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Optimal pricing and lot sizing in VMI model

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A B S T R A C T

Vendor Managed Inventory (VMI) is one of the effective techniques for managing the inventory in supply chain. VMI models have been proven to reduce the cost of inventory compared with traditional economic order quantity method under some conditions such as constant demand and production expenditure. However, the modeling of the VMI problem has never been studied under some realistic assumptions such as price dependent demand. In this paper, three problem formulations are proposed. In the first problem formulation, we study a VMI problem with one buyer and one supplier when demand is considered to be a function of price and price elasticity to demand, and production cost is also a function of demand. The proposed model is formulated and solved in a form of geometric programming. For the second and the third models, we consider VMI problem with two buyers and two suppliers assuming that each buyer centre is relatively close to the other buyer centre. Each supplier has only one product which is different from the product of the other supplier. Two suppliers cooperate in customer relationship management and two buyers cooperate in supplier relationship management as well, so the suppliers send the orders of two buyers by one vehicle, simultaneously. For the third model, an additional assumption which is practically applicable and reasonable is considered. For all the proposed models, the optimal solution is compared with the traditional one. We demonstrate the implementation of our proposed models using some numerical examples.

1. Introduction

During the past two decades, Vendor Managed Inventory (VMI) has become one of the most effective and popular approaches in supply chain management (VMI) (Lee & Chu, 2005; De Toni & Zamolo, 2005; Rusdiansyah & Tsao, 2005;Yao & Evers, 2007). VMI can shift the retailers' responsibility for planning and replenishment activities to the manufacturers based on demand information sent by the customers or retailers to the manufacturers. There are the following advantages for VMI model: 1) There is an improvement on performance of the supply chain by collaborative partnerships between buyer(s) and supplier(s), 2) The customer service levels increase, 3) The inventory levels decrease and fill rates increase, 4) Total costs and lead-time are minimized. This approach has been widely used in the literature (Cachon and Fisher, 1997; Cachon, 2001; Dong & Xu, 2002). Fry et al. (2001) study the effects of logistics in VMI for problems with one buyer and one supplier. Luca Bertazzi et al. (2005) compare the VMI policies with more traditional retailer-managed inventory (RMI) policy and show that the VMI policies significantly reduce the average cost compared with RMI policy. Yan Dong et al. (2002) study VMI effects on total costs in long-term and short-term planning horizons and show that in certain conditions, VMI may decrease the supplier's profit. The VMI problem with one buyer and one supplier has been well studied in the context of some assumptions such as constant demand and production expenditure. However, these assumptions are often unrealistic for real world

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problems. We first consider the one buyer-one supplier VMI problem when demand is a function of price and price elasticity to demand. This assumption has already been used for different problems (Lee & Kim, 1993; Lee & Kim, 1993; Hoon & Cerry, 2005). Lee and Kim (1993) are believed to be the first people who model and solve some traditional problems under this kind of assumption. Sadjadi et al. (2005) extend the modelling of the problem under some different conditions. Sadjadi and Ziaee (2006) study the effects of price for a price discrimination model. The reminder of this paper is organized as follows. The first proposed model of this paper studies a one-buyer one-supplier VMI model in which we assume when demand for a product increases, the production is less costly. We propose a solution procedure for the proposed model which is based on Geometric Programming (GP). In sections 3 and 4, we consider the two-buyer two-supplier VMI problem. For both models, we assume that each buyer centre is relatively close to other buyer centre but there is a relatively a big distance between the buyer and all other suppliers. Two suppliers collaborate with each other in a form of a customer relationship management and two buyers also cooperate in a form of a supplier relationship management as well, so the suppliers send the orders of two buyers by one vehicle, simultaneously. Note that this kind of conditions holds for many real-world problems. For all three proposed models, the optimal solution is compared with the traditional one and a numerical example is presented in order to illustrate the implementation of the proposed method. Finally, the conclusion remarks are given to summarize the contribution of this paper.

2. Model 1

Consider a single product where demand is affected by the selling price. Let P, α be the selling price per unit and elasticity of price with respect to demand in the market, respectively. We assume that the demand or production (D) is a function of price per unit (P), that is,

$$D = kP^{-\alpha} \qquad \alpha > 0, \tag{1}$$

The scaling parameter k represents the effect of other related factors and $\alpha > 0$ implies that D increases as P decreases. We also assume that the unit production cost (C) can be discounted with β. Therefore we have,

$$C = uD^{-\beta} \qquad \beta > 0, \tag{2}$$

where D is the amount of demand or production in the planning period and u is a scaling parameter. The exponent β represents total production elasticity of production unit cost with a small value for β (say $\beta = 0.01$). We consider a traditional inventory model (EOQ[1]) where no shortage is permitted. The following assumptions hold in this paper,

- The consumption rate is constant.
- The amount of order is constant.
- Each new order arrives as soon as the inventory level is equal to zero.
- Transportation times are negligible and each order is delivered at once.
- The setup and holding costs in VMI model are paid by the supplier.

The other notations of our model are as follows,

- K_0**:** Total inventory costs for the supply chain which includes the costs of the supplier and the buyer in traditional model.
- K_1**:** Total inventory costs for the supply chain which includes the costs of the supplier and the buyer in VMI model.
- K_0^*, K_1^*: The optimal values of K_0, K_1, respectively.
- KS_0, KS_1**:** Total costs of the supplier in traditional and VMI models, respectively.
- KB_0, KB_1**:** Total costs of the buyer in traditional and VMI models, respectively.
- Q: The size of each order.
- Q_0^*, Q_1^*: The optimal value of Q in traditional and VMI models, respectively.
- A_S: The cost of each order for the supplier.

1- Economic Order Quantity

- A_B: The cost of each order for the buyer.
- D: The demand of buyer or the amount of production by supplier in each period.
- H: The inventory holding cost per unit of product and per period.

In a traditional inventory model, the costs of inventory and procurement are computed as follows,

$$K_0 = KS_0 + KB_0, \tag{3}$$

where

$$\mathrm{KB}_0 = PD + \frac{A_B.D}{Q} + \frac{H}{2}Q, \tag{4}$$

and

$$\mathrm{KS}_0 = C.D + \frac{A_S.D}{Q}. \tag{5}$$

Using (1) and (2), the problem of minimizing the total costs of inventory for the buyer in traditional model can be written as follows,

$$\min\ \mathrm{KB}_0 = kP^{(1-\alpha)} + A_B kP^{-\alpha}Q^{-1} + \frac{H}{2}Q. \tag{6}$$

Problem (6) is a posynomial form of geometric programming and its degree of difficulty (Beightler & Phillips 1976; Dembo, 1982; Freedland, 1982) is 3-(2+1) = 0. Suppose we assume,

$$\delta_1 = kP^{(1-\alpha)}, \qquad \delta_2 = A_B kP^{-\alpha}Q^{-1}, \qquad \delta_3 = \frac{H}{2}Q. \tag{7}$$

Also let w_i , i = 1,2,3 be the dual variables associated with δ_i , i = 1,2,3, respectively. Therefore, the dual problem formulation for the problem (6) is as follows,

$$\mathrm{d(KB_0)} = \max\ \mathrm{f(w)} = \left(\frac{\mathrm{k}}{\mathrm{w_1}}\right)^{\mathrm{w_1}} \left(\frac{\mathrm{A_B.k}}{\mathrm{w_2}}\right)^{\mathrm{w_2}} \left(\frac{\mathrm{H}}{\mathrm{2w_3}}\right)^{\mathrm{w_3}} \tag{8}$$

subject to

$$w_1 + w_2 + w_3 = 1,\ \ (1-\alpha)w_1 - \alpha w_2 = 0,\ \ -w_2 + w_3 = 0.\ \ w_i \geq 0 \qquad i=1,2,3 \tag{9}$$

Thus,

$$\mathrm{w_1} = \frac{\alpha}{2-\alpha} \qquad \text{and} \qquad \mathrm{w_2} = \mathrm{w_3} = \frac{1-\alpha}{2-\alpha}\ . \tag{10}$$

Note that in order to have $w_i \geq 0$, for i = 1, 2, 3 we must limit α to 0 < α < 1 and the optimal value of $f(w)$ can be determined using w_i, i = 1, 2, 3 as follows,

$$\mathrm{f^*} = \mathrm{f(w^*)} = \mathrm{KB_0}^* = (2-\alpha)\mathrm{k}^{\left(\frac{1}{2-\alpha}\right)} \alpha^{\left(\frac{\alpha}{\alpha-2}\right)} \left(\frac{\mathrm{A_B}.H}{2}\right)^{\left(\frac{1-\alpha}{2-\alpha}\right)} (1-\alpha)^{\left(\frac{2\alpha-2}{2-\alpha}\right)} \tag{11}$$

Now, we can use the following equations to determine the optimal value of the primal variables, P^*, Q^*,

$$\delta_1 = kP^{(1-\alpha)} = w_1 f^*, \qquad \delta_3 = \frac{H}{2} Q = w_3 f^* \tag{12}$$

Thus,

$$P^* = \left(\frac{A_B . H . \alpha^2}{2k(1-\alpha)^2} \right)^{\left(\frac{1}{2-\alpha}\right)}, \quad Q^* = \left(\frac{2k . A_B^{(1-\alpha)} . (1-\alpha)^{\alpha}}{\mathrm{H} . \alpha^{\alpha}} \right)^{\left(\frac{1}{2-\alpha}\right)}. \tag{13}$$

The total inventory cost for the supply chain in traditional model which is the sum of the costs of buyer and supplier is computed as follows,

$$\mathrm{K}_0 = \mathrm{KS}_0 + \mathrm{KB}_0 = \frac{AD}{Q} + \frac{H}{2} Q + CD + PD. \tag{14}$$

In (14), the parameter A is the sum of A_B and A_S (*i.e.* $A = A_B + A_S$). Using equations (1), (2) and (13) yields,

$$\mathrm{K}_0{}^* = u.\mathrm{k}^{(1-\beta)} \left(\frac{A_B . H . \alpha^2}{2k(1-\alpha)^2} \right)^{\left(\frac{\alpha\beta-\alpha}{2-\alpha}\right)} + A.k \left(\frac{A_B . H . \alpha^2}{2k(1-\alpha)^2} \right)^{\left(\frac{-\alpha}{2-\alpha}\right)} \left(\frac{2\mathrm{k.A_B}^{(1-\alpha)} (1-\alpha)^{\alpha}}{H . \alpha^{\alpha}} \right)^{\left(\frac{-1}{2-\alpha}\right)}$$
$$+ \frac{H}{2} \left(\frac{2\mathrm{k.A_B}^{(1-\alpha)} (1-\alpha)^{\alpha}}{H . \alpha^{\alpha}} \right)^{\left(\frac{1}{2-\alpha}\right)} + k \left(\frac{A_B . H . \alpha^2}{2k(1-\alpha)^2} \right)^{\left(\frac{1-\alpha}{2-\alpha}\right)}. \tag{15}$$

Similarly, the total cost of inventory in VMI model is calculated as follows,

$$\mathrm{K}_1 = \mathrm{KS}_1 + \mathrm{KB}_1 = \frac{AD}{Q} + \frac{H}{2} Q + CD + PD. \tag{16}$$

Using (1) and (2), the problem of minimizing the total costs of inventory in VMI model is computed as follows,

$$\min \ \mathrm{K}_1 = uk^{(1-\beta)} P^{(\alpha\beta-\alpha)} + AkP^{-\alpha} Q^{-1} + \frac{H}{2} Q + kP^{1-\alpha}. \tag{17}$$

Problem (17) is a posynomial form of geometric programming and its degree of difficulty is 4-(2+1) = 1. Let w_i for i = 1,2,3,4 be the dual variables. Therefore, the dual problem formulation for the problem (17) is as follows,

$$\mathrm{d(K_1)} = \max \ \mathrm{g(w)} = \left(\frac{u.k^{(1-\beta)}}{\mathrm{w}_1} \right)^{\mathrm{w}_1} \left(\frac{\mathrm{A.k}}{\mathrm{w}_2} \right)^{\mathrm{w}_2} \left(\frac{\mathrm{H}}{2\mathrm{w}_3} \right)^{\mathrm{w}_3} \left(\frac{\mathrm{k}}{\mathrm{w}_4} \right)^{\mathrm{w}_4} \tag{18}$$

subject to

$$w_1 + w_2 + w_3 + w_4 = 1, \ (\alpha\beta - \alpha).w_1 - \alpha.w_2 + (1-\alpha).w_4 = 0, \ -w_2 + w_3 = 0 \ w_i \geq 0 \qquad i = 1,2,3,4 \tag{19}$$

We rewrite the equation (18) in terms of only one dual variable, w. Therefore we have,

$$\mathrm{w}_1 = \frac{(\alpha-1) + (2-\alpha) w_2}{(\alpha\beta-1)}, \quad \mathrm{w}_3 = \mathrm{w}_2, \quad w_4 = \frac{(\alpha\beta-\alpha) + (\alpha - 2\alpha\beta) w_2}{(\alpha\beta-1)}. \tag{20}$$

According to the assumptions concerning the values of α and β which were mentioned earlier, we can now assume $\alpha\beta - 1 < 0$ and the constraints $w_i \geq 0$, for i = 1,2,3,4 require that $0 < \alpha < 1$ and $0 < \beta < 0.5$ and therefore $w_2 < 0.5$.

$$d(K_1) = \left(\frac{u.k^{(1-\beta)}.(\alpha\beta-1)}{(\alpha-1)+(2-\alpha)w_2}\right)^{\left(\frac{(\alpha-1)+(2-\alpha)w_2}{(\alpha\beta-1)}\right)} \times \left(\frac{A.k.H}{2.w_2^{\,2}}\right)^{w_2} \times \left(\frac{k.(\alpha\beta-1)}{(\alpha\beta-\alpha)+(\alpha-2\alpha\beta)w_2}\right)^{\left(\frac{(\alpha\beta-\alpha)+(\alpha-2\alpha\beta)w_2}{(\alpha\beta-1)}\right)}$$

Therefore, we can use a simple linear search to find the optimal solution ($d^*(K_1) = K_1^*$). Then the values of w_1, w_3 and w_4 can be easily calculated using (20) with the values of K_1^* and corresponding w_2 (that is w_2^*). P^* and Q^* are also obtained as follows,

$$P^* = \left(\frac{w_4 K_1^*}{k}\right)^{\left(\frac{1}{1-\alpha}\right)}, \qquad Q^* = \frac{2w_3 K_1^*}{H}. \tag{11}$$

Obviously, the VMI model is preferred to the traditional one if

$$K_1^* \leq K_0^*. \tag{22}$$

Numerical example

We now illustrate the proposed model using a numerical example. The parameter values for this example are as follows,

$\alpha = 0.3$, $\beta = 0.2$, $u = 4$, $k = 1000$, $A_B = 15$, $A_S = 10$, $H = 8$.

The results obtained from solving the test problem are as follows.

For traditional model: $P^* = 0.070526$, $Q^* = 91.151306$, $K_0^* = 3027.23676$.

For VMI model: $P^* = 0.099$, $Q^* = 114.62$, $K_1^* = 1118.247$, $(w_2 = 0.41)$.

The above results show that the total costs of VMI model are much lower than those of traditional model (about one-third).

3. Model 2

In this model, we assume that there are two buyers and two suppliers. Each buyer centre is relatively close to the other buyer centre and so are the two suppliers but any buyer is far away from any supplier. Other assumptions of this model are as follows,

- Each supplier procures only one product which is different from the product of other supplier. Therefore, we have two products in our model. For each product, two buyers have the same holding cost.
- Two suppliers cooperate in customer relationship management and two buyers cooperate in supplier relationship management, therefore they send the orders of two buyers by only one vehicle, simultaneously, and therefore for each buyer, the two suppliers have the same fixed ordering cost; and for each supplier, the two buyers have also the same fixed ordering cost (see Fig. 1).
- The demands of suppliers and buyers are constant and given.
- Transportation times are negligible and each order is delivered at once.
- Shortage is not allowed.
- Each new order arrives as soon as the inventory level is equal to zero.
- We assume that each supplier has a constant selling price for his product (as opposed to the first model) and there is no discounting.

- The consumption rate is constant.
- The Setup and holding costs in VMI model are paid by supplier.

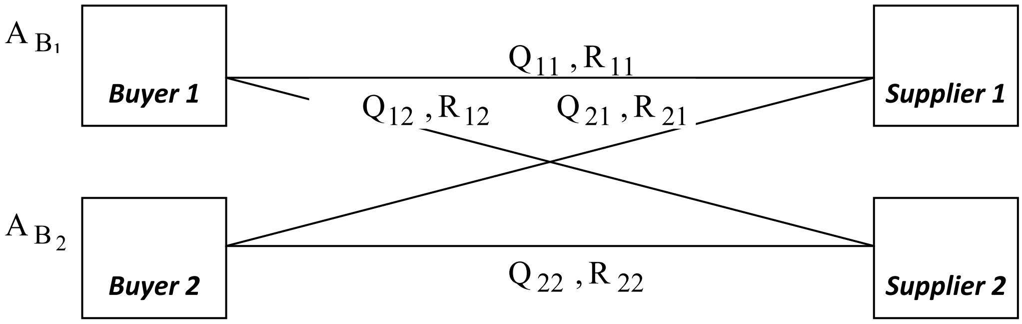

Fig. 1. Two-buyer, two-supplier system

Additional notations used for the second model are as follows,

A_{S_j} : The cost of each order (fixed ordering cost) for the supplier *j*

A_{B_i} : The cost of each order (fixed ordering cost) for the buyer *i*

H_j : The inventory holding cost per unit of product and per period for supplier j

$KS_{o_{ij}}$: Total costs of supplier *j* for procuring all demands of buyer *i* in traditional model

$KB_{o_{ij}}$: Total costs of buyer *i* for procuring all his demands from supplier *j* in traditional model

$K_{o_{ij}}$: The sum of $KS_{o_{ij}}$ and $KB_{o_{ij}}$

K_o : Total costs of supply chain including the costs of all suppliers and buyers in traditional model

K_v : Total costs of supply chain including the costs of all suppliers and buyers in VMI model

KS_v : Total costs of two suppliers in VMI model

KB_v : Total costs of two buyers in VMI model

R_{ij} : The demand of buyer i for the product of supplier j in each period

$Q_{o_{ij}}$: The order quantity of the product of supplier *j* for buyer *i* in traditional model

$Q_{v_{ij}}$: The order quantity of the product of supplier *j* for buyer *i* in VMI model

$Q_{o_{ij}}$ and $Q_{v_{ij}}$ are decision variables; $Q^*_{o_{ij}}$ and $Q^*_{v_{ij}}$ are the optimal values of $Q_{o_{ij}}$ and $Q_{v_{ij}}$, respectively. For traditional model, we have,

$$Q^*_{o_{ij}} = \sqrt{\frac{2A_{B_i}R_{ij}}{H_j}} \qquad i = 1,2 \; ; \; j = 1,2 \tag{23}$$

$$KS^*_{o_{ij}} = \frac{A_{S_j}R_{ij}}{Q^*_{o_{ij}}}, \; KB^*_{o_{ij}} = \frac{A_{B_i}R_{ij}}{Q^*_{o_{ij}}} + \frac{H_j}{2}Q^*_{o_{ij}}. \tag{24}$$

Therefore,

$$K^*_{o_{ij}} = KS^*_{o_{ij}} + KB^*_{o_{ij}} = \sqrt{\frac{H_j R_{ij}}{2A_{B_i}}} A_{S\,j} + \sqrt{2A_{B_i} R_{ij} H_j} == \sqrt{2A_{B_i} R_{ij} H_j}\,(1+\frac{A_{S_j}}{2A_{B_i}}) \tag{25}$$

$$K^*_o = \sum_{i=1}^{2}\sum_{j=1}^{2} K^*_{o_{ij}} = \sum_{i=1}^{2}\sum_{j=1}^{2}\left[\sqrt{2A_{B_i} R_{ij} H_j}\,(1+\frac{A_{S_j}}{2A_{B_i}})\right] \tag{26}$$

In VMI model, the number of orders for the supplier *j* is equal to the following expression,

$$\sum_{i=1}^{2} R_{ij} / \sum_{i=1}^{2} Q_{ij} \,. \tag{27}$$

Based on the assumptions mentioned earlier, the two suppliers have the same number of orders, that is,

$$\frac{R_{11} + R_{21}}{Q_{11} + Q_{21}} = \frac{R_{12} + R_{22}}{Q_{12} + Q_{22}} \,. \tag{28}$$

We assume that for each product and for each buyer, the proportion of the buyer order quantity with respect to the total order quantity is equal to the proportion of the buyer demand with respect to the total demand, that is,

$$\frac{Q_{11}}{Q_{11} + Q_{21}} = \frac{R_{11}}{R_{11} + R_{21}}, \quad \text{or} \quad \frac{R_{11} + R_{21}}{Q_{11} + Q_{21}} = \frac{R_{11}}{Q_{11}}. \tag{29}$$

Similarly we have,

$$\frac{R_{12} + R_{22}}{Q_{12} + Q_{22}} = \frac{R_{12}}{Q_{12}}. \tag{30}$$

Therefore, the number of orders in VMI model is equal to $\frac{R_{11}}{Q_{11}}$ or $\frac{R_{12}}{Q_{12}}$. Therefore,

$$KS_V = A_{S_1}\frac{R_{11}}{Q_{11}} + A_{S_2}\frac{R_{11}}{Q_{11}}, \tag{31}$$

$$KB_V = A_{B_1}\frac{R_{11}}{Q_{11}} + A_{B_2}\frac{R_{11}}{Q_{11}} + \sum_{i=1}^{2}\sum_{j=1}^{2}\left[\frac{H_j}{2}Q_{ij}\right]. \tag{32}$$

Thus,

$$K_V = KS_V + KB_V = (A_{S_1} + A_{S_2} + A_{B_1} + A_{B_2})\frac{R_{11}}{Q_{11}} + \frac{H_1}{2}(Q_{11} + Q_{21}) + \frac{H_2}{2}(Q_{12} + Q_{22}). \tag{33}$$

For the sake of the convenience, we assume that,

$$A_{S_1} + A_{S_2} + A_{B_1} + A_{B_2} = A, \quad and \quad \frac{R_{11}}{Q_{11}} = x. \tag{34}$$

Therefore,

$$K_v = A\frac{R_{11}}{Q_{11}} + \frac{H_1}{2}(R_{11} + R_{21})\frac{Q_{11}}{R_{11}} + \frac{H_2}{2}(R_{12} + R_{22})\frac{Q_{12}}{R_{12}}, \quad (35)$$

Recall that we have $\frac{Q_{11}}{R_{11}} = \frac{Q_{12}}{R_{12}}$. Therefore we have,

$$K_v = Ax + \frac{H_1}{2}(R_{11} + R_{21})\frac{1}{x} + \frac{H_2}{2}(R_{12} + R_{22})\frac{1}{x}. \quad (36)$$

The optimal value of x is obtained by taking the derivative of K_v with respect to x and equating it to zero.

$$\frac{\partial K_v}{\partial x} = 0, \rightarrow A - \left[\frac{H_1}{2}(R_{11} + R_{21}) + \frac{H_2}{2}(R_{12} + R_{22})\right]\frac{1}{x^2} = 0, \quad (37)$$

$$x^* = \sqrt{\frac{H_1(R_{11} + R_{21}) + H_2(R_{12} + R_{22})}{2A}}, \quad (38)$$

And therefore, the optimal value of K_v is obtained as follows:

$$K_v^* = \sqrt{2(A_{S_1} + A_{S_2} + A_{B_1} + A_{B_2})[H_1(R_{11} + R_{21}) + H_2(R_{12} + R_{22})]}, \quad (39)$$

The difference between the minimum total costs of traditional model and those of VMI model is calculated as follows,

$$K_o^* - K_v^* = \sum_{i=1}^{2}\sum_{j=1}^{2}\left[\sqrt{2A_{B_i}R_{ij}H_j}\,(1 + \frac{A_{S_j}}{2A_{B_i}})\right] - \quad (39)$$

$$\sqrt{2(A_{S_1} + A_{S_2} + A_{B_1} + A_{B_2})[H_1(R_{11} + R_{21}) + H_2(R_{12} + R_{22})]},$$

Numerical example

The general notion behind the problem formulation is best illustrated by a numerical example. The parameter values for the test problem are as follows:

$A_{S_1} = 8$, $A_{S_2} = 10$, $A_{B_1} = 4$, $A_{B_2} = 3$, $R_{11} = 12$, $R_{12} = 15$, $R_{21} = 14$, $R_{22} = 6$, $H_1 = 2$, $H_2 = 3$.

The optimal results are as follows,

For traditional model:

$Q_{11}^* = 6.928203$, $Q_{12}^* = 6.324555$, $Q_{21}^* = 6.480741$, $Q_{22}^* = 3.464102$, $K^* = 128.359831$.

For VMI model:

$Q_{11}^* = 7.912566$, $Q_{12}^* = 9.890707$, $K^* = 75.828754$.

The above results show that the total costs of the VMI model are less than those of the traditional one.

4. Model 3

This model is similar to the second model, but we now consider an additional assumption which is practically applicable and profitable. Assume that the number of orders for the supplier j (j=1, 2) is equal to n_j and one of them (say n_2) is greater than the other one, that is $n_2 > n_1$. In such a situation, it is better to send n_1 orders by the method of model 2 (i.e. the orders of two suppliers are sent by one vehicle, simultaneously), and $n_2 - n_1$ orders (which belongs to the supplier 2) are sent separately by supplier 2. Suppose that the cost of each order sent by the second model method is equal to Ac. As previously mentioned, the following relations hold for the number of orders for suppliers 1 and 2:

$$\frac{R_{11}+R_{21}}{Q_{11}+Q_{21}} = \frac{R_{11}}{Q_{11}} \quad \text{and,} \quad \frac{R_{12}+R_{22}}{Q_{12}+Q_{22}} = \frac{R_{12}}{Q_{12}}.$$

If $\frac{R_{11}}{Q_{11}} \geq \frac{R_{12}}{Q_{12}}$**,** then the total costs of two suppliers in VMI model is calculated as follows,

$$KS_V = (A_{S_1} + A_{S_2} + Ac)\times \min\left(\frac{R_{11}}{Q_{11}}, \frac{R_{12}}{Q_{12}}\right) + \left(\frac{R_{11}}{Q_{11}} - \frac{R_{12}}{Q_{12}}\right)A_{S_1} \tag{40}$$
$$= A_{S_1}\frac{R_{11}}{Q_{11}} + A_{S_2}\frac{R_{12}}{Q_{12}} + Ac\frac{R_{12}}{Q_{12}},$$

And if $\frac{R_{11}}{Q_{11}} < \frac{R_{12}}{Q_{12}}$**,** then the total costs of two suppliers in VMI model is obtained as follows,

$$KS_V = (A_{S_1} + A_{S_2} + Ac)\frac{R_{11}}{Q_{11}} + \left(\frac{R_{12}}{Q_{12}} - \frac{R_{11}}{Q_{11}}\right)A_{S_2} = A_{S_1}\frac{R_{11}}{Q_{11}} + A_{S_2}\frac{R_{12}}{Q_{12}} + Ac\frac{R_{11}}{Q_{11}}. \tag{41}$$

Let

$$KS_V = A_{S_1}\frac{R_{11}}{Q_{11}} + A_{S_2}\frac{R_{12}}{Q_{12}} + Ac\times \min\left(\frac{R_{11}}{Q_{11}}, \frac{R_{12}}{Q_{12}}\right). \tag{42}$$

And the total costs of two buyers in VMI model is obtained as follows,

$$KB_V = (A_{B_1} + A_{B_2})\times \max\left(\frac{R_{11}}{Q_{11}}, \frac{R_{12}}{Q_{12}}\right) + \sum_{i=1}^{2}\sum_{j=1}^{2}\left[\frac{H_j}{2}Q_{ij}\right], \tag{43}$$

Therefore, the total costs in VMI model is calculated as follows:

$$K_V = KS_V + KB_V = A_{S_1}\frac{R_{11}}{Q_{11}} + A_{S_2}\frac{R_{12}}{Q_{12}} + Ac\times \min\left(\frac{R_{11}}{Q_{11}}, \frac{R_{12}}{Q_{12}}\right) +$$
$$(A_{B_1} + A_{B_2})\times \max\left(\frac{R_{11}}{Q_{11}}, \frac{R_{12}}{Q_{12}}\right) + \frac{H_1}{2}(R_{11}+R_{21})\frac{Q_{11}}{R_{11}} + \frac{H_2}{2}(R_{12}+R_{22})\frac{Q_{12}}{R_{12}}, \tag{44}$$

$$i.Assumption\ 1:\ \min\left(\frac{R_{11}}{Q_{11}}, \frac{R_{12}}{Q_{12}}\right) = \frac{R_{11}}{Q_{11}} \tag{45}$$

Based on the above assumption, the following relationship holds,

$$K_{v(1)} = A_{S_1}\frac{R_{11}}{Q_{11}} + A_{S_2}\frac{R_{12}}{Q_{12}} + Ac\frac{R_{11}}{Q_{11}} + (A_{B_1} + A_{B_2})\frac{R_{12}}{Q_{12}} + \frac{H_1}{2}(R_{11} + R_{21})\frac{Q_{11}}{R_{11}} + \frac{H_2}{2}(R_{12} + R_{22})\frac{Q_{12}}{R_{12}}, \tag{46}$$

$K_{v(1)}$ denotes the value of K_v under the assumption 1. Therefore we have,

$$K_{v(1)} = (A_{S_1} + Ac)\frac{R_{11}}{Q_{11}} + (A_{S_2} + A_{B_1} + A_{B_2})\frac{R_{12}}{Q_{12}} + \frac{H_1}{2}(R_{11} + R_{21})\frac{Q_{11}}{R_{11}} + \frac{H_2}{2}(R_{12} + R_{22})\frac{Q_{12}}{R_{12}}. \tag{47}$$

To obtain the optimal value of $K_{v(1)}$ we use the first order necessary conditions,

$$\begin{cases} \dfrac{\partial K_{v(1)}}{\partial Q_{11}} = -\dfrac{(A_{S_1} + Ac)R_{11}}{Q_{11}^2} + \dfrac{H_1}{2}(R_{11} + R_{21})\dfrac{1}{R_{11}} = 0 \\ \dfrac{\partial K_{v(1)}}{\partial Q_{12}} = -\dfrac{(A_{S_2} + A_{B_1} + A_{B_2})R_{12}}{Q_{12}^2} + \dfrac{H_2}{2}(R_{12} + R_{22})\dfrac{1}{R_{12}} = 0 \end{cases} \tag{48}$$

Therefore we have,

$$\begin{cases} Q_{11}^* = \sqrt{\dfrac{2(A_{S_1} + Ac)R_{11}}{H_1(1 + \dfrac{R_{21}}{R_{11}})}} \\ Q_{12}^* = \sqrt{\dfrac{2(A_{S_2} + A_{B_1} + A_{B_2})R_{12}}{H_2(1 + \dfrac{R_{22}}{R_{12}})}} \end{cases} \tag{49}$$

and

$$K_{v(1)}^* = \sqrt{2(A_{S_1} + Ac)H_1(R_{11} + R_{21})} + \sqrt{2(A_{S_2} + A_{B_1} + A_{B_2})H_2(R_{12} + R_{22})} \tag{50}$$

ii. Assumption 2 : $\min(\frac{R_{11}}{Q_{11}}, \frac{R_{12}}{Q_{12}}) = \frac{R_{12}}{Q_{12}}$ (51)

Based on the above assumption, the following relationship holds,

$$K_{v(2)} = A_{S_1}\frac{R_{11}}{Q_{11}} + A_{S_2}\frac{R_{12}}{Q_{12}} + Ac\frac{R_{12}}{Q_{12}} + (A_{B_1} + A_{B_2})\frac{R_{11}}{Q_{11}} + \frac{H_1}{2}(R_{11} + R_{21})\frac{Q_{11}}{R_{11}} + \frac{H_2}{2}(R_{12} + R_{22})\frac{Q_{12}}{R_{12}}, \tag{52}$$

$K_{v(2)}$ denotes the value of K_v under the assumption 2. Therefore we have:

$$K_{v(2)} = (A_{S_1} + A_{B_1} + A_{B_2})\frac{R_{11}}{Q_{11}} + (A_{S_2} + Ac)\frac{R_{12}}{Q_{12}} + \frac{H_1}{2}(R_{11} + R_{21})\frac{Q_{11}}{R_{11}} + \frac{H_2}{2}(R_{12} + R_{22})\frac{Q_{12}}{R_{12}}, \tag{53}$$

And the optimal value of $K_{v(2)}$ is calculated as follows:

$$\begin{cases} \dfrac{\partial K_{v_{(2)}}}{\partial Q_{11}} = -\dfrac{(A_{S_1}+A_{B_1}+A_{B_2})R_{11}}{{Q_{11}}^2} + \dfrac{H_1}{2}(R_{11}+R_{21})\dfrac{1}{R_{11}} = 0 \\ \dfrac{\partial K_{v_{(2)}}}{\partial Q_{12}} = -\dfrac{(A_{S_2}+Ac)R_{12}}{{Q_{12}}^2} + \dfrac{H_2}{2}(R_{12}+R_{22})\dfrac{1}{R_{12}} = 0 \end{cases} \tag{54}$$

$$\begin{cases} {Q_{11}}^* = \sqrt{\dfrac{2(A_{S_1}+A_{B_1}+A_{B_2})R_{11}}{H_1(1+\dfrac{R_{21}}{R_{11}})}}, \\ {Q_{12}}^* = \sqrt{\dfrac{2(A_{S_2}+Ac)R_{12}}{H_2(1+\dfrac{R_{22}}{R_{12}})}}, \end{cases} \tag{55}$$

$$K_{v_{(2)}}{}^* = \sqrt{2(A_{S_1}+A_{B_1}+A_{B_2})H_1(R_{11}+R_{21})} + \sqrt{2(A_{S_2}+Ac)H_2(R_{12}+R_{22})} \tag{56}$$

To solve the third model, we first obtain the values of ${Q_{11}}^*$ and ${Q_{12}}^*$ using (49) and (55), and then select the one which satisfies the corresponding assumption. Another way is to solve the following mathematical programming:

$$K_v = KS_v + KB_v = A_{S_1}\frac{R_{11}}{Q_{11}} + A_{S_2}\frac{R_{12}}{Q_{12}} + Ac \times (\frac{R_{11}}{Q_{11}} + \frac{R_{12}}{Q_{12}} - y) +$$
$$(A_{B_1} + A_{B_2}) \times y + \frac{H_1}{2}(R_{11} + R_{21})\frac{Q_{11}}{R_{11}} + \frac{H_2}{2}(R_{12} + R_{22})\frac{Q_{12}}{R_{12}},$$
$$\textit{subject to} : y \geq \frac{R_{11}}{Q_{11}}, y \geq \frac{R_{12}}{Q_{12}}, Q_{11}, Q_{12} \geq 0.$$

Numerical example

We now use the numerical example of model 2 (with Ac = 8) to illustrate the approach. Under the assumptions 1 and 2, ${K_{v_{(1)}}}^*$, ${K_{v_{(2)}}}^*$, ${Q_{11}}^*$ and ${Q_{12}}^*$ are obtained as follows:

Under assumption 1:

$$K_{v_{(1)}}{}^* = 87.0739, \quad {Q_{11}}^* = 9.4136, \quad {Q_{12}}^* = 11.0195, \frac{R_{11}}{{Q_{11}}^*} = 1.2748, \quad \frac{R_{12}}{{Q_{12}}^*} = 1.361.$$

Under assumption 2:

$$K_{v_{(2)}}{}^* = 87.12, \quad {Q_{11}}^* = 9.11, \quad {Q_{12}}^* = 11.34, \quad \frac{R_{11}}{{Q_{11}}^*} = 1.32, \quad \frac{R_{12}}{{Q_{12}}^*} = 1.322.$$

Therefore, the optimal solution of the problem is the data for assumptions and the total costs for the VMI model is equal to 82.970966. (Note that if AC=0 then the value of total costs of VMI in this model is equal to 74.993314 which is less than the corresponding value in the model (2).

5. Conclusions

In this paper, we have presented three problem formulations for Vendor Managed Inventory. The first proposed model of this paper considers the demand as a function of price and price elasticity to demand and the production cost is also considered to be a function of demand. The optimal solutions of both VMI and traditional models are derived and compared. We have also explained through a numerical example how VMI could reduce the total inventory cost compared with traditional method. In the second and third models, we have studied VMI problem with two buyers and two suppliers and discussed the optimal solutions

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