \psi\left(t,\mathbf{x}\right)\cdot\nabla\bar{\psi}\left(t,\mathbf{x}\right)\right) && (46) \\ &= \beta \cdot \mathbf{S}^{[2]}\left(t,\mathbf{x}\right), && (47) \end{aligned}$$

where $\hbar \equiv \frac{h}{2\pi}$, $\hat{m} \equiv m^{[3]}_{measured} \equiv$ the measured mass of the $[particle, wave]$, β is the constant of proportionality from the preceding Proposition 3, and $\mathbf{S}^{[2]}\left(t,\mathbf{x}\right)$ is the Poynting vector apportioned to $\mathbf{B} \subset \mathcal{M}^{[2]}$.

Proof. Without loss of generality as based on (linear) superpositions of fields, consider a free photon that travels in the direction of $(x > 0, 0, 0)$ with

$$\begin{aligned}\psi(t, \mathbf{x}) &= z_0 \cdot \|\mathbb{E}(t, \mathbf{x})\|_{\mathbb{C}^3} &(48)\\ &= z_0 \cdot \left\| \left(0, e^{-i(\omega t - kx)}, 0\right)^T \right\|_{\mathbb{C}^3} &(49)\\ &= z_0 e^{-i(\omega t - kx)}. &(50)\end{aligned}$$

Then

$$\begin{aligned}\nabla\psi &= \left(z_0 e^{-i(\omega t - kx)} \cdot ki, 0, 0\right)^T \quad \text{and} &(51)\\ \nabla\bar{\psi} &= \left(z_0 e^{i(\omega t - kx)} \cdot (-ki), 0, 0\right)^T, &(52)\end{aligned}$$

so that $\mathbf{j} := \left(\frac{\hbar}{2\hat{m}i}\right)\left(\bar{\psi} \cdot \nabla\psi - \psi \cdot \nabla\bar{\psi}\right) = \frac{1}{2\hat{m}}\left(\bar{\psi} \cdot \frac{\hbar}{i}\nabla\psi - \psi \cdot \frac{\hbar}{i}\nabla\bar{\psi}\right)$

$$= \frac{1}{2\hat{m}}\left(\bar{\psi}\psi \cdot (\hbar k, 0, 0)^T + \psi\bar{\psi} \cdot (\hbar k, 0, 0)^T\right) \equiv \frac{1}{\hat{m}} \cdot |\psi|^2 \cdot \hat{\mathbf{p}} \tag{53}$$

(where $\hat{\mathbf{p}}$ denotes the measured momentum vector of unit $[\frac{\text{kilogram} \cdot \text{meter}}{\text{second}}]$)

$$= \frac{1}{\hat{m}} \cdot (\beta \cdot (0.4\hat{u})) \cdot \frac{\hat{\mathbf{S}} \cdot \text{meter}^3}{c^2} \tag{54}$$

(where $|\psi|^2$ equal to $\beta \cdot (0.4\hat{u})$ is from the above Axiom Equation (40), and $\hat{\mathbf{S}}$ denotes the measured Poynting vector, cf. $[4]$, II-27-9, so that $\frac{\hat{\mathbf{S}}}{c^2}$ equals the momentum density of unit $\left[\frac{\text{kilogram}}{\text{second} \cdot \text{meter}^2}\right]$)

$$= \left(\frac{\hat{u}}{\hat{m}c^2/\text{meter}^3}\right) \cdot \beta \cdot \left(0.4\hat{\mathbf{S}}\right) = 1 \cdot \beta \cdot \mathbf{S}^{[2]} \tag{55}$$

(due to the uniform probability density for a free photon).

■

Remark 4 *Our geometry of $\mathcal{M}^{[1]} \times \mathbf{B}$ serves to explain the following: (1) quantum tunneling, (2) vacuum polarization, where we provide a different geometric structure for this phenomenon in comparison with that of the "infinite sea of invisible negative energy particles" by Dirac (for a recent treatment on this subject, see, e.g., [3]), and (3) the existence of dark matter and energy $\left[0, E^{[2]}\right]$, where we note that electromagnetic waves can form standing waves (making the collection of waves "matter-like") by superposition.*

Remark 5 *In addition to the above, our $\mathcal{M}^{[1]} \times \mathbf{B} \subset \mathcal{M}^{[1]} \times \mathcal{M}^{[2]}$ resolves the pervasive problem of singularities at $r = 0$ in both the classical and the quantum* *domains by considering a neighborhood N of $r = 0$ that transfers uncertainty energies between $\mathcal{M}^{[1]}$ and $\mathcal{M}^{[2]}$. In this connection (cf.* [4], *II-28-4 through 10), we assert that (1) an electron e^- is a point particle in $\mathcal{M}^{[1]}$ that carries an electromagnetic wave in $\mathbf{B} \subset \mathcal{M}^{[2]}$, (2) in calculating the electromagnetic energy of e^-, one stops at Bdry N , and (3) as such, e^- has no "self force."*

Remark 6 *We also note that a periodic electromagnetic field (in $\mathbf{B}$) renders itself a quotient space, displaying the phenomenon of "instantaneous communication," a feature serving as potential reference for quantum computing.*

3 Summary

In this paper, we have settled the previously undetermined two parameters, $\left(E^{[1]}/E^{[2]}\right)$ and $G^{[2]}$. Our geometry of $\mathcal{M}^{[1]} \times \mathbf{B}$ has contributed physical logic to quantum mechanics, in particular, providing an energy interpretation to probabilities; as a closing example, consider the fine structure constant,

$$\alpha := \frac{\frac{e^2}{4\pi\epsilon_o}}{\hbar c} = \frac{\frac{e^2}{4\pi\epsilon_o}}{\frac{h}{2\pi} \cdot \nu\lambda} = \frac{\frac{e^2}{4\pi\epsilon_o\lambda}}{E^{[3]}_{measured}/2\pi} \tag{56}$$

= (the electrostatic potential energy between two electrons separated by a distance of λ) / (the energy $E^{[3]}_{measured}$ of the virtual photon needed to mediate the two electrons divided by 2π) = the constant α, or, $E^{[3]}_{measured} \cdot \lambda =$ constant, i.e., a uniform probability for any two electrons to interact across all space.

4 References

[1] J.B. Barbour, Time and complex numbers in canonical quantum gravity, Phys. Rev. D 47 (1993), 5422-5429.

[2] J. Cheeger and G. Tian, Curvature and injectivity radius estimates for Einstein 4-manifolds, J. Amer. Math. Soc., 19 (2006), 487-525.

[3] M.J. Esteban, M. Lewin and E. Sere, Variation methods in relativistic quantum mechanics, Bull. Amer. Math. Soc., 45 No. 4 (2008), 535-593.

[4] R.P. Feynman, R.B. Leighton and M. Sands, The Feynman Lectures on Physics, Addison-Wesley, Reading, 1963.

[5] V.P. Frolov, M.A. Markov and V.F. Mukhanov, Black holes as possible sources of closed and semiclosed worlds, Phys. Rev. D 41 (1990), 383-394.

[6] V.G. Ivancevic and T.T. Ivancevic, Complex Dynamics - - Advanced System Dynamics in Complex Variables, Springer, Dordrecht 2007.

[7] P. Kraus, H. Ooguri and S. Shenker, Inside the horizon with AdS/CFT, Phys. Rev. D 67 (2003), 124022-124037.

[8] G.L. Light, An introduction of a combined space-time manifold, Appl. Math. Sci., 1 No. 27 (2007), 1341-1346.

[9] G.L. Light, Energy inquiry, Can. J. Phys., 68 (1990), 242-243.

[10] M.S. Longair, Theoretical Concepts in Physics, Cambridge University Press, Cambridge, 1986.

[11] G. Maschler, Central Kähler metrics, Trans. Amer. Math. Soc., 355 (2003), 2161-2182.

[12] D. Töben, Parallel focal structure and singular Riemannian foliations, Trans. Amer. Math. Soc., 358 (2006), 1677-1704.

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