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Solving a Nonlinear System of Second Order Two-Point Boundary Value Problem

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Abstract

In this paper, we give a new method to solve a nonlinear system of second order two-point boundary value problem. Its exact solutions are represented in the form of series in the reproducing kernel space. The n-term approximation $u_n(x), v_n(x)$ are proved to converge to the exact solutions $u(x), v(x)$, respectively. An example is given to demonstrate the application of the new method.

Keywords: Nonlinear second-order differential system ; Reproducing kernel space;Two-point boundary problem; Exact Solutions

1 Introduction

The purpose of this paper is to study a general nonlinear second-order differential system with the Dirichlet boundary conditions

$$\begin{cases} u''(x) = f(x, u(x), u'(x), v(x), v'(x)), \quad 0 < x < 1, \\ v''(x) = g(x, u(x), u'(x), v(x), v'(x)), \quad 0 < x < 1, \\ u(0) = 0, u(1) = 0, \\ v(0) = 0, v(1) = 0. \end{cases} \tag{1.1}$$

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In recent years, boundary value problems of the system of ordinary differential equations have been of interest in mathematics, physics, engineering, biology and so on(see Refs[1-4]). It should be pointed out that the authors [5-11] obtained the existence results of solution of Eqs(1.1), but discussion on numerical solutions of Eqs (1.1) is scare.

In this paper, we will give the presentation of the exact solutions of Eqs.(1.1)and approximate solutions in the reproducing kernel space. The method has the following advantages: Firstly, the conditions for determining solution in Eqs(1.1) can be imposed on the reproducing kernel space and therefore reproducing kernel satisfying the conditions for determining solution can be calculated. We will use the kernel to solve problems. Secondly, the iterative sequences $u_n(x), v_n(x)$ of approximate solutions converge in $C^2[0,1]$ to the solutions $u(x), v(x)$ respectively .

2 Preliminaries

2.1 The reproducing kernel space $W_2^3[0,1]$

The inner product space $W_2^3[0,1]$ is defined as $W_2^3[0,1] = \{u(x) \mid u''(x)$ is a absolutely continuous real value function, $u'''(x) \in L^2[0,1], u(0) = u(1) = 0\}$. The inner product in $W_2^3[0,1]$ is given by

$$\langle u(x), v(x)\rangle_{W_2^3} = \sum_{i=0}^{2} u^{(i)}(0)v^{(i)}(0) + \int_0^1 u'''(x)v'''(x)\,\mathrm{d}x, \tag{2.1}$$

and the norm $\| u \|_{W_2^3}$ is denoted by $\| u \|_{W_2^3} = \sqrt{\langle u, u\rangle_{W_2^3}}$, where $u, v \in W_2^3[0,1]$.

Theorem 2.1. *The space $W_2^3[0,1]$ is a complete reproducing kernel space. That is, for each fixed $x \in [0,1]$, there exists $R_x(y) \in W_2^3[0,1]$, such that $\langle u(y), R_x(y)\rangle_{W_2^3} = u(x)$ for any $u(y) \in W_2^3[0,1]$ and $y \in [0,1]$. The reproducing kernel $R_x(y)$ can be written as*

$$R_x(y) = \begin{cases} \sum\limits_{i=1}^{6} c_i y^{i-1}, & y \le x, \\ \sum\limits_{i=1}^{6} d_i y^{i-1}, & y > x. \end{cases} \tag{2.2}$$

The representation of $R_x(y)$ is given in Appendix. The method of obtaining coefficients of the reproducing kernel $R_x(y)$ and the proof of Theorem 2.1 are given in Theorem 1.3.1 and Theorem 1.3.2 in [12].

2.2 The reproducing kernel space $W_2^1[0,1]$

The inner product space $W_2^1[0,1]$ is defined by $W_2^1[0,1] = \{u(x) \mid u$ is a absolutely continuous real value function, $u' \in L^2[0,1]\}$. The inner product and norm in $W_2^1[0,1]$ are given respectively by

$$\langle u(x), v(x)\rangle_{W_2^1} = u(0)v(0) + \int_0^1 u'(x)v'(x)dx, \ \parallel u \parallel_{W_2^1} = \sqrt{\langle u, u\rangle_{W_2^1}}$$

In [12], it has been proved that $W_2^1[0,1]$ is also a complete reproducing kernel space and its reproducing kernel is

$$K_x(y) = \begin{cases} 1+y, & y \le x, \\ 1+x, & y > x. \end{cases}$$

2.3 Introduction into a linear operator

Let $Lu = u''$, $L : W_2^3[0,1] \to W_2^1[0,1]$, then Eqs.(1.1)can be converted into the form as follows

$$\begin{cases} Lu(x) = f(x, u(x), u'(x), v(x), v'(x)), & 0 < x < 1, \\ Lv(x) = g(x, u(x), u'(x), v(x), v'(x)), & 0 < x < 1, \end{cases} \tag{2.3}$$

where $u(x), v(x) \in W_2^3[0,1]$ and $f(x,y,z,w,v) \in W_2^1[0,1], g(x,y,z,w,v) \in W_2^1[0,1]$ as $y = y(x), z = z(x), w = w(x), v = v(x) \in W_2^3[0,1]$. Therefore, we have

$$u(0) = u(1) = 0, v(0) = v(1) = 0.$$

It is easy to prove that L is a bounded linear operator.

Now, we construct an orthogonal system of functions first.

Let $\varphi_i(x) = K_{x_i}(x)$and $\psi_i(x) = L^*\varphi_i(x)$, where L^* is the conjugate operator of L. In terms of the properties of reproducing kernel $K_x(y)$, one obtains

$$\langle u(x), \psi_i(x)\rangle_{W_2^3} = \langle Lu(x), \varphi_i(x)\rangle_{W_2^1} = Lu(x_i), \ i = 1, 2, \dots .$$

The normal orthogonal system of functions $\{\overline{\psi}_i(x)\}_{i=0}^{\infty}$ in $W_2^3[0,1]$ can be derived from Gram-Schmidt orthogonalization process of $\{\psi_i(x)\}_{i=0}^{\infty}$,

$$\overline{\psi}_i(x) = \sum_{k=1}^{i} \beta_{ik}\psi_k(x),$$

where β_{ik} are orthogonalization coefficients,$\beta_{ii} > 0, i = 1, 2, \dots$.

We collect the following lemmas in [13] for future use.

Lemma 2.1. *If $\{x_i\}_{i=1}^{\infty}$ is dense on $[0,1]$, then $\{\psi_i(x)\}_{i=1}^{\infty}$ is a complete system of $W_2^3[0,1]$ and $\psi_i(x) = L_y R_x(y)|_{y=x_i}$.*

Lemma 2.2. *If $u(x) \in W_2^3[0,1]$, then there exists $M_1 > 0$, such that*

$$\|u\|_{C^2[0,1]} \leq M_1 \|u\|_{W_2^3},$$

where $\|u\|_{C^2[0,1]} = \max\limits_{x\in[0,1]} |u(x)| + \max\limits_{x\in[0,1]} |u'(x)| + \max\limits_{x\in[0,1]} |u''(x)|$.

Lemma 2.3. *If $\|u_n - u\|_{W_2^3} \to 0, \|v_n - v\|_{W_2^3} \to 0, x_n \to x, (n \to \infty)$ and $f(x,y,z,w,v)$, $g(x,y,z,w,v)$ for $x \in [0,1], y,z,w,v \in (-\infty,+\infty)$ are continuous with respect to x,y,z,w,v, then*

$$f(x_n, u_{n-1}(x_n), u'_{n-1}(x_n), v_{n-1}(x_n), v'_{n-1}(x_n)) \to f(x, u(x), u'(x), v(x), v'(x)) \text{ as n} \to \infty,$$

$$g(x_n, u_{n-1}(x_n), u'_{n-1}(x_n), v_{n-1}(x_n), v'_{n-1}(x_n)) \to g(x, u(x), u'(x), v(x), v'(x)) \text{ as n} \to \infty.$$

3 The exact and approximate solutions of Eqs.(1.1)

Theorem 3.1. *If $\{x_i\}_{i=1}^{\infty}$ is dense on $[0,1]$and $u(x), v(x) \in W_2^3[0,1]$ are the solutions of Eq.(2.3), then $u(x), v(x)$ satisfy the following form , respectively*

$$u(x) = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} f(x_k, u(x_k), u'(x_k), v(x_k), v'(x_k)) \bar{\psi}_i(x), \tag{3.1}$$

$$v(x) = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} g(x_k, u(x_k), u'(x_k), v(x_k), v'(x_k)) \bar{\psi}_i(x). \tag{3.2}$$

Proof. $u(x)$ can be expanded to Fourier series in terms of normal orthogonal basis $\bar{\psi}_i(x)$ in $W_2^3[0,1]$.

$$\begin{aligned} u(x) &= \sum_{i=1}^{\infty} \langle u(x), \bar{\psi}_i(x) \rangle_{W_2^3} \bar{\psi}_i(x) = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \langle u(x), \psi_k(x) \rangle_{W_2^3} \bar{\psi}_i(x) \\ &= \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \langle u(x), L^* \varphi_k(x) \rangle_{W_2^3} \bar{\psi}_i(x) = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \langle Lu(x), \varphi_k(x) \rangle_{W_2^1} \bar{\psi}_i(x) \\ &= \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \langle f(x, u(x), u'(x), v(x), v'(x)), \varphi_k(x) \rangle_{W_2^1} \bar{\psi}_i(x) \\ &= \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} f(x_k, u(x_k), u'(x_k), v(x_k), v'(x_k)) \bar{\psi}_i(x). \end{aligned}$$

In the same way, we can get

$$v(x)=\sum_{i=1}^{\infty}\sum_{k=1}^{i}\beta_{ik}g(x_k,u(x_k),u'(x_k),v(x_k),v'(x_k))\bar{\psi}_i(x).$$

□

Remark:(i) If Eqs.(2.3) are linear, that is, $f(x,u(x),u'(x),v(x),v'(x)) = f(x)$, $g(x,u(x),u'(x),v(x),v'(x)) = g(x)$, then the analytical solutions of Eq.(2.3) can be obtained directly by (3.1),(3.2).
(ii) If Eqs.(2.3) are nonlinear, then the solutions of Eqs.(2.3) can be obtained by the following iterative method.
3.1 The iterative sequence

We construct the iterative sequences $u_n(x), v_n(x)$, putting

$$\begin{cases}\forall \text{ fixed } u_0(x), v_0(x)\in W_2^3[0,1],\\ u_n(x)=\sum\limits_{i=1}^{n}A_i\bar{\psi}_i,\\ v_n(x)=\sum\limits_{i=1}^{n}B_i\bar{\psi}_i,\end{cases}\tag{3.3}$$

where

$$\begin{cases}A_1=\beta_{11}f(x_1,u_0(x_1),u_0'(x_1),v_0(x_1),v_0'(x_1)),\\ A_2=\sum\limits_{k=1}^{2}\beta_{2k}f(x_k,u_{k-1}(x_k),u_{k-1}'(x_k),v_{k-1}(x_k),v_{k-1}'(x_k)),\\ \cdots\\ A_n=\sum\limits_{k=1}^{n}\beta_{nk}f(x_k,u_{k-1}(x_k),u_{k-1}'(x_k),v_{k-1}(x_k),v_{k-1}'(x_k)),\end{cases}\tag{3.4}$$

$$\begin{cases}B_1=\beta_{11}g(x_1,u_0(x_1),u_0'(x_1),v_0(x_1),v_0'(x_1)),\\ B_2=\sum\limits_{k=1}^{2}\beta_{2k}g(x_k,u_{k-1}(x_k),u_{k-1}'(x_k),v_{k-1}(x_k),v_{k-1}'(x_k)),\\ \cdots\\ B_n=\sum\limits_{k=1}^{n}\beta_{nk}g(x_k,u_{k-1}(x_k),u_{k-1}'(x_k),v_{k-1}(x_k),v_{k-1}'(x_k)).\end{cases}\tag{3.5}$$

Next, we proof $u_n(x), v_n(x)$ in iterative formula (3.3) are convergent to the exact solutions of Eqs.(2.3)

Theorem 3.2. *Suppose the following conditions are satisfied:*
(i) $\|u_n\|_{W_2^3}, \|v_n\|_{W_2^3}$ *are bounded ;*
(ii) $\{x_i\}_{i=1}^{\infty}$ *is dense on* $[0,1]$*;*
(iii) $f(x, y(x), z(x), w(x), v(x)), g(x, y(x), z(x), w(x), v(x)) \in W_2^1[0,1]$ *for any* $y(x), z(x), w(x), v(x) \in W_2^3[0,1]$.
Then $u_n(x), v_n(x)$ *in iterative formula (3.3) are convergent to the exact solution* $u(x), v(x)$ *of Eq.(2.3) in* $W_2^3[0,1]$ *and*

$$u(x) = \sum_{i=1}^{\infty} A_i \bar{\psi}_i, v(x) = \sum_{i=1}^{\infty} B_i \bar{\psi}_i,$$

where A_i, B_i *are given by (3.4),(3.5),respectively*

Proof. (1) First, we will prove the convergence of $u_n(x), v_n(x)$.
By (3.3), we have

$$\begin{cases} u_{n+1}(x) = u_n(x) + A_{n+1}\bar{\psi}_{n+1}(x), \\ v_{n+1}(x) = v_n(x) + B_{n+1}\bar{\psi}_{n+1}(x). \end{cases} \tag{3.6}$$

From the orthogonality of $\{\bar{\psi}(x)\}_{i=1}^{\infty}$, it follows that

$$\begin{aligned} \|u_{n+1}\|_{W_2^3}^2 &= \|u_n\|_{W_2^3}^2 + (A_{n+1})^2 = \|u_{n-1}\|_{W_2^3}^2 + (A_n)^2 + (A_{n+1})^2, \\ &\cdots \\ &= \|u_0\|_{W_2^3}^2 + \sum_{i=1}^{n+1} (A_i)^2, \\ \|v_{n+1}\|_{W_2^3}^2 &= \|v_n\|_{W_2^3}^2 + (B_{n+1})^2 = \|v_{n-1}\|_{W_2^3}^2 + (B_n)^2 + (B_{n+1})^2, \\ &\cdots \\ &= \|v_0\|_{W_2^3}^2 + \sum_{i=1}^{n+1} (B_i)^2. \end{aligned} \tag{3.7}$$

From boundedness of $\|u_n\|_{W_2^3}$ and $\|v_n\|_{W_2^3}$,we have

$$\sum_{i=1}^{\infty} (A_i)^2 < \infty, \sum_{i=1}^{\infty} (B_i)^2 < \infty,$$

i.e.

$$\{A_i\} \in l^2, \{B_i\} \in l^2 (i = 1, 2, \cdots).$$

Let $m > n$, for $(u_m - u_{m-1}) \perp (u_{m-1} - u_{m-2}) \perp \cdots \perp (u_{n+1} - u_n), (v_m - v_{m-1}) \perp (v_{m-1} - v_{m-2}) \perp \cdots \perp (v_{n+1} - v_n)$, it follows that

$$\begin{aligned}
\|u_m(x) - u_n(x)\|^2_{W_2^3} &= \|u_m(x) - u_{m-1}(x) + u_{m-1}(x) - u_{m-2}(x) + \cdots + u_{n+1}(x) - u_n(x)\|^2_{W_2^3} \\
&\leq \|u_m(x) - u_{m-1}(x)\|^2_{W_2^3} + \cdots + \|u_{n+1}(x) - u_n(x)\|^2_{W_2^3} \\
&= \sum_{i=n+1}^{m} (A_i)^2 \to 0, \ (n \to \infty) \qquad (3.8) \\
\|v_m(x) - v_n(x)\|^2_{W_2^3} &= \|v_m(x) - v_{m-1}(x) + v_{m-1}(x) - v_{m-2}(x) + \cdots + v_{n+1}(x) - v_n(x)\|^2_{W_2^3} \\
&\leq \|v_m(x) - v_{m-1}(x)\|^2_{W_2^3} + \cdots + \|v_{n+1}(x) - v_n(x)\|^2_{W_2^3} \\
&= \sum_{i=n+1}^{m} (B_i)^2 \to 0, \ (n \to \infty).
\end{aligned}$$

Considering the completeness of $W_2^3[0,1]$, there exists $u(x), v(x) \in W_2^3[0,1]$, such that

$$u_n(x) \overset{\|\cdot\|_{W_2^3}}{\longrightarrow} u(x), \text{as n} \to \infty,$$

$$v_n(x) \overset{\|\cdot\|_{W_2^3}}{\longrightarrow} v(x), \text{as n} \to \infty.$$

(2) Second, we will prove $u(x), v(x)$ are the solutions of Eqs(2.3).

By Lemma 2.2 and (i) of Theorem 3.2, we know $u_n(x), v_n(x)$ converge uniformly to $u(x), v(x)$,respectively. It follows that, on taking limits in (3.3), we have

$$u(x) = \sum_{i=1}^{\infty} A_i \bar{\psi}_i, v(x) = \sum_{i=1}^{\infty} B_i \bar{\psi}_i.$$

Since

$$(Lu)(x_j) = \sum_{i=1}^{\infty} A_i \langle L\bar{\psi}_i(x), \varphi_j(x)\rangle_{W_2^1} = \sum_{i=1}^{\infty} A_i \langle \bar{\psi}_i(x), L^*\varphi_j(x)\rangle_{W_2^3} = \sum_{i=1}^{\infty} A_i \langle \bar{\psi}_i(x), \psi_j(x)\rangle_{W_2^3},$$

and

$$(Lv)(x_j) = \sum_{i=1}^{\infty} B_i \langle L\bar{\psi}_i(x), \varphi_j(x)\rangle_{W_2^1} = \sum_{i=1}^{\infty} B_i \langle \bar{\psi}_i(x), L^*\varphi_j(x)\rangle_{W_2^3} = \sum_{i=1}^{\infty} B_i \langle \bar{\psi}_i(x), \psi_j(x)\rangle_{W_2^3}.$$

It follows that

$$\sum_{j=1}^{n} \beta_{nj}(Lu)(x_j) = \sum_{i=1}^{\infty} A_i \langle \bar{\psi}_i(x), \sum_{j=1}^{n} \beta_{nj}\psi_j(x)\rangle_{W_2^3} = \sum_{i=1}^{\infty} A_i \langle \bar{\psi}_i(x), \bar{\psi}_n(x)\rangle_{W_2^3} = A_n,$$

and

$$\sum_{j=1}^{n} \beta_{nj}(Lv)(x_j) = \sum_{i=1}^{\infty} B_i \langle \bar{\psi}_i(x), \sum_{j=1}^{n} \beta_{nj}\psi_j(x)\rangle_{W_2^3} = \sum_{i=1}^{\infty} B_i < \bar{\psi}_i(x), \bar{\psi}_n(x) >_{W_2^3} = B_n.$$

If $n = 1$, then

$$(Lu)(x_1) = f(x_1, u_0(x_1), u_0'(x_1), v_0(x_1), v_0'(x_1)),$$
$$(Lv)(x_1) = g(x_1, u_0(x_1), u_0'(x_1), v_0(x_1), v_0'(x_1)). \tag{3.9}$$

If $n = 2$, then

$$\beta_{21}(Lu)(x_1) + \beta_{22}(Lu)(x_2) = \beta_{21} f(x_1, u_0(x_1), u_0'(x_1), v_0(x_1), v_0'(x_1)) + \beta_{22} f(x_2, u_1(x_2), u_1'(x_2), v_1(x_2), v_1'(x_2)),$$
$$\beta_{21}(Lv)(x_1) + \beta_{22}(Lv)(x_2) = \beta_{21} g(x_1, u_0(x_1), u_0'(x_1), v_0(x_1), v_0'(x_1)) + \beta_{22} g(x_2, u_1(x_2), u_1'(x_2), v_1(x_2), v_1'(x_2)). \tag{3.10}$$

From (3.9) and (3.10), it is clear that

$$(Lu)(x_2) = f(x_2, u_1(x_2), u_1'(x_2), v_1(x_2), v_1'(x_2)),$$

$$(Lv)(x_2) = g(x_2, u_1(x_2), u_1'(x_2), v_1(x_2), v_1'(x_2)).$$

Furthermore, it is easy to see by induction that

$$(Lu)(x_j) = f(x_j, u_{j-1}(x_j), u_{j-1}'(x_j), v_{j-1}(x_j), v_{j-1}'(x_j)),$$
$$(Lv)(x_j) = g(x_j, u_{j-1}(x_j), u_{j-1}'(x_j), v_{j-1}(x_j), v_{j-1}'(x_j)). \tag{3.11}$$

Since $\{x_i\}_{i=1}^{\infty}$ is dense on interval $[0, 1]$, for any $y \in [0, 1]$, there exists subsequence $\{x_{n_j}\}$, such that

$$x_{n_j} \to y, \text{as j} \to \infty.$$

Hence, let $j \to \infty$ in (3.11),by the convergence of $u_n(x), v_n(x)$ and lemma 2.3 , we have

$$(Lu)(y) = f(y, u(y), u'(y), v(y), v'(y)),$$
$$(Lv)(y) = g(y, u(y), u'(y), v(y), v'(y)). \tag{3.12}$$

That is, $u(x), v(x)$ are the solutions of Eqs.(2.3) and

$$u(x) = \sum_{i=1}^{\infty} A_i \bar{\psi}_i, v(x) = \sum_{i=1}^{\infty} B_i \bar{\psi}_i, \tag{3.13}$$

where A_i, B_i are given by (3.4),(3.5),respectively. □

From Lemma 2.2, we have the following the corollary.

Corollary 3.1. *Assume that the conditions of Theorem 3.2 hold, then* $u_n(x), v_n(x)$ *in (3.3) satisfy* $\|u_n(x) - u(x)\|_{C^2} \to 0, \|v_n(x) - v(x)\|_{C^2} \to 0,\ n \to \infty$ *, where* $u(x), v(x)$ *are the exact solutions of Eqs.(2.3).*

4 Example

Consider the system of equation

$$\begin{cases} u''(x) = \frac{\cos[u'(x)]\cos[v(x)]}{1+u^2(x)} + \sin[v'(x)] + f(x), \ \ 0 < x < 1, \\ v''(x) = \frac{\sin[u'(x)]}{1+v'^2(x)} - \cos[v(x)] + g(x), \ \ 0 < x < 1, \\ u(0) = 0, u(1) = 0 \\ v(0) = 0, v(1) = 0 \end{cases}$$

The exact solution is $u(x) = (x-1)\sin(\pi x), v(x) = 2x\cos(\frac{\pi}{2}x)$. The numerical results are displayed in Table 1 ,Table 2 ,Table 3 ,Table 4.

Table 1: The numerical results for Example

Node	True solution $u(x)$	Approximate solution $u_{100}(x)$	Absolute error	Relative error
0.1	-0.278115	-0.278108	6.99E-6	2.51E-5
0.2	-0.470228	-0.470217	1.12E-5	2.40E-5
0.3	-0.566312	-0.566299	1.27E-5	2.25E-5
0.4	-0.570634	-0.570622	1.16E-5	2.03E-5
0.5	-0.5	-0.499991	8.58E-6	1.71E-5
0.6	-0.380423	-0.380418	4.70E-6	1.23E-5
0.7	-0.242705	-0.242704	1.09E-6	4.51E-6
0.8	-0.117557	-0.117558	1.26E-6	1.07E-5
0.9	-0.0309017	-0.0309034	1.72E-6	5.57E-5
1	0	0	0	

Table 2: The mean square deviations for the derivatives for u_{100}

$\sqrt{\frac{\sum_{i=1}^{10} u'(0.1i)-u'_{100}(0.1i)}{10}}$	$\sqrt{\frac{\sum_{i=1}^{10} u''(0.1i)-u''_{100}(0.1i)}{10}}$
3.07E-5	3.02E-5

7 Appendix

$$R_x(y) = \begin{cases} -\frac{x}{156}(-36+30x+10x^2-5x^3+x^4)y - \frac{x}{624}(120-126x+10x^2-5x^3+x^4)y^2 \\ -\frac{1}{1872}(x(120-126x+10x^2-5x^3+x^4)y^3) + \frac{1}{3744}(x(-36+30x+10x^2-5x^3+x^4)y^4) \\ +\frac{1}{18720}((156-120x-30x^2-10x^3+5x^4-x^5)y^5) \quad y \leq x, \\ \frac{x^5}{120} - \frac{x}{312}(-72+60x+20x^2+3x^3+2x^4)y - \frac{x}{624}(120-126x-42x^2-5x^3+x^4)y^2 \\ -\frac{1}{1872}(x(120+30x+10x^2-5x^3+x^4)y^3) + \frac{1}{3744}(x(120+30x+10x^2-5x^3+x^4)y^4) \\ -\frac{1}{18720}(x(120+30x+10x^2-5x^3+x^4)y^5) \quad y > x. \end{cases}$$

Table 3: The numerical results for Example

Node	True solution $v(x)$	Approximate solution $v_{100}(x)$	Absolute error	Relative error
0.1	0.197538	0.197536	1.49E-6	7.54E-6
0.2	0.380423	0.38042	2.92E-6	7.68E-6
0.3	0.534604	0.5346	4.11E-6	7.69E-6
0.4	0.647214	0.647209	4.95E-6	7.65E-6
0.5	0.707107	0.707101	5.44E-6	7.70E-6
0.6	0.705342	0.705337	5.56E-6	7.88E-6
0.7	0.635587	0.635582	5.15E-6	8.11E-6
0.8	0.494427	0.494423	4.11E-6	8.32E-6
0.9	0.281582	0.28158	2.40E-6	8.53E-6
1	0	0	0	

Table 4: The mean square deviations for the derivatives for v_{100}

$\sqrt{\frac{\sum_{i=1}^{10} v'(0.1i)-v'_{100}(0.1i)}{10}}$	$\sqrt{\frac{\sum_{i=1}^{10} v''(0.1i)-v''_{100}(0.1i)}{10}}$
1.41E-5	9.30E-6

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