

## Integrated model for line balancing with workstation inventory management

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### ABSTRACT

In this paper, we address the optimization of an integrated line balancing process with workstation inventory management. While doing so, we have studied the interconnection between line balancing and its conversion process. Almost each and every moderate to large manufacturing industry depends on a long and integrated supply chain, consisting of inbound logistic, conversion process and outbound logistic. In this sense an approach addresses a very general problem of integrated line balancing. Research works reported in the literature so far mainly deals with minimization of cost for inbound and outbound logistic subsystems. In most of the cases conversion process has been ignored. We suggest a generic approach for linking the balancing of the line of production in the conversion area with the customers' rate of demand in the market and for configuring the related stock chambers. Thus, the main aim of this paper is to translate the underlying problem in the form of mixed nonlinear programming problem and design the optimum supply chain so that the total inventory cost and the cost of balancing loss of the conversion process is jointly minimized and ideal cycle time of the production process is determined along with ideal sizes of the stock chambers. A numerical example has been added to demonstrate the suitability of our approach.

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## 1. Introduction

During Integrated line balancing involves a network of interconnected activities for the ultimate provision of products and service packages required by the end customers. The domain of integrated line balancing covers all movements and storage of raw materials, inventory of work-in-progress and finished goods from point-of-origin to point-of-consumption. Basically, integrated line balancing is the planning, organizing and controlling of sourcing, procurement, conversion and logistic activities. It is used to characterize all the inter-related components and processes required to ensure that the right amount of product is in the right locations at the right time at the lowest possible cost. In the recent past, integrated line balancing has become a very important part of any business activity. This importance is going to increase further due to growing uncertainty in the global business environment and cost minimization has again become the bull's eye. However, the form of integrated line balancing depends on the type of industry under study. Industries can be broadly divided into two types on the basis of their final offers. One is the manufacturing sector and other one is the service sector. So far as manufacturing sector is concerned, there exists a very close relationship between the increase in productivity and increase in resultant profit. Since integrated line balancing is a management technique through which the better quality products are delivered to the customers at lower costs by making a balance

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between the inbound and the outbound logistics, this increases productivity, improves quality and adds to profit. For most of the manufacturing sector, a line of production is located between inbound and outbound logistics. Therefore, this line of production plays an important role in the overall integrated line balancing process. The details may differ, but the basic flow line principles remain the same. Items are processed as they pass through a series of workstations along a production line, i.e., different tasks are performed in different workstations. One or more tasks can be performed in a workstation but the activity time for each workstation should be near about equal so that the line will be balanced. In a nutshell, the problem of line balancing deals with the distribution of activities among the workstations so that there will be maximum utilization of human resources and facilities without disturbing the work sequence. But this problem should not be addressed in isolation. This is because in each workstation supply of materials and planning of inventory play important roles for ensuring smooth functioning of the assembly line. So, the excellence in integrated line balancing can only be achieved by undertaking a system approach and by simultaneously minimizing the total inventory cost and balancing the line of production. Better coordination of different links that connect source with the destination can create competitive advantage for a company. In fact, along with the company, its suppliers, its channel members and its customers can all benefit from better coordination and usage of such linkages. For example, if we consider the case of modern rice milling facilities and if we examine the conversion area, we can classify the activities involved with the conversion process into four different groups such as steeping, parboil, dryer and hulling. Though, each group of activity may consist of one or more activities, those four groups are performed in four separate areas. For each separate area, we may consider sub-operative areas forming a network of workstations and a network of stock chambers. Therefore, to maximize the efficiency of the entire chain of operation, balancing of the assembly line is needed. And for each workstation, configuration of separate stock chamber is also needed to feed the respective workstation. For these stock chambers, the need for inventory planning, which is one of the major decision areas in integrated line balancing can be highly felt. Since we have to perform all these sub-activities under an overall system activity, the approach should be an integrated system approach where the inventory decision of integrated line balancing and balancing decision of assembly line will be considered jointly as one set of decisions.

## **2. Literature Review**

In view of the importance of integrated line balancing, many researchers have worked on the problem of supply chain management. In the recent past, researchers started working with the impact of product life cycle on supply chain (see Aitken et al., 2001) and then gradually entered into the details of it. Zhao and Xie (2002) addressed the forecasting part and worked on forecasting errors and the value of information sharing in a supply chain. Zhao et al. (2002) also discussed on the impact of forecasting models on the value of information sharing in a supply chain. Childerhouse and Towill (2002) analyzed the factors affecting real-world value stream performance. Reinforcement learning approach for Global supply chain management may be seen from the works of Pontrandolfo et al. (2002). Wadhwa et al. (2003) introduced the concept of flexibility in dynamic supply chain management. The concept of a collaborative agent-based infrastructure for internet-enabled collaborative enterprise, proposed by Shen et al. (2003), provided supply chain a new form. Mondal and Tiwari (2003) formulated mobile agents for integration of various parts of a supply chain. Talluri et al. (2007) worked on how Information technologies influence procurement decisions. The effects of schedule volatility on supply chain performance may be seen from the works of Childerhouse et al. (2007). For the complicated nature of supply chain management in the current world business scenario, Bohme et al. (2008a) have worked on stability of the supply chain uncertainty. Balancing power and dependency in buyer-supplier relationships in a supply chain have been discussed by Bohme et al. (2008b). But none of these studies interlink flow of materials with conversion process. Similarly, if we consider the works done in the field of assembly line balancing, we will find three streams of attack such as heuristic approach, simulation approach and programming approach where interlinking with stock chambers has hardly been considered. For a detailed review of the existing literature on generalized assembly line balancing, one may refer to Scholl (1999) and

Scholl and Becker (2006). Our aim of this present work is to establish a link between these two fields of work so that there could be positive synergetic effect on business efficiency.

### 3. Notation

$K$	number of jobs
$N$	number of workstations
$t_i$	task time or assembly time of $i^{\text{th}}$ job
$W_j$	$j^{\text{th}}$ workstation
$a(i, j)$	assignment variable taking value 1 if task $i$ is assigned to workstation $j$ and taking value 0, otherwise
$L_j$	idle time of $j^{\text{th}}$ work station
$C$	cycle time
$C_t$	trial cycle time
$C_{\min}$	minimum cycle time
$R_L$	lowest rate of market demand
$R_H$	highest rate of market demand
$Q_j$	ordered quantity for materials to be used in workstation $j$ for each workelement
$C_o$	ordering cost of materials
$CH_i$	holding cost of materials for $i^{\text{th}}$ job
$WH_j$	total cost of holding of materials for $j^{\text{th}}$ workstation
$M$	constant multiplying factor, to be determined from average cost of one man-hour

### 4. Problem Description

To study the integrated problem of supply chain, there is a need to link line balancing and customers' rate of demand with the entire supply chain. This linking has an important role towards achieving excellence in the complete task of moving from source to destination. Our objective in this current work is to design the integrated model for line balancing with workstation inventory where, given the rate of customers' demand, the total of inventory cost and the cost of balancing loss of the assembly line will be jointly minimized so that the entire optimization approach can be a holistic one.

### 5. Methodology and Mathematical formulation

To formulate the integrated problem of cost minimization, we like to split the same into three interrelated parts, viz. determination of customers' rate of demand, deciding about the stock of materials for consumption during production activity and balancing of the workstations. At the end, we propose to rejoin their part-wise measures on a common scale and undertake the joint optimization task.

For us, on one hand the assembly line will be balanced, when the idle time in each work station is in minimum level and in turn, the cost of the production will be minimized when the balancing loss is minimized. The

measure of balancing loss of an assembly line is defined as the loss resulting from allocation of work elements to workstations and is given by (see Ray Wild, 2004)

$$B = \left\{ (NC - \sum_{i=1}^K t_i) / NC \right\} 100\%,$$

where the numerator of which indicates the idle time per  $C$  unit time of work. The corresponding cost per unit time can be expressed as,

$$\frac{(NC - \sum_{i=1}^K t_i)}{C} M. \quad (1)$$

Inventory cost that mainly deals with ordering cost, holding cost and the shortage cost is minimized under the plan of optimum ordering quantity. If we do not permit any shortage that badly affects the total assembly line, the total cost of the supply chain for our purpose will be the sum total of ordering cost, average holding cost and the cost of balancing loss to be observed per unit time. To link the entire system with the rate of customers' demand we consider the inverse of the same to arrive at the cycle time of operation. Due to variations in the market demand it is preferable to express the same in terms of lower and upper bounds,  $R_L$  and  $R_H$  respectively. Then the cycle time must lie in the interval  $[1/R_H, 1/R_L]$ . Now, given a cycle time  $C$ , demand,

$D_j$ , for the materials may be expressed as  $D_j = \sum_{i=1}^K a(i, j) / C$  under the assumption that each work element has

one unit of consumption. Holding cost for a particular workstation  $j$  will be  $WH_j = \sum_{i=1}^K (a(i, j) * CH_i)$ , under a similar assumption and argument. With the help of our proposed notation, we can express the ordering cost for the  $j^{\text{th}}$  workstation as,

$$\frac{\sum_{i=1}^K a(i, j)}{Q_j} * \frac{C_o}{C},$$

Therefore, the total ordering cost for the  $N$  stock chambers of the  $N$ -workstation line balancing system is

$$\sum_{j=1}^N \left[ \frac{\sum_{i=1}^K a(i, j)}{Q_j} * \frac{C_o}{C} \right] \quad (2)$$

Average holding cost for the  $j^{\text{th}}$  workstation will be  $Q_j \sum_{i=1}^K (a(i, j) * CH_i) / 2$ . Hence, the total average holding cost for the system is as follows,

$$\sum_{j=1}^N \left[ Q_j \sum_{i=1}^K (a(i, j) * CH_i) \right] / 2 \quad (3)$$

To this end we add the cost of balancing loss which is  $\left[ NC - \sum_{i=1}^K t_i \right] * M / C$ . The cycle time  $C$ , being linked with the average rate of demand of the customers, may be determined in terms of an interval corresponding to interval estimator of average rate of demand of the customers. Thus,  $C$  varies between two points  $C_{\min} = 1/R_H$  and  $C_{\max} = 1/R_L$  and the optimum  $C$  along with optimum  $Q_j$  values will be determined from the total cost of the

plan. Thus, our objective is to minimize total cost of the supply chain including  $N$  stock chambers and  $N$  workstations where the integrated objective function, as obtained from (1), (2) and (3), is

$$Z = \sum_{j=1}^N \left[ \frac{\sum_{i=1}^K a(i, j)}{Q_j} * \frac{C_o}{C} \right] + \sum_{j=1}^N \left[ \frac{1}{2} Q_j \sum_{i=1}^K (a(i, j) * CH_i) \right] + \left[ NC - \sum_{i=1}^K t_i \right] * M / C$$

and our objective is to minimize  $Z$ , subject to (i) precedence constraints as given by the technology and (ii) cycle time constraints as determined from the market demand. Under the condition that the  $i^{\text{th}}$  task can be assigned to only one workstation, we must have,

$$\sum_{j=1}^N a(i, j) = 1 \quad i = 1, 2, \dots, K.$$

Also, according to precedence constraints if task  $i'$  is to be assigned before assigning task  $i$ , that is  $i' < i$ , then

$$a(i, j) \leq \sum_{r=1}^j a(i', r). \quad \forall i' < i$$

Further, since each workstation can at the most be assigned  $C$  unit of time we have,

$$\sum_{i=1}^K a(i, j) t_i \leq C \quad \forall j=1, 2, \dots, N$$

Thus, a mathematical programming formulation of the integrated optimization problem can be written as,

$$\text{minimize } z = \sum_{j=1}^N \left[ \frac{\sum_{i=1}^K a(i, j)}{Q_j} * \frac{C_o}{C} \right] + \sum_{j=1}^N \left[ \frac{1}{2} Q_j \sum_{i=1}^K (a(i, j) * CH_i) \right] + \left[ NC - \sum_{i=1}^K t_i \right] * M / C$$

subject to,

$$\sum_{j=1}^N a(i, j) = 1 \quad i = 1, 2, \dots, K$$

$$a(i, j) \leq \sum_{r=1}^j a(i', r) \quad \forall i' < i \text{ and } i', i = 1, 2, \dots, K$$

$$\sum_{i=1}^K a(i, j) t_i \leq C \quad \forall j=1, 2, \dots, N \quad (4)$$

$$C_{\min} \leq C \leq C_{\max}$$

$$a(i, j) = 0, 1 \quad \forall i, j \quad i = 1, 2, \dots, K. \quad j = 1, 2, \dots, N.$$

Obviously, this is a mixed nonlinear programming problem. To arrive at the optimum solution we may take the course of iterative algorithm. For this, we formulate holding cost, ordering cost and cost of balancing loss and add them up to get the total cost of the supply chain and denote it by  $Z$ . Iteratively we minimize  $Z$  by optimization procedure to get the minimum cost of the chain. Inputs of the system include the values of ordering cost, holding cost and average cost of one man hour. Highest and lowest rate of market demand is

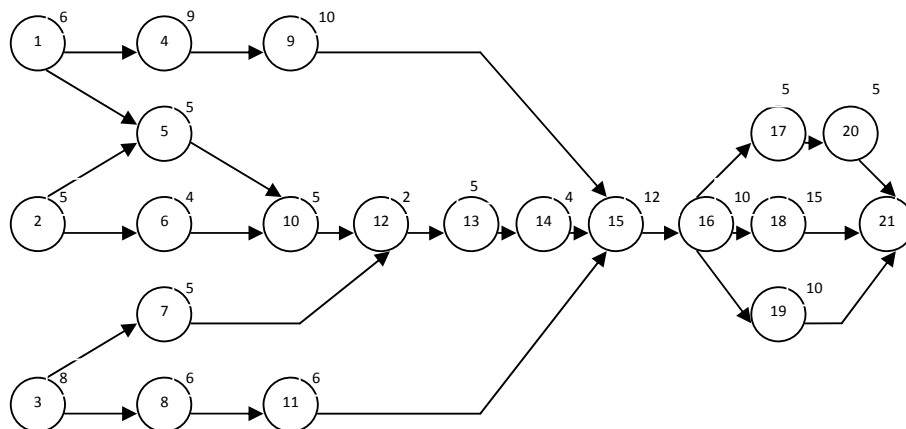
also fed into the program. After successful iterations we get the desired cycle time, assignments of work elements to workstations and order quantity for each stock chamber associated with each workstation. The next section includes a worked out example to demonstrate the functioning of the proposed method.

## 6. The Algorithm

1. Formulate the objective function
2. Complete the model formulation by restricting the objective function using precedence constraints, keeping in mind the zoning constraints and nonnegative constraints
3. Set the cycle time  $C$ , determine the minimum number of workstations  $N_{min}$  and calculate the  $C_{min}$  value
4. Set the trial cycle time  $C_t$  at  $C_{min}$
5. Solve the formulated problem for the particular  $C_t$  value
6. Complete the distribution of tasks to workstations and calculate the objective function
7. For each  $C_t$  value, compare the new value of the objective function with the previous one, if the new value of the objective function is less than the older one, replace the new solution with the older one and keep it for the next comparison, Otherwise keep the previous one as the basis for the comparison
8. Increase the trial cycle time  $C_t$  by one unit until it crosses  $C$  value. If  $C$  value is crossed, go to step 11
9. Repeat steps 5 to 8
10. Check whether all the work elements are assigned to specified number of workstations. If not, increase the value of  $N_{min}$  by 1 and go to step 4
11. Print the best solution in terms of overall minimum value of objective function

## 7. Worked Out Example

To explain how the proposed method works, consider Fig. 1, where circles represent the task numbers with their duration times and the arrows represent the precedence constraints. Work elements and precedence constraints as obtained from this conversion process are fed into the program developed for this purpose.



**Fig. 1:** Precedence diagram of workstations along with the task times

In addition, we have given the values of ordering cost ( $C_o$ ), holding cost for each job ( $CH_i$ ), task time or assembly time of each job ( $t_i$ ), trial cycle time ( $C_t$ ) and minimum cycle time ( $C_{min}$ ) and average cost of one manhour ( $M$ ) as input data. Cycle time ( $C$ ) and ordered quantity for each work station ( $Q_i$ ) are the decisions

variables. We will determine the optimum values of those variables from the iterative run of our program. The conversion process can be summarized in a tabular form in terms of the binary variables  $a(i,j)$ s and is given in Table 1 along with the choices of the cost parameters.

**Table 1**

Precedence relation, task times of work elements and cost parameters

Work Element	Activity Time	Immediate Predecessor	CHi	Work Station				
				1	2	3	4	5
1	6	-	15	a(1,1)	a(1,2)	a(1,3)	a(1,4)	a(1,5)
2	5	-	10	a(2,1)	a(2,2)	a(2,3)	a(2,4)	a(2,5)
3	8	-	10	a(3,1)	a(3,2)	a(3,3)	a(3,4)	a(3,5)
4	9	1	20	a(4,1)	a(4,2)	a(4,3)	a(4,4)	a(4,5)
5	5	1,2	18	a(5,1)	a(5,2)	a(5,3)	a(5,4)	a(5,5)
6	4	2	25	a(6,1)	a(6,2)	a(6,3)	a(6,4)	a(6,5)
7	5	3	22	a(7,1)	a(7,2)	a(7,3)	a(7,4)	a(7,5)
8	6	3	9	a(8,1)	a(8,2)	a(8,3)	a(8,4)	a(8,5)
9	10	4	10	a(9,1)	a(9,2)	a(9,3)	a(9,4)	a(9,5)
10	5	5,6	12	a(10,1)	a(10,2)	a(10,3)	a(10,4)	a(10,5)
11	6	8	10	a(11,1)	a(11,2)	a(11,3)	a(11,4)	a(11,5)
12	2	10,7	13	a(12,1)	a(12,2)	a(12,3)	a(12,4)	a(12,5)
13	5	12	15	a(13,1)	a(13,2)	a(13,3)	a(14,4)	a(13,5)
14	4	13	30	a(14,1)	a(14,2)	a(14,3)	a(14,4)	a(14,5)
15	12	9, 11, 14	20	a(15,1)	a(15,2)	a(15,3)	a(15,4)	a(15,5)
16	10	15	10	a(16,1)	a(16,2)	a(16,3)	a(16,4)	a(16,5)
17	5	16	40	a(17,1)	a(17,2)	a(17,3)	a(17,4)	a(17,5)
18	15	16	10	a(18,1)	a(18,2)	a(18,3)	a(18,4)	a(18,5)
19	10	16	17	a(19,1)	a(19,2)	a(19,3)	a(19,4)	a(19,5)
20	5	17	13	a(20,1)	a(20,2)	a(20,3)	a(20,4)	a(20,5)
21	6	18,19,20	10	A(21,1)	A(21,2)	A(21,3)	A(21,4)	A(21,5)
Co = 50		M=25						

The optimum policy and configuration is presented in Table 2. As we can observe from the table there will be five workstations with five stock chambers with the balancing loss of the system works out of 7.412% and the total cost of operation per unit time of 4667.7 units.

**Table 2**

Final Optimum Configuration and policy

Work Station	Assigned tasks	Ordered quantity	Cycle time
1	1, 2, 3, 8, 11	17	31.00
2	5, 6, 7, 10, 12, 13, 14	13	
3	4, 9, 15	14	
4	16, 18	18	
5	17, 19, 20, 21	12	

## 8. Conclusion

We have presented a mathematical programming approach for solving an integrated model of line balancing with workstation inventory management problem with the objective of integrated cost minimization, resulting

in optimum provision of materials in each stock chamber and determination of ideal cycle time so that the demand of the market can be met in time. The proposed method of this paper can easily calculate the ideal cycle time for a system according to demand in the market. It is also possible to adjust the cycle time, when needed and it minimizes the inventory raw materials as well as finished goods inventory and the total cost of the chain. We know that the cost of the supply chain is mainly the running cost of the chain and the primary objective of this research is devoted to the minimization of item. As a result of this point wise optimization and chronological improvement in the system, the quality of the supply chain could be increased providing distinctive competence to the company.

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