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Stability Analysis of Impulsive Fuzzy Recurrent Neural Networks with Hybrid Delays[1]

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Abstract. In this paper, the impulsive fuzzy recurrent neural network with both time-varying delays and distributed delays is considered. Applying the idea of vector Lyapunov function, M-matrix theory and analytic methods, several sufficient conditions are obtained to ensure the existence, uniqueness and global exponential stability of equilibrium point for the addressed neural network. Moreover, the estimation of the exponential convergence rate index is provided. These results generalize a few previous known results and remove some restrictions on the neural networks. Two examples are given to show the effectiveness of the obtained results. The method of this paper, which does not make use of Lyapunov functional, is simple and valid for the stability analysis of fuzzy recurrent neural networks with variable delays or/and distributed delays, it is believed that these results are significant and useful for the design and applications of fuzzy neural networks.

1. Introduction

Recurrent neural networks have been extensively studied in the past decades. Two popular examples of such kinds of neural networks are the Hopfield neural

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networks and cellular neural networks. They have been successfully applied in signal processing, pattern recognition, associative memories, optimization solvers, and other engineering and scientific areas [1], [2]. In such applications, it is of prime importance to ensure that the designed neural networks are stable.

In an hardware implementation of a recurrent neural network using analog electronic circuits, the time delay will be inevitable and occur in the signal transmission among the neurons, which will affect the stability of the neural system and may lead to some complex dynamic behaviors such as oscillation, divergence, chaos, instability or other poor performance of the neural networks [2]. In this case, the time delay may substantially affect the performance of the recurrent neural networks. Thus, the study of stability for delayed recurrent neural networks is of both theoretical and practical importance. In recent years, the dynamical behaviors of recurrent neural networks with constant delays or time-varying delays or/and distributed delays have been deeply investigated, for example, see [2]-[20] and references therein.

However, besides delay effect, impulsive effects are also likely to exist in the recurrent neural networks [21]. For instance, in implementation of electronic networks, the state of the networks is subject to instantaneous perturbations and experiences abrupt change at certain instants, which may be caused by switching phenomenon, frequency change or other sudden noise, that is, does exhibit impulsive effects. Therefore, it is necessary to consider both impulsive effect and delay effect on the stability of the recurrent neural networks. Several interesting results on impulsive effect have been gained for neural networks with delays, for example, see [21]-[32] and references therein.

It is well-known that the fuzzy cellular neural networks (FCNN), was introduced by Yang in 1996 [33], is also a kind of important recurrent neural networks. It combines fuzzy logic with the traditional cellular neural networks. Studies have shown that the FCNN is very useful paradigm for image processing problems, which is a cornerstone in image processing and pattern recognition. Recently, some results on stability have been derived for the FCNN without time delays and with time delays, for example, see [34]-[37]. In [34], Yang and Yang obtained some conditions for the existence and the global stability of the equilibrium point of the FCNN without delay. In [35], Liu and Tang considered the FCNN with either constant delays or time-varying delays, several sufficient conditions were obtained to ensure the existence and uniqueness of the equilibrium point and its global exponential stability. In [36], Yuan, Cao and Deng gave several criteria of exponential stability and periodic solutions for FCNN with time-varying delays. In [37], Huang considered the stability of FCNN with diffusion terms and time-varying delay. To the best of our knowledge, few authors have considered fuzzy recurrent neural network model with both time delays and impulsive effects. From the view of mathematical model, the fuzzy recurrent neural networks with both time delays and impulsive effects belongs to new category of dynamical systems, which is neither purely continuous-time nor purely discrete-time one. Such a model displays a combination of characteristics of both the continuous-time and discrete-time systems and has complex dynamical behaviors. Therefore, it is necessary to further investigate the dynamical behaviors of fuzzy recurrent neural network model with both time delays and impulsive effects.

Motivated by the above discussions, the objective of this paper is to study the global exponential stability of the impulsive fuzzy recurrent neural network with both time-varying delays and distributed delays, and estimate the exponential convergence rate index.

2. Model description and preliminaries

In this paper, we consider the following model

$$\begin{cases} \frac{du_i(t)}{dt} = -c_i u_i(t) + \sum\limits_{j=1}^{n} a_{ij} f_j(u_j(t)) + \sum\limits_{j=1}^{n} b_{ij} v_j + J_i \\ \qquad + \bigwedge\limits_{j=1}^{n} \alpha_{ij} f_j(u_j(t-\tau_{ij}(t))) + \bigvee\limits_{j=1}^{n} \beta_{ij} f_j(u_j(t-\tau_{ij}(t))) \\ \qquad + \bigwedge\limits_{j=1}^{n} \delta_{ij} \int\limits_{-\infty}^{t} K_{ij}(t-s) f_j(u_j(s))ds \\ \qquad + \bigvee\limits_{j=1}^{n} \eta_{ij} \int\limits_{-\infty}^{t} K_{ij}(t-s) f_j(u_j(s))ds \\ \qquad + \bigwedge\limits_{j=1}^{n} T_{ij} v_j + \bigvee\limits_{j=1}^{n} H_{ij} v_j, \quad t \neq t_k, \quad t \geq 0, \\ \triangle u_i(t_k) = u_i(t_k^+) - u_i(t_k^-) = I_{ik}(u_i(t_k)), \\ \qquad i = 1, 2, \cdots, n; \quad k = 1, 2, \cdots, \end{cases} \tag{1}$$

where n corresponds to the number of units in a neural network; $u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T$, $u_i(t)$ corresponds to the state of the ith unit at time t; f_j denotes the activation function; $\tau_{ij}(t)$ corresponds to the transmission delay along the axon of the jth unit from the ith unit and satisfies $0 \leq \tau_{ij}(t) \leq \tau_{ij}$ (τ_{ij} is a constant); $K_{ij}(t)$ is delay kernel function; $C = diag(c_1, c_2, \cdots, c_n)$, c_i represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, a_{ij} and b_{ij} are elements of feedback template and feed forward template, respectively; $\alpha = (\alpha_{ij})_{n\times n}$, $\beta = (\beta_{ij})_{n\times n}$, $\delta = (\delta_{ij})_{n\times n}$, $\eta = (\eta_{ij})_{n\times n}$, α_{ij}, β_{ij}, δ_{ij} and η_{ij} are elements of the discrete fuzzy feedback MIN template, the discrete fuzzy feedback MAX template, the distributed fuzzy feedback MIN template and the distributed fuzzy feedback MAX template, respectively; $T = (T_{ij})_{n\times n}$, $H = (H_{ij})_{n\times n}$, T_{ij} and H_{ij} are elements of fuzzy feed forward MIN template and fuzzy feed forward MAX template, respectively; $V = (v_1, v_2, \cdots, v_n)^T$, $J = (J_1, J_2, \cdots, J_n)^T$, v_i and J_i denote input and bias of the ith neuron, respectively; t_k is called impulsive moments and satisfy $0 \leq t_0 < t_1 < t_2 < \cdots$, $\lim_{k\to+\infty} t_k = +\infty$; $u_i(t_k^-)$ and $u_i(t_k^+)$ denote the left limit and right limit at t_k, respectively;

$I_k(u(t_k)) = (I_{1k}(u_1(t_k)), I_{2k}(u_2(t_k)), \cdots, I_{nk}(u_n(t_k)))^T$, $I_{ik}(u_i(t_k))$ shows impulsive perturbation of the ith neuron at t_k.

Remark 1. *Since the solution $(u_1(t,x), \cdots, u_n(t,x))^T$ of model (1) is discontinuous at the point t_k, by theory of impulsive differential equations, we assume that $(u_1(t_k), u_2(t_k), \cdots, u_n(t_k))^T \equiv (u_1(t_k+0,x), u_2(t_k+0,x), \cdots, u_n(t_k+0,x))^T$. It is clear that, in general, the derivatives $\frac{du_i(t_k)}{dt}$ do not exist. On the other hand, according to the first equation of model (1) there exist the limits $\frac{du_i(t_k\mp 0)}{dt}$. According to the above convention, we assume $\frac{du_i(t_k)}{dt} \equiv \frac{du_i(t_k+0)}{dt}$.*

Remark 2. *If $I_{ik}(u_i(t_k)) = 0$ for $i = 1, 2, \cdots, n$; $\quad k = 1, 2, \cdots$, then model (1) becomes continuous fuzzy recurrent neural network*

$$\begin{aligned}
\frac{du_i(t)}{dt} &= -c_i u_i(t) + \sum_{j=1}^{n} a_{ij} f_j(u_j(t)) + \sum_{j=1}^{n} b_{ij} v_j + J_i \\
&\quad + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(u_j(t-\tau_{ij}(t))) + \bigvee_{j=1}^{n} \beta_{ij} f_j(u_j(t-\tau_{ij}(t))) \\
&\quad + \bigwedge_{j=1}^{n} \delta_{ij} \int_{-\infty}^{t} K_{ij}(t-s) f_j(u_j(s)) ds \\
&\quad + \bigvee_{j=1}^{n} \eta_{ij} \int_{-\infty}^{t} K_{ij}(t-s) f_j(u_j(s)) ds + \bigwedge_{j=1}^{n} T_{ij} v_j + \bigvee_{j=1}^{n} H_{ij} v_j \qquad (2)
\end{aligned}$$

for $t \geq 0, \quad i = 1, 2, \cdots, n$.

For convenience, we introduce several notations. $u = (u_1, u_2, \cdots, u_n)^T \in R^n$ denotes a column vector; $|u|$ denotes the absolute-value vector given by $|u| = (|u_1|, |u_2|, \cdots, |u_n|)^T$. For matrix $A = (a_{ij})_{n\times n} \in R^{n\times n}$, $|A|$ denotes the absolute-value matrix given by $|A| = (|a_{ij}|)_{n\times n}$; $diag(b_1, b_2, \cdots, b_n)$ denotes the diagonal matrix with diagonal entries $b_1, b_2, \cdots, b_n$; $\rho(A)$ denotes the spectral radius of A; $\|u\|$ denotes a vector norm defined by $\|u\| = \max_{1\leq i\leq n} |u_i|$, while $\|A\|$ denotes a matrix norm defined by $\|A\| = \max_{1\leq i\leq n}\{\sum_{j=1}^{n} |a_{ij}|\}$. For $A, B \in R^{m\times n}$, $A \geq B (A > B)$ means that each pair of corresponding elements of A and B satisfies the inequality "$\geq (>)$". $C[X, Y]$ denotes the space of continuous mappings from the topological space X to the topological space Y. $PC[I, R^n] = \{\psi : I \to R^n \big| \ \psi(t^+) = \psi(t)$ for $t \in I, \psi(t^-)$ exists for $t \in (t_0, +\infty), \psi(t^-) = \psi(t)$ for all but points $t_k \in (t_0, +\infty)\}$, where $I \subseteq R$ is an interval.

Definition 1. *A function $u(t) : R \to R^n$ is called a solution of model (1) with the initial condition $u(s) = \phi(s) \in PC((-\infty, t_0], R^n)$, if $u(t)$ is continuous at $t \neq t_k$ and $t \geq t_0$, $u(t_k) = u(t_k^+)$ and $u(t_k^-)$ exists, $u(t)$ satisfies model (1) for*

$t \geq t_0$ under the initial condition. Especially, a point $u^* \in R^n$ is called an equilibrium point of model (1), if $u(t) = u^*$ is a solution of model (1).

Definition 2. *An equilibrium point $u^* = (u_1^*, u_2^*, \cdots, u_n^*)^T$ of model (1) is said to be globally exponentially stable, if there exist constants $\varepsilon > 0$ and $M > 0$ such that*

$$\|u(t) - u^*\| \leq M\|\phi - u^*\|e^{-\varepsilon(t-t_0)}$$

for all $t > t_0$, where $u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T$ is any solution of model (1) with initial value $u_i(s) = \phi_i(s) \in PC((-\infty, t_0], R)$, $i = 1, 2, \cdots, n$, and $\|\phi - u^*\| = \max_{1\leq i\leq n} |\phi_i(s) - u_i^*|_\infty$ with $|\phi_i(s) - u_i^*|_\infty = \sup_{s\in(-\infty,t_0]} |\phi_i(s) - u_i^*|$.

Definition 3. [32] *A real matrix $A = (a_{ij})_{n\times n}$ is said to be an M-matrix if $a_{ij} \leq 0 (i, j = 1, 2, \cdots, n; i \neq j)$ and $A^{-1} \geq 0$.*

To prove our results, the following lemmas are necessary.

Lemma 1. [32] *Let Q be $n\times n$ matrix with non-positive off-diagonal elements, then Q is an M-matrix if and only if one of the following conditions holds.*

(i) There exists a vector $\xi > 0$ such that $\xi^T Q > 0$.
(ii) There exists a vector $\xi > 0$ such that $Q\xi > 0$.

when A is an M-matrix, denote

$$\Omega(A) = \{\xi \in R^n | A\xi > 0, \xi > 0\}, \tag{3}$$

from Lemma 1, we know that $\Omega(A)$ is nonempty.

Lemma 2. [38] *Let A be a nonnegative matrix, then $\rho(A)$ is a eigenvalue of A, and A has at least one positive eigenvector which is provided by $\rho(A)$.*

When A is an nonnegative matrix, denote

$$\Gamma(A) = \{\xi \in R^n | A\xi = \rho(A)\xi\}, \tag{4}$$

from Lemma 2, we know that $\Gamma(A)$ is nonempty.

Lemma 3. [34] *Suppose u and u' are two state of model (1), then we have*

$$\Big|\bigwedge_{j=1}^{n} \alpha_{ij} f_j(u_j) - \bigwedge_{j=1}^{n} \alpha_{ij} f_j(u'_j)\Big| \leq \sum_{j=1}^{n} \Big|\alpha_{ij}\Big| \cdot \Big|f_j(u_j) - f_j(u'_j)\Big|,$$

$$\Big|\bigvee_{j=1}^{n} \beta_{ij} f_j(u_j) - \bigvee_{j=1}^{n} \beta_{ij} f_j(u'_j)\Big| \leq \sum_{j=1}^{n} \Big|\beta_{ij}\Big| \cdot \Big|f_j(u_j) - f_j(u'_j)\Big|.$$

Throughout this paper, we make the following assumptions:

(**H**1) If $(u_1^*, u_2^*, \cdots, u_n^*)^T$ is an equilibrium point of model (2), then impulsive jumps I_k of model (1) satisfy the following conditions [23]-[26]

$$I_{ik}(u_i^*) = 0, \quad k = 1, 2, \cdots, ; i = 1, 2, \cdots, n.$$

(**H**2) For function f_j, there exists a positive diagonal matrix $F = diag(F_1, F_2, \cdots, F_n)$ such that

$$F_j = \sup_{x_1 \neq x_2} \left| \frac{f_j(x_1) - f_j(x_2)}{x_1 - x_2} \right|$$

for all $x_1 \neq x_2$, $j = 1, 2, \cdots, n$.

(**H**3) The delay kernel $K_{ij} : [0, +\infty) \to [0, +\infty)$ is real valued nonnegative continuous function and satisfies [18]

$$\int_0^{+\infty} e^{\beta s} K_{ij}(s) ds = r_{ij}(\beta),$$

where $p_{ij}(\beta)$ is continuous function in $[0,\sigma)$, $\sigma > 0$, and $p_{ij}(0) = 1$, $i, j = 1, 2, \cdots, n$.

(**H**4) Let $u + I_k(u) := p_k(u)$, then there exist nonnegative matrices $P_k = (p_{ij}^{(k)})_{n \times n}$ such that

$$|p_k(x_1) - p_k(x_2)| \leq P_k |x_1 - x_2|$$

for all $x_1, x_2 \in R^n$, $k = 1, 2, \cdots$.

3. Main results

Theorem 1. *Under assumptions (**H**1)- (**H**4), model (1) has a unique equilibrium point, which is globally exponentially stable and the exponential convergence rate index equals $\varepsilon - \lambda$, if the following conditions are satisfied*

(i) $W = C - (|A| + |\alpha| + |\beta| + |\delta| + |\eta|)F$ *is an* $M-$*matrix.*

(ii) $\Delta = \Omega(W) \bigcap_{k=1}^{\infty} \Gamma(P_k)$ *is nonempty.*

(iii) Let

$$\gamma_k \geq \max\{1, \rho(P_k)\}, \tag{5}$$

and there exists a constant λ such that

$$\frac{\ln \gamma_k}{t_k - t_{k-1}} \leq \lambda < \varepsilon, \quad k = 1, 2, \cdots, \tag{6}$$

where the scalar $\varepsilon > 0$ is determined by the inequality

$$\xi_i(\varepsilon - c_i) + \sum_{j=1}^{n} \xi_j F_j \Big(|a_{ij}| + e^{\varepsilon \tau}(|\alpha_{ij}| + |\beta_{ij}|) + r_{ij}(\varepsilon)(|\delta_{ij}| + |\eta_{ij}|) \Big) < 0 \tag{7}$$

for a given $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T \in \Delta$, $\tau = \max_{1 \leq i \leq n, 1 \leq j \leq n} \{\tau_{ij}\}$.

Proof. Since equilibrium point $u^* = (u_1^*, u_2^*, \cdots, u_n^*)^T$ of model (2) satisfy the following equation

$$\begin{aligned} c_i u_i^* \;=\; & \sum_{j=1}^{n} a_{ij} f_j(u_j^*) + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(u_j^*) + \bigvee_{j=1}^{n} \beta_{ij} f_j(u_j^*) \\ & + \bigwedge_{j=1}^{n} \delta_{ij} \int_{-\infty}^{t} K_{ij}(t-s) f_j(u_j^*) ds + \bigvee_{j=1}^{n} \eta_{ij} \int_{-\infty}^{t} K_{ij}(t-s) f_j(u_j^*) ds \\ & + \sum_{j=1}^{n} b_{ij} v_j + I_i + \bigwedge_{j=1}^{n} T_{ij} v_j + \bigvee_{j=1}^{n} H_{ij} v_j \end{aligned}$$

for $i = 1, 2, \cdots, n$. From assumption (**H**3), we get

$$\begin{aligned} c_i u_i^* \;=\; & \sum_{j=1}^{n} a_{ij} f_j(u_j^*) + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(u_j^*) + \bigvee_{j=1}^{n} \beta_{ij} f_j(u_j^*) + \bigwedge_{j=1}^{n} \delta_{ij} f_j(u_j^*) \\ & + \bigvee_{j=1}^{n} \eta_{ij} f_j(u_j^*) + \sum_{j=1}^{n} b_{ij} v_j + I_i + \bigwedge_{j=1}^{n} T_{ij} v_j + \bigvee_{j=1}^{n} H_{ij} v_j \end{aligned}$$

for $i = 1, 2, \cdots, n$. By using of Lemma 3 and theory of homeomorphism, the proof for the existence and uniqueness of the equilibrium point of model (2) is similar to the proof of Theorem 1 in [18], we omit it. From assumption (**H**1), we know that model (1) have a unique equilibrium point. So, we will only prove that this unique equilibrium point of model (1) is global exponentially stable.

Let $u^* = (u_1^*, u_2^*, \cdots, u_n^*)^T$ be the unique equilibrium point of model (1). Denote

$$y_i(t) = u_i(t) - u_i^*, \quad \widetilde{f}_j(y_j(t)) = f_j(y_j(t) + u_j^*) - f_j(u_j^*),,$$
$$\widetilde{p}_{ik}(y_i(t)) = p_{ik}(y_i(t) + u_i^*) - p_{ik}(u_i^*)$$

then model (1) can be rewritten as

$$\begin{cases} \frac{dy_i(t)}{dt} = -c_i y_i(t) + \sum\limits_{j=1}^{n} a_{ij} \widetilde{f}_j(y_j(t)) + \bigwedge\limits_{j=1}^{n} \alpha_{ij} \widetilde{f}_j(y_j(t-\tau_{ij}(t))) \\ \qquad + \bigvee\limits_{j=1}^{n} \beta_{ij} \widetilde{f}_j(y_j(t-\tau_{ij}(t))) + \bigwedge\limits_{j=1}^{n} \delta_{ij} \int\limits_{-\infty}^{t} K_{ij}(t-s) \widetilde{f}_j(y_j(s)) ds \\ \qquad + \bigvee\limits_{j=1}^{n} \eta_{ij} \int\limits_{-\infty}^{t} K_{ij}(t-s) \widetilde{f}_j(y_j(s)) ds, \\ y_i(t_k^+) = \widetilde{p}_{ik}(y_i(t_k^-)). \end{cases} \tag{8}$$

Since W is an M-matrix and the set Δ is nonempty, from lemma 1, there exists a positive vector $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T \in \Delta \subseteq \Omega(W)$ such that

$$-\xi_i c_i + \sum_{j=1}^{n} \xi_j F_j \Big(|a_{ij}| + (|\alpha_{ij}| + |\beta_{ij}| + |\delta_{ij}| + |\eta_{ij}| \Big) < 0. \tag{9}$$

Considering functions

$$\begin{aligned} L_i(x) &= \xi_i(x-c_i)+\sum_{j=1}^{n}\xi_jF_j\Big(|a_{ij}|+e^{x\tau}(|\alpha_{ij}|+|\beta_{ij}|)+r_{ij}(x)(|\delta_{ij}|+|\eta_{ij}|)\Big),\\ & \quad i=1,2,\cdots,n. \end{aligned}$$

From (9) and assumption (**H**3), we know that $L_i(0)<0$ and $L_i(x)$ is continuous. Since $\frac{dL_i(x)}{dx}>0$, $L_i(x)$ is strictly monotone increasing, there exist $\varepsilon_i>0$ such that

$$\begin{aligned} L_i(\varepsilon_i) &= \xi_i(\varepsilon_i-c_i)\\ &\quad +\sum_{j=1}^{n}\xi_jF_j\Big(|a_{ij}|+e^{\varepsilon_i\tau}(|\alpha_{ij}|+|\beta_{ij}|)+r_{ij}(\varepsilon_i)(|\delta_{ij}|+|\eta_{ij}|)\Big)<0, \end{aligned}$$

$i=1,2,\cdots,n.$

Choosing $0<\varepsilon<\min\{\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_n\}$, then

$$\xi_i(\varepsilon-c_i)+\sum_{j=1}^{n}\xi_jF_j\Big(|a_{ij}|+e^{\varepsilon\tau}(|\alpha_{ij}|+|\beta_{ij}|)+r_{ij}(\varepsilon)(|\delta_{ij}|+|\eta_{ij}|)\Big)<0, \quad (10)$$

$i=1,2,\cdots,n.$

Let

$$x_i(t)=e^{\varepsilon(t-t_0)}|y_i(t)|, \qquad i=1,2,\cdots,n.$$

Calculating the upper right derivative $D^+x_i(t)$ of $x_i(t)$ along the solutions of (8), from Lemma 3 and the assumption (**H**2), we can get

$$\begin{aligned}
D^+x_i(t) &= \varepsilon e^{\varepsilon(t-t_0)}|y_i(t)| + e^{\varepsilon(t-t_0)} sgn(y_i(t))\Big\{ -c_i y_i(t) + \sum_{j=1}^{n} a_{ij}\widetilde{f}_j(y_j(t)) \\
&\quad + \bigwedge_{j=1}^{n} \alpha_{ij}\widetilde{f}_j(y_j(t-\tau_{ij}(t))) + \bigvee_{j=1}^{n} \beta_{ij}\widetilde{f}_j(y_j(t-\tau_{ij}(t))) \\
&\quad + \bigwedge_{j=1}^{n} \delta_{ij} \int_{-\infty}^{t} K_{ij}(t-s)\widetilde{f}_j(y_j(s))ds \\
&\quad + \bigvee_{j=1}^{n} \eta_{ij} \int_{-\infty}^{t} K_{ij}(t-s)\widetilde{f}_j(y_j(s))ds \Big\} \\
&\le e^{\varepsilon(t-t_0)}\Big\{(\varepsilon - c_i)|y_i(t)| + \sum_{j=1}^{n} |a_{ij}|F_j|y_j(t)| \\
&\quad + \sum_{j=1}^{n}(|\alpha_{ij}| + |\beta_{ij}|)F_j|y_j(t-\tau_{ij}(t))| \\
&\quad + \sum_{j=1}^{n}(|\delta_{ij}| + |\eta_{ij}|)F_j \int_{-\infty}^{t} K_{ij}(t-s)|y_j(s)|ds\Big\} \\
&= (\varepsilon - c_i)x_i(t) + \sum_{j=1}^{n} |a_{ij}|F_j x_j(t) \\
&\quad + \sum_{j=1}^{n}(|\alpha_{ij}| + |\beta_{ij}|)F_j e^{\varepsilon\tau_{ij}(t)} x_j(t-\tau_{ij}(t)) \\
&\quad + \sum_{j=1}^{n}(|\delta_{ij}| + |\eta_{ij}|)F_j \int_{-\infty}^{t} e^{\varepsilon(t-s)} K_{ij}(t-s)x_j(s)ds \\
&\le (\varepsilon - c_i)x_i(t) + \sum_{j=1}^{n} |a_{ij}|F_j x_j(t) \\
&\quad + \sum_{j=1}^{n}(|\alpha_{ij}| + |\beta_{ij}|)F_j e^{\varepsilon\tau} x_j(t-\tau_{ij}(t)) \\
&\quad + \sum_{j=1}^{n}(|\delta_{ij}| + |\eta_{ij}|)F_j \int_{-\infty}^{t} e^{\varepsilon(t-s)} K_{ij}(t-s)x_j(s)ds \qquad (11)
\end{aligned}$$

for $i = 1, 2, \cdots, n; t_{k-1} < t < t_k, k = 1, 2, \cdots$.

Let $l_0 = \frac{\|\phi - u^*\|}{\min\limits_{1\le i\le n}\{\xi_i\}}$, then

$$x_i(s) = e^{\varepsilon(s-t_0)}|y_i(s)| \le |y_i(s)| = |\phi_i(s) - u_i^*| \le \|\phi - u^*\| \le \xi_i l_0 \qquad (12)$$

for $-\infty < s \le t_0 \quad i = 1, 2, \cdots, n$. We can prove that

$$x_i(t) \le \xi_i l_0, \quad t_0 \le t < t_1, \quad i = 1, 2, \cdots, n. \qquad (13)$$

In fact, if inequality (13) is not true, then there must exist some i and $t^* \in [t_0, t_1)$ such that

$$x_i(t^*) = \xi_i l_0, \quad D^+ x_i(t^*) \ge 0 \quad \text{and} \quad x_j(t) \le \xi_j l_0$$

for $-\infty < t \le t^*$, $j = 1, 2, \cdots, n$. However, from (10), (11) and (**H**2), we get

$$\begin{aligned} D^+ x_i(t^*) \le & \Big((\varepsilon - c_i)\xi_i + \sum_{j=1}^{n} |a_{ij}| F_j \xi_j + \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|) F_j e^{\varepsilon\tau} \xi_j \\ & + \sum_{j=1}^{n} (|\delta_{ij}| + |\eta_{ij}|) F_j r_{ij}(\varepsilon) \xi_j \Big) l_0 < 0, \end{aligned} \qquad (12)$$

this is a contradiction. So inequality (13) is true. Thus, we have

$$|y_i(t)| \le \xi_i l_0 e^{-\varepsilon(t-t_0)}, \quad t_0 \le t < t_1, \quad i = 1, 2, \cdots, n. \qquad (14)$$

In the following, we will use the mathematical induction to prove that

$$\begin{aligned} |y_i(t)| \le \gamma_0 \gamma_1 \cdots \gamma_{k-1} \xi_i l_0 e^{-\varepsilon(t-t_0)}, \quad t_{k-1} \le t < t_k, \\ i = 1, 2, \cdots, n; k = 1, 2, \cdots, \end{aligned} \qquad 15$$

hold, where $\gamma_0 = 1$.

When $k = 1$, from inequality (14) we know that inequality (15) hold.

Suppose that the inequalities

$$|y_i(t)| \le \gamma_0 \gamma_1 \cdots \gamma_{m-1} \xi_i l_0 e^{-\varepsilon(t-t_0)}, \quad t_{k-1} \le t < t_k, \quad i = 1, 2, \cdots, n, \qquad (16)$$

hold for $k = 1, 2, \cdots, m$.

From assumption (**H**4) and (16), we know that the second equation of model (8) satisfies

$$\begin{aligned} |y_i(t_m)| & \le \sum_{j=1}^{n} p_{ij}^{(m)} |y_j(t_m^-)| \\ & \le \sum_{j=1}^{n} p_{ij}^{(m)} \gamma_0 \gamma_1 \cdots \gamma_{m-1} \xi_j l_0 e^{-\varepsilon(t_m - t_0)} \end{aligned} \qquad (17)$$

for $i = 1, 2, \cdots, n$.

From $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T \in \Delta$, we know that $\xi \in \Gamma(P_m)$. That is

$$P_m \xi = \rho(P_m) \xi,$$

i.e.,

$$\sum_{j=1}^{n} p_{ij}^{(m)}\xi_j = \rho(P_m)\xi_i, \qquad i = 1, 2, \cdots, n. \tag{18}$$

Applying (18) to (17), we get

$$|y_i(t_m)| \le \gamma_0\gamma_1\cdots\gamma_{m-1}\rho(P_m)\xi_i l_0 e^{-\varepsilon(t_m-t_0)}, \quad i = 1, 2, \cdots, n. \tag{19}$$

From (5) and (19), we have

$$|y_i(t_m)| \le \gamma_0\gamma_1\cdots\gamma_{m-1}\gamma_m\xi_i l_0 e^{-\varepsilon(t_m-t_0)}, \quad i = 1, 2, \cdots, n. \tag{20}$$

This, together with both (12), (16) and (20), lead to

$$|y_i(t)| \le \gamma_0\gamma_1\cdots\gamma_{m-1}\gamma_m\xi_i l_0 e^{-\varepsilon(t-t_0)}, \quad i = 1, 2, \cdots, n; t \in (-\infty, t_m], \tag{21}$$

i.e.,

$$x_i(t) \le \gamma_0\gamma_1\cdots\gamma_{m-1}\gamma_m\xi_i l_0, \quad i = 1, 2, \cdots, n; t \in (-\infty, t_m]. \tag{22}$$

In the following, we will prove that

$$x_i(t) \le \gamma_0\gamma_1\cdots\gamma_{m-1}\gamma_m\xi_i l_0, \quad i = 1, 2, \cdots, n; t \in [t_m, t_{m+1}), \tag{23}$$

hold.

If (23) is not true, then there must exists some i and $t^{**} \in [t_m, t_{m+1})$ such that

$$x_i(t^{**}) = \gamma_0\gamma_1\cdots\gamma_{m-1}\gamma_m\xi_i l_0, \quad D^+x_i(t^{**}) \ge 0$$

and

$$x_j(t) \le \gamma_0\gamma_1\cdots\gamma_{m-1}\gamma_m\xi_j l_0$$

for $-\infty < t \le t^{**}$, $j = 1, 2, \cdots, n$. However, from (11) and (10), we get

$$\begin{aligned} D^+x_i(t^{**}) \;\; &\le \;\; \Big[(\varepsilon - c_i)\xi_i + \sum_{j=1}^{n}|a_{ij}|F_j\xi_j + \sum_{j=1}^{n}(|\alpha_{ij}| + |\beta_{ij}|)F_j e^{\varepsilon\tau}\xi_j \\ &\quad + \sum_{j=1}^{n}(|\delta_{ij}| + |\eta_{ij}|)F_j r_{ij}(\varepsilon)\xi_j\Big]\gamma_0\gamma_1\cdots\gamma_{m-1}\gamma_m l_0 \\ &< \;\; 0, \end{aligned}$$

this is a contradiction. So (23) holds.

By the mathematical induction, we can conclude that (15) holds.

From (6), we have

$$\gamma_k \le e^{\lambda(t_k-t_{k-1})}, \qquad k = 1, 2, \cdots .$$

From (15), we get

$$\begin{aligned}|y_i(t)| &\leq e^{\lambda(t_1-t_0)}e^{\lambda(t_2-t_1)}\cdots e^{\lambda(t_{k-1}-t_{k-2})}\xi_i l_0 e^{-\varepsilon(t-t_0)}\\ &= \frac{\xi_i}{\min\limits_{1\leq i\leq n}\{\xi_i\}}\|\phi-u^*\|e^{\lambda(t_{k-1}-t_0)}e^{-\varepsilon(t-t_0)}\\ &\leq \frac{\xi_i}{\min\limits_{1\leq i\leq n}\{\xi_i\}}\|\phi-u^*\|e^{\lambda(t-t_0)}e^{-\varepsilon(t-t_0)}\\ &= \frac{\xi_i}{\min\limits_{1\leq i\leq n}\{\xi_i\}}\|\phi-u^*\|e^{-(\varepsilon-\lambda)(t-t_0)}\end{aligned}$$

for any $t\in[t_{k-1},t_k), k=1,2,\cdots$, that is

$$|u_i(t)-u_i^*|\leq \frac{\xi_i}{\min\limits_{1\leq i\leq n}\{\xi_i\}}\|\phi-u^*\|e^{-(\varepsilon-\lambda)(t-t_0)}$$

for $t\geq t_0$. So

$$\|u(t)-u^*\|\leq M\|\phi-u^*\|e^{-(\varepsilon-\lambda)(t-t_0)}$$

for $t\geq t_0$, where $M=\max\limits_{1\leq i\leq n}\{\xi_i\}/\min\limits_{1\leq i\leq n}\{\xi_i\}\geq 1$. This means that the equilibrium point u^* of model (1) is globally exponentially stable, and the exponential convergence rate index equals $\varepsilon-\lambda$. The proof is completed. □

Remark 3. *We may properly choose the matrix P_k in assumption (**H**4) to guarantee Δ in Theorem 1 be nonempty. Especially, when $P_k=p_kE$ (p_k is nonnegative constant), Δ is certainly nonempty. So, by using Theorem 1, we easily obtain the following corollary.*

Corollary 1. *Under assumptions (**H**1)- (**H**3) and $P_k=P=diag(p_1,p_2,\cdots,p_n)$, model (1) has a unique equilibrium point, which is globally exponentially stable and the exponential convergence rate index equals $\varepsilon-\lambda$, if the following conditions are satisfied*

(i) $W=C-(|A|+|\alpha|+|\beta|+|\delta|+|\eta|)F$ is an $M-$matrix.

(ii) Let $\gamma_k\geq\max\{1,p_k\}$, and there exists a constant λ such that

$$\frac{\ln\gamma_k}{t_k-t_{k-1}}\leq\lambda<\varepsilon,\quad k=1,2,\cdots,$$

where the scalar $\varepsilon>0$ is determined by the inequality

$$\xi_i(\varepsilon-c_i)+\sum_{j=1}^{n}\xi_jF_j\Big(|a_{ij}|+e^{\varepsilon\tau}(|\alpha_{ij}|+|\beta_{ij}|)+r_{ij}(\varepsilon)(|\delta_{ij}|+|\eta_{ij}|)\Big)<0$$

for a given $\xi=(\xi_1,\xi_2,\cdots,\xi_n)^T\in\Omega(W)$, $\tau=\max\limits_{1\leq i\leq n,1\leq j\leq n}\{\tau_{ij}\}$.

Remark 4. *Theorem 1 and Corollary 1 show the fact that the exponential stability of model (1) still remains even under strong impulsive perturbations if model (2) exponentially converges as fast as possible.*

Corollary 2. *Let assumptions (***H***2) and (***H***3) hold, if*

$$W = C - (|A| + |\alpha| + |\beta| + |\delta| + |\eta|)F$$

is an $M-$matrix, then model (2) has a unique equilibrium point, which is globally exponentially stable, and the exponential convergence rate index $\varepsilon > 0$ is determined by the inequality

$$\xi_i(\varepsilon - c_i) + \sum_{j=1}^{n} \xi_j F_j \Big(|a_{ij}| + e^{\varepsilon\tau}(|\alpha_{ij}| + |\beta_{ij}|) + r_{ij}(\varepsilon)(|\delta_{ij}| + |\eta_{ij}|) \Big) < 0$$

for $i = 1, 2, \cdots, n$, and $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T \in \Omega(W)$, $\tau = \max_{1 \le i \le n, 1 \le j \le n} \{\tau_{ij}\}$.

Proof. From W is an $M-$ matrix, we know that model (2) has one unique equilibrium point. Since model (2) is a special case of model (1) with $I_k(u) = 0$, we have $p_k(u) = u$. So, (**H**1) hold, and (**H**4) with $P_k = diag\{1, 1, \cdots, 1\}$ hold. From Corollary 1, we can get that the unique equilibrium point of model (2) is globally exponentially stable. The proof is completed. □

Corollary 3. *Let assumption (***H***2) hold, if*

$$W = C - (|A| + |\alpha| + |\beta|)F$$

is an $M-$matrix, then model

$$\begin{aligned} \frac{du_i(t)}{dt} &= -c_i u_i(t) + \sum_{j=1}^{n} a_{ij} f_j(u_j(t)) + \sum_{j=1}^{n} b_{ij} v_j + J_i \\ &\quad + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(u_j(t - \tau_{ij}(t))) + \bigvee_{j=1}^{n} \beta_{ij} f_j(u_j(t - \tau_{ij}(t))) \\ &\quad + \bigwedge_{j=1}^{n} T_{ij} v_j + \bigvee_{j=1}^{n} H_{ij} v_j, \quad t \ge 0, \quad i = 1, 2, \cdots, n \end{aligned} \tag{24}$$

has a unique equilibrium point, which is globally exponentially stable, and the exponential convergence rate index $\varepsilon > 0$ is determined by the inequality

$$\xi_i(\varepsilon - c_i) + \sum_{j=1}^{n} \xi_j F_j \Big(|a_{ij}| + e^{\varepsilon\tau}(|\alpha_{ij}| + |\beta_{ij}|) \Big) < 0$$

for $i = 1, 2, \cdots, n$, and $\xi = (\xi_1, \xi_2, \cdots, \xi_n)^T \in \Omega(W)$, $\tau = \max_{1 \le i \le n, 1 \le j \le n} \{\tau_{ij}\}$.

Remark 5. Corollary 3 extends and improves the corresponding results on the stability of model (24) in [35]-[37]. In [35] and [36], the differentiability on time-varying delays is required. In addition, the following two examples show that the results obtained in this paper have a less restriction than those in [35] and [37].

4. Examples

Example 1. *Consider the following model*

$$\begin{aligned}\frac{du_i(t)}{dt} &= -c_i u_i(t) + \sum_{j=1}^{2} a_{ij} f_j(u_j(t)) + \sum_{j=1}^{2} b_{ij} v_j + J_i \\ &\quad + \bigwedge_{j=1}^{2} \alpha_{ij} f_j(u_j(t-\tau_{ij}(t))) + \bigvee_{j=1}^{2} \beta_{ij} f_j(u_j(t-\tau_{ij}(t))) \\ &\quad + \bigwedge_{j=1}^{2} T_{ij} v_j + \bigvee_{j=1}^{2} H_{ij} v_j, \quad t \geq 0, \quad i = 1, 2, \qquad (25)\end{aligned}$$

where

$$C = \begin{pmatrix} 2.2 & 0 \\ 0 & 9.6 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & -1.1 \\ 0.8 & 1 \end{pmatrix},$$

$$\alpha = \begin{pmatrix} 0.3 & 0.9 \\ 0.7 & 0.2 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0.2 & 0.5 \\ 0.6 & 0.4 \end{pmatrix},$$

$$f_1(\theta) = f_2(\theta) = \frac{1}{2}(|\theta+1| + |\theta-1|), \quad \tau_1(t) = \tau_2(t) = 3 + 2|\sin t|.$$

*Obviously, assumption (***H***2) hold, and* $F = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\tau = 5$. *It is easy computing that* $C - (|A| + |\alpha| + |\beta|)F = \begin{pmatrix} 0.7 & -2.5 \\ -2.1 & 8 \end{pmatrix}$ *is an* M*-matrix. By Corollary 3, we know that model (25) has a unique equilibrium point which is globally exponentially stable, and the exponential converging index* $\lambda = 0.01326$.

Since $\tau_1(t)$ *and* $\tau_2(t)$ *are not differentiable, the conclusions in* [35] *and* [36] *are not applicable to ascertain the stability of model (25).*

Since

$$\begin{aligned} C - (|A| + diag\Big(\max_{1\leq j\leq 2}\{|\alpha_{1j}F_j|\}, \max_{1\leq j\leq 2}\{|\alpha_{2j}F_j|\}\Big) \\ + diag\Big(\max_{1\leq j\leq 2}\{|\beta_{1j}F_j|\}, \max_{1\leq j\leq 2}\{|\beta_{2j}F_j|\}\Big)) = \begin{pmatrix} -0.2 & -1.1 \\ -0.8 & 7.3 \end{pmatrix}\end{aligned}$$

is not an M*-matrix, the conclusions in* [37] *are not applicable to ascertain the stability of model (25).*

Example 2. *Consider the following model*

$$\begin{cases} \frac{du_i(t)}{dt} = -c_i u_i(t) + \sum\limits_{j=1}^{2} a_{ij} f_j(u_j(t)) + \sum\limits_{j=1}^{2} b_{ij} v_j + J_i \\ \qquad + \bigwedge\limits_{j=1}^{2} \alpha_{ij} f_j(u_j(t-\tau_{ij}(t))) + \bigvee\limits_{j=1}^{2} \beta_{ij} f_j(u_j(t-\tau_{ij}(t))) \\ \qquad + \bigwedge\limits_{j=1}^{2} \delta_{ij} \int\limits_{-\infty}^{t} K_{ij}(t-s) f_j(u_j(s)) ds \\ \qquad + \bigvee\limits_{j=1}^{2} \eta_{ij} \int\limits_{-\infty}^{t} K_{ij}(t-s) f_j(u_j(s)) ds \\ \qquad + \bigwedge\limits_{j=1}^{2} T_{ij} v_j + \bigvee\limits_{j=1}^{2} H_{ij} v_j, \quad t \neq t_k, \quad t \geq 0, \\ \triangle u_i(t_k) = u_i(t_k^+) - u_i(t_k^-) = I_{ik}(u_i(t_k)), \quad i = 1, 2; \quad k = 1, 2, \cdots, \end{cases} \tag{26}$$

where

$$C = \begin{pmatrix} 4.9 & 0 \\ 0 & 4.8 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & -1.2 \\ 0.3 & 2 \end{pmatrix}, \quad \alpha = \begin{pmatrix} 0.3 & 0.4 \\ 0.1 & 0.5 \end{pmatrix},$$

$$\beta = \begin{pmatrix} 0.1 & 0.6 \\ 0.3 & 0.2 \end{pmatrix}, \quad \delta = \begin{pmatrix} 0.1 & 0.2 \\ 0.4 & 0.5 \end{pmatrix}, \quad \eta = \begin{pmatrix} 0.1 & 0.6 \\ 0.3 & 0.2 \end{pmatrix},$$

$$B = T = H = \begin{pmatrix} 0.3 & 0.7 \\ 0.2 & 0.9 \end{pmatrix}, \quad V = (0.3, 0.7)^T, \quad J = (2.34, 2.02)^T,$$

$f_1(x) = f_2(x) = x$, $K_{ij}(t) = te^{-t}$, $\tau_{ij}(t) = 0.01(|\cos t| + 1)$, $t_0 = 0.5$, $t_k = t_{k-1} + 0.5$, $I_{1k}(u_1(t_k)) = 0.005(u_1(t_k) - 1)$, $I_{2k}(u_2(t_k)) = -0.005(u_2(t_k) - 1)$ *for* $k = 1, 2, 3, \cdots$.

One can verify that the point $(1, 1)^T$ *is an equilibrium point of model (26), and assumptions (***H***1)-(***H***4) hold, and* $\tau = 0.02$, $F = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $P_k = \begin{pmatrix} 1.005 & 0 \\ 0 & 1.005 \end{pmatrix}$ *for* $k = 1, 2, 3, \cdots$.

It is easily computing that

$$W = C - (|A| + |\alpha| + |\beta| + |\delta| + |\eta|)F = \begin{pmatrix} 3.3 & -3 \\ -1.4 & 1.4 \end{pmatrix}$$

is an M-matrix, and $\Gamma(P_k) = R^2$, $\Omega(W) = \{(z_1, z_2)^T | z_1 < z_2 < 1.1z_1, z_1 > 0, z_2 > 0\}$, *so* $\Delta = \{(z_1, z_2)^T | z_1 < z_2 < 1.1z_1, z_1 > 0, z_2 > 0\}$ *is nonempty. Let* $\xi = (1, 1.05)^T \in \Delta$ *and* $\varepsilon = 0.01753$, *which satisfy inequalities*

$$\xi_i(\varepsilon - c_i) + \sum_{j=1}^{n} \xi_j F_j \Big(|a_{ij}| + e^{\varepsilon\tau}(|\alpha_{ij}| + |\beta_{ij}|) + r_{ij}(\varepsilon)(|\delta_{ij}| + |\eta_{ij}|) \Big) < 0, \quad i, j = 1, 2.$$

Taking $\gamma_k = 1.005$, $\lambda = 0.01$, *which satisfy inequalities*

$$\gamma_k \geq \max\{1, \rho(P_k)\},$$

and

$$\frac{\ln \gamma_k}{t_k - t_{k-1}} = \frac{\ln 1.005}{0.5} \leq \lambda < \varepsilon, \quad k = 1, 2, \cdots .$$

Clearly, all conditions of Theorem 1 are satisfied. From Theorem 1, we know that the unique equilibrium point $(1,1)^T$ *of model (26) is globally exponentially stable, and the exponential convergence rate index equals 0.00753.*

5. Conclusions

In this paper, the global exponential stability of the impulsive fuzzy recurrent neural network with both time-varying delays and distributed delays has been studied. Several sufficient conditions have been obtained to ensure the existence, uniqueness, and global exponential stability of equilibrium point for impulsive fuzzy recurrent neural networks model with both time-varying delays and distributed delays. The sufficient conditions obtained are delay-independent, which implies that the strong self-regulation is dominant in the networks. In particular, the estimate of the exponential converging index was also provided, which depends on the system parameters. Compared with the method of Lyapunov functional that is employed by previous publications, our method is simpler and more effective for stability analysis of the fuzzy recurrent neural networks with both time-varying delays and distributed delays. Two examples have shown that our results are less restrictive than previously known criteria.

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