Applied Mathematical Sciences, Vol. 3, 2009, no. 26, 1295 - 1302

On Poisson Repairmen Model:M^X/M/2/K/N with Balk Arrival, Balking, Reneging and Hetrogeneous Repairmen

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Abstract

The aim of this Paper is to derive the analytical Solution of the queue: M^X/M/2/N/k for repairmen model with bulk arrival, balking, reneging and two heterogeneous in which (i) units (machines) arrive in bulk of random size with the arrival bulk following poisson distribution, (ii) the queue discipline is FCFS, it is being assumed that the bulk are Pre-ordered for service purpose; and (iii) the service time distribution is exponential. Recurrence relation connecting the various probabilities introduced is founded. Some measures of effectiveness are deduced. Also, some special cases are obtained.

Keywords: Heterogeneous repairmen, repairmen model, bulk arrival, balking and reneging.

1 Introduction

The system: M/M/2/K/N with balking, reneging and two heterogeneous repairmen but without bulk arrival has been treated by Shawky [1]. Kleinrock [7] studied the queue: M/M/C/K/N and White et al. [5] treated the system: M/M/C/K/K. These studies were obtained without bulk arrival .The present paper treats the analytical solution of the queue: $M^X/M/2/K/N$ for repairmen model with bulk arrival, balking, reneging and two heterogeneous repairmen.

2 Description of the Model

In the present model, it is assumed that the machines arrive at the system in bulkes of random size X, i.e,at each moment of arrival, there is probability $c_j = p(x=j)$ that j machines arrive simultaneously , $c_o = 0$, the bulk arrival follows the poisson distribution with rate λ and the service time distribution is an exponential with service rates μ_1 and μ_2. Let $\lambda c_j \Delta t$, (j =1,2,…k) , be the first order probability that a bulk of j machines comes in time Δt . We assume that we have a finite source of N machines, two heterogeneous repairmen are available and the model has a finite storage room such that the total number of machines in the system is no more than k. The queue discipline considered here is a modification of both Singh [8] and Krishnamurthi [3] and it is:

(i) If both repairmen are free, the head machine of the queue goes to the first repairman with probability π_1 or to the second repairman with probability π_2, $\pi_1 + \pi_2 = 1$.

(ii) If only one repairman is free, the head machine goes to it.

(iii) If the two repairmen are busy, the machines wait in their order until any repairman becomes vacant.

Consider the balk Concept with probability:

$$\beta = \text{prob. \{a customer joins the queue\}},$$

where $0 \le \beta < 1$ if n = 2,3,…,k and $\beta = 1$ if n = 0,1 .

We assume that the customer may renege according to an exponential distribution, f(t) = $\alpha e^{-\alpha t}, t > 0$, with parameter α . The probability of reneging in a short period of time Δt is given by $r_n = (n-2)\alpha\Delta t$, for $2 < n \le k$ and $r_n = 0$ for $n = 0,1,2$.

3 The steady – state equation and their solution.

We define the equilibrium probabilities:

$p_{0,0}$ = prob. {there is no machines in the system},

$p_{1,0}$ = prob. {there is one machine in repairman I},

$p_{0,1}$ = prob. {there is one machine in repairman II},

p_n = prob. {there are n machines in the system}, n = 2, 3,…, k .

Also, $p_0 = p_{0,0}$, $p_1 = p_{1,0} + p_{0,1}$ and $p_2 = p_{1,1}$.

Consequently, the steady–state probability deference equations are:

$$N\lambda\, p_0 = \mu_1 p_{1,0} + \mu_2 p_{0,1}, \qquad \text{n = 0,} \tag{1}$$

$$\left.\begin{aligned} \left[(N-1)\lambda + \mu_1\right] p_{1,0} &= \mu_2 p_{1,1} + N\lambda c_1 \pi_1 p_0 \\ &, \\ \left[(N-1)\lambda + \mu_2\right] p_{0,1} &= \mu_2 p_{1,1} + N\lambda c_1 \pi_2 p_0 \end{aligned}\right\} \text{n = 1 ,} \tag{2}$$

$$\left[(N-2)\beta\lambda + \mu\right] p_2 = (\mu+\alpha) p_3 + (N-1)\lambda c_1 p_1 + N\lambda c_2 p_0 \quad \text{, n = 2} \tag{3}$$

$$\left[(N-n)\beta\lambda + \mu + (n-2)\alpha\right] p_n = \left[\mu + (n-1)\alpha\right] p_{n+1}$$

$$+ \sum_{j=1}^{n-2} (N-n+j)\beta\lambda c_j p_{n-j} + (N-1)\lambda c_{n-1} p_1 + N\lambda c_n p_o \quad ,$$

$$\text{n = 3, 4,… ,k-1} \tag{4}$$

$$\left[\mu + (k-2)\alpha\right] p_k = \sum_{j=1}^{k-2} \beta\lambda c_j (N-k+j) p_{k-j} + \sum_{j=2}^{k-1} \sum_{i=k-j+1}^{k-1} \beta\lambda (N-i) c_j p_i$$

$$+ \sum_{j=2}^{k-1} \beta\lambda (N-j) c_k p_j + (N-1)\lambda c_{k-1} p_1 + N\lambda c_k p_0 \qquad \text{, n=k} \tag{5}$$

Where : $\mu = \mu_1 + \mu_2$.

Form equations (1) and (2) we have:

$$p_{1,0} = \frac{N\lambda\left\{(N-1)\lambda + \mu_2(1-c_1) + c_1\pi_1\mu\right\}}{\mu_1\left\{2(N-1)\lambda + \mu\right\}} \quad , \tag{6}$$

$$p_{0,1} = \frac{N\lambda\left\{(N-1)\lambda + \mu_1(1-c_1) + c_1\pi_2\mu\right\}}{\mu_2\left\{2(N-1)\lambda + \mu\right\}} \quad , \tag{7}$$

Therefore,

$$p_1 = p_{1,0} + p_{0,1} = N\,\Delta\, p_0 \quad , \tag{8}$$

Where,

$$\Delta = \frac{\lambda \left[(N-1)\lambda\mu + (1-c_1)\left(\mu_1^2 + \mu_2^2\right) + c_1\mu\left(\pi_1\mu_2 + \pi_2\mu_1\right) \right]}{\mu_1\mu_2 \left[2(N-1)\lambda + \mu \right]}.$$

Summing (1) and (2) we obtain:

$$p_2 = \frac{(N-1)\lambda}{\mu}\left[p_1 + (1-c_1)p_o\right] . \tag{9}$$

We can write the steady – state equations as follows

$$p_1 = \Delta\, p_0 , \qquad n = 1 \tag{10}$$

$$p_2 = \varphi\,(0,1)\left[p_1 + (1-c_1)p_0\right] , \qquad n = 2 \tag{11}$$

$$p_3 = [1+\theta_1]p_2 - \varphi(1,1)c_1 p_1 - \varphi(1,0)c_2 p_0 , \qquad n = 3 \tag{12}$$

$$p_n = [1+\theta_{n-2}]p_{n-1} - \beta\sum_{s=2}^{n-2}\varphi(n-2,s)\,p_s c_{n-s-1}$$

$$- \varphi\,(n-2,1)\; c_{n-2}\;\; p_1 - \varphi\,(n-2,0)\; c_{n-1}p_o, \qquad n = 4,5,\ldots k \tag{13}$$

where,

$$\theta_n = \frac{(N-n-1)\beta\;\;\lambda - \alpha}{\mu + n\alpha} \quad and \quad \varphi(n,s) = \frac{(N-s)\lambda}{\mu + n\alpha} .$$

If we put $p_n = g_n\, p_0$ in the equations (1o) – (13) we deduce that:

$$g_n = \begin{cases} 1, & n=0 \\ N\Delta, & n=1 \\ \varphi\;(0,1)\;\left[N\Delta + (1-c_1)\right], & n=2 \\ (1+\theta_1)g_2 \;-\; \varphi\;(1,1)\;\; g_1 c_1 \;-\; \varphi(1,0)\;\; c_2 g_o, & n=3 \\ (1+\theta_{n-2})\;\; g_{n-1} \;-\; \beta\sum_{s=2}^{n-2}\varphi(n-2,s)\;\; c_{n-s-1}\;\; g_s - \varphi\;(n-2,1)\;\; c_{n-2}\;\; g_1 - \varphi\;(n-2,0)\;\; c_{n-1} g_0, & n=4,5,\ldots,K \end{cases} \tag{14}$$

From the boundary condition: $\sum_{n=o}^{k} p_n = 1,$ we get

$$p_0^{-1} = \left[1 + \sum_{n=1}^{k} g_n\right].$$

Thus, the expected number of failed machines in the system and in the queue is, respectively,

$$L = \left(\sum_{n=1}^{k} n g_n\right) p_0 , \qquad Lq = L + 2p_0 - p_0 - 2,$$

and the expected waiting time of failed machines in the system and in the queue are, respectively,

$$w = \frac{L}{\lambda'} \qquad \text{and} \qquad wq = \frac{Lq}{\lambda'}$$

Where

$$\lambda' = \frac{\mu}{2}(L - Lq) \ .$$

The machine availability is: MA $= 1 - \frac{L}{k}$.

The operative utilization is: OU $= 1 - \frac{E(I)}{2}$,

$$E(I) = \sum_{n=0}^{2}(2-n)p_n = 2p_0 + p_1$$

Or $OU = 1 - p_0 - \frac{1}{2}p_1$,

Where

$E(I)$: is expected number of idle permanent repairmen in the system.

4 Special cases

Some queuing models can be obtained as special cases of this model.

(i) If $c_1 = 1, c_n = o \ , n > 1$ (i.e. the queue: M/M/2/K/N) in equations (6),(8) and (14) We get:

$$p_{1,0} = \frac{N\lambda\{(N-1)\lambda + \pi_1\mu\}}{\mu_1\{2(N-1)\lambda + \mu\}} p_0 \ ,$$

$$p_{0,1} = \frac{N\lambda\{(N-1)\lambda + \pi_2\mu\}}{\mu_2\{2(N-1)\lambda + \mu\}} p_0 \ , \quad \text{and} \quad p_n = g_n p_0 \ ,$$

Where

$$g_n = \begin{cases} N\Delta' & , \ n = 1 \\ \dfrac{N(N-1)\lambda\Delta'}{\mu} & , \ n = 2 \\ \dfrac{N(n)\beta^{n-2}\gamma^{n-1}}{(\theta)n-1}\Delta' & , n = 3,4,\ldots,k \ , \end{cases}$$

$$\Delta' = \frac{\lambda\left[(N-1)\lambda\mu + \mu(\pi_1\mu_2 + \pi_2\mu_1)\right]}{\mu_1\mu_2\left[2(N-1)\lambda + \mu\right]} \ ,$$

$$\gamma = \frac{\lambda}{\alpha} \quad , \theta = \frac{\mu}{\alpha} \quad , \ N_{(n)} = N(N-1)\ldots(N-n+1), n \geq 1 ,$$
$$N_{(0)} = 1 \ and \ (\theta_n) = \theta(\theta+1)\ldots(\theta+n-1) , \ n \geq 1 \ , (\theta)_o = 1.$$

These results are as in Shawky [1].

(ii) If $c_1 = 1$, $c_n = 0$, $n > 1$, $\mu_1 = \mu_2$ and $\pi_1 = \pi_2 = \frac{1}{2}$

We get the homogeneous repairmen model: M/M/2/K/N with balking and reneging, which discussed by Shawky [2] at c = 2.

(iii) Also, if we put $\alpha = 0$ and $\beta = 1$, we can get the model : M/M/2/K/N without balking and reneging , which studied by kleinrok [7] . Thus, if N = k, the system becomes: M/M/2/K/K without any concept, which was discussed by white et al. [5], Medhi [6] and Bunday [4].

5 Numerical example

In the above model, letting K= 4 and N = 7, i.e, the model: M^x/M/2/4/7, the results are:

$$p_0^{-1} = 1 + 7\Delta + \varphi(0,1) \ \left[7\Delta + (1-c_1)\right] \ \left[1 + (1+\theta_1) + (1+\theta_1) \ (1+\theta_2) \ - \ \beta \ c_1 \varphi(2,2)\right]$$
$$- \left[1 + (1+\theta_2)\right] \ \left[7c_1\Delta\varphi(1,1) + \varphi(1,o) \ c_2\right] \ - \ 7c_2\Delta\varphi(2,1) \ - \ c_3\varphi(2,o) \ .$$
$$L = \{7\Delta + \varphi \ (0,1) \ \left[7\Delta + 1 - c_1\right] \ \left[2 + 3(1+\theta_1) \ + 4 \ (1+\theta_1) \ (1+\theta_2) \ - \ 4\beta \ c_1\varphi(2,2)\right]$$
$$- \ \left[7c_1\Delta\varphi(1,1) + \varphi(1,0) \ c_2\right] \ \left[3 + 4(1+\theta_2)\right] \ -4 \ \left[7\Delta c_2\varphi \ (2,1) \ + c_3\varphi \ (2,0)\right] \} \ p_0 \ ,$$
$$Lq = L + 2p_o + 7\Delta p_o - 2 \ ,$$
$$w = \frac{L}{\frac{\mu}{2} \ (2 - 2p_o - 7\Delta p_o)} \ , \ wq = \frac{Lq}{\frac{\mu}{2}(2 - 2p_o - 7\Delta p_o)} \ ,$$
$$MA = 1 - \frac{L}{4} \ \ and \ \ \ OU = 1 - p_0 - \frac{7}{2}\Delta \ \ p_0 \ .$$

New, we introduce the two tables for some measures of effectiveness at $\mu_1 = 6$, $\mu_2 = 8$ $(\mu = 14)$, $\pi_1 = 0.2(\pi_2 = 0.8), c_1 = 0.27$, $c_2 = 0.26$, $c_3 = 0.25$, $c_4 = 0.22$ and $\lambda = 2$ for the different values of β and α when α or β is fixed.

β	0.100000	0.200000	0.300000	0.400000	0.500000	0.600000	0.700000	0.800000	0.900000	1.000000
p_0	0.104741	0.100636	0.096678	0.092868	0.089209	0.085699	0.082336	0.079117	0.076040	0.073100
L	2.262850	2.322405	2.380204	2.436164	2.490234	2.542386	2.592612	2.640923	2.687342	2.731904
L_q	0.685527	0.728518	0.770343	0.810931	0.850233	0.888219	0.924875	0.960199	0.994200	1.026898
MA	0.434288	0.419399	0.404949	0.390959	0.377441	0.364404	0.351847	0.339769	0.328164	0.317024
OU	0.788661	0.796943	0.804930	0.812616	0.820000	0.827083	0.833869	0.840362	0.846571	0.852503

Table (I), $\alpha = 0.5$

α	0.100000	0.200000	0.300000	0.400000	0.500000	0.600000	0.700000	0.800000	0.900000	1.000000
p_0	0.087042	0.087594	0.088139	0.088678	0.089209	0.089734	0.090252	0.090764	0.091269	0.091769
L	2.529189	2.519195	2.509374	2.499722	2.490234	2.480906	2.471732	2.462711	2.453836	2.445105
L_q	0.880445	0.872679	0.865058	0.857577	0.850233	0.843022	0.835941	0.828985	0.822151	0.815436
MA	0.367703	0.370201	0.372656	0.375069	0.377441	0.379774	0.382067	0.384322	0.386541	0.388724
OU	0.824372	0.823258	0.822158	0.821072	0.820000	0.818942	0.817896	0.816863	0.815843	0.814835

Table (II) $\beta = 0.5$

6 Conclusion

In the present paper, the repairmen model: $M^x/M/2/K/N$ is studied with bulk arrival, balking, reneging and heterogeneous repairmen. The steady state probabilities and some measures of effectiveness are derived in explicit forms. We treated the numerical example and deduced the machine availability and operative utilization.

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Received: May 4, 2008