Applied Mathematical Sciences, Vol. 2, 2008, no. 46, 2259 - 2269

Applications of Lie Group Analysis to the Equations of Motion of Inclined Unsagged Cables

Waraporn Chatanin

Department of Mathematics
King Mongkut's University of Technology Thonburi, Bangkok, Thailand
s7501303@st.kmutt.ac.th

Anatoli Loutsiouk

Department of Mathematics
King Mongkut's University of Technology Thonburi, Bangkok, Thailand
loutsiouk.ana@kmutt.ac.th

Somchai Chucheepsakul

Department of Civil Engineering
King Mongkut's University of Technology Thonburi, Bangkok, Thailand
somchai.chu@kmutt.ac.th

Abstract

Analytical solutions to the equations of motion of inclined unsagged cables have been obtained by using the Lie group theory. The infinitesimal generators corresponding to the problem are calculated and then used to construct some invariant solutions. Invariant solutions can be found as solutions of a reduced system of differential equations that has fewer independent variables. Consequently, the system of second - order partial differential equations is reduced to a system of ordinary differential equations and some of these systems can be solved analytically.

Mathematics Subject Classification: 35Q80, 35Q72,70G65, 35A25

Keywords: exact solutions, inclined unsagged cables, Lie group theory, invariant solutions

1 Introduction

Lie group theory was originally designed for constructing exact solutions of differential equations. This theory was originated by a great mathematician of 19th century, Sophus Lie (Norway, 1842-1899). Group analysis provides two basic ways for constructing exact solutions of differential equations, group transformation of known solutions and construction of invariant solutions. The symmetry Lie groups and invariant solutions for the wave propagation in gas and the transonic equation were obtained by Ames et al. [10]. After their work, Torissi and Valenti [5] applied the infinitesimal group analysis to a second - order nonlinear wave equation describing a non - homogeneous process. Bluman and Kumei [2] studied the wave equations with two and four-parameter symmetry groups and the corresponding invariant (similarity) solutions. Peters and Ames [4] applied Lie group method to the nonlinear dynamic equations of elastic string. Group invariant solutions for a second - order hyperbolic wave equation involving an axially transported speed and nonlinear terms were constructed by Fung et al. [9]. Ozkaya and Pakdemirli [1] investigated exact solutions of transverse vibrations of a string moving with time-dependent velocity. In this paper, the Lie group theory is applied to the nonlinear equations of motion of inclined unsagged cables. In section 3, a method for seeking the infinitesimal generators admitted by the equations is demonstrated and the construction of some invariant solutions is presented in section 4.

2 Physical Model and Equations of Motion

2.1 Cable Model

The configuration $(\tilde{x}, \tilde{y})$ in the global coordinates (X, Y) describes the position of the inclined unsagged cable as shown in Figure 1. The angle θ shows the inclination of the cable with respect to X axis. After the disturbance of an external excitation, the cable configuration is changed into the dynamic configuration which is displaced from the initial configuration. The displacement vectors in X and Y directions are represented by $\tilde{u}(\tilde{x}, \tilde{t})$ and $\tilde{v}(\tilde{x}, \tilde{t})$ respectively. The horizontal span X_H is fixed and the cable's vertical span Y_H is varied to attain specified values of θ.

2.2 Equation of Motion

In deriving the equation of motion by the virtual work - energy principle the cable is considered to be perfectly flexible, homogeneous, linearly elastic with negligible torsional, bending, and shear rigidities. Therefore, the strain energy is due only to the stretching of the cable axis. The total strain of the cable at

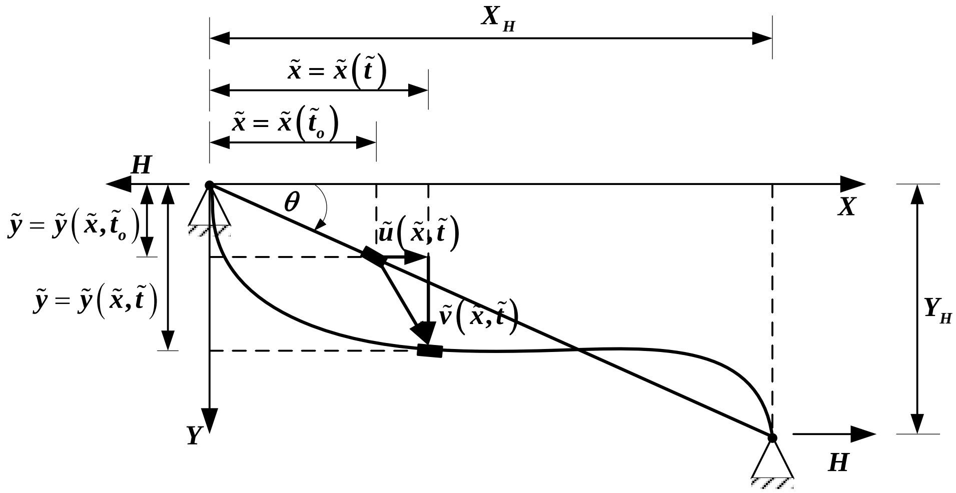

Figure 1: Configurations of an inclined unsagged cable.

the displaced state, with assumption of moderately large vibration amplitudes, can be expressed as follows

$$\bar{e} = e + (1+\tilde{y}'^2)^{-1}[\tilde{u}' + \tilde{y}'\tilde{v}' + \frac{1}{2}(\tilde{u}'^2 + \tilde{v}'^2)]. \tag{1}$$

In this study, the initial strain e is assumed to have a small value and is neglected. The prime ($'$) is used to denote differentiation with respect to $\tilde{x}$. From [6] and [3], the governing equations of motion are

$$\rho\ddot{u} = \{u' + \frac{\bar{E}}{\rho^3}(u' + y'v') + \frac{\bar{E}}{\rho^3}(u'^2 + y'u'v' + \frac{1}{2}(1+u')(u'^2+v'^2))\}', \tag{2}$$

$$\rho\ddot{v} = \{v' + \frac{\bar{E}}{\rho^3}(y'u' + y'^2v') + \frac{\bar{E}}{\rho^3}(u'v' + y'v'^2 + \frac{1}{2}(y'+v')(u'^2+v'^2))\}', \tag{3}$$

where the dot denotes differentiation with respect to t ,

$x = \frac{\tilde{x}}{X_H}, \quad y = \frac{\tilde{y}}{X_H}, \quad u = \frac{\tilde{u}}{X_H}, \quad v = \frac{\tilde{v}}{X_H}, \quad t = \frac{\tilde{t}}{X_H}\sqrt{\frac{gH}{w_c}},$

$\rho = \sqrt{1+\tilde{y}'^2},$

$\tilde{y} = \tilde{x}\tan\theta, \quad \tilde{y}' = \tan\theta,$

$\overline{E} = \frac{EA}{H}$, EA is the cable axial stiffness,

$H = \frac{T_s}{\rho}$ is the horizontal component of the cable static tension T_s,

w_c is the cable weight per unit length,

g is the gravity constant.

A method for solving the nonlinear equations of motion for horizontal and inclined elastic sagged cables by using a numerical technique is demonstrated in Srinil [7]. Here, the unsagged suspended cables are studied. Because the cable is unsagged we can neglect the second derivative of y , and so the simplified non-linear equations of motion of inclined unsagged cables take the form

$$\rho u_{tt} - \{u_{xx} + \frac{\overline{E}}{\rho^3}(u_{xx} + \psi v_{xx} + 3u_x u_{xx} + \psi u_x v_{xx} + \psi u_{xx} v_x + v_x v_{xx})\} = 0, \quad (4)$$

$$\rho v_{tt} - \{v_{xx} + \frac{\overline{E}}{\rho^3}(\psi u_{xx} + \psi^2 v_{xx} + u_x v_{xx} + u_{xx} v_x + 3\psi v_x v_{xx} + \psi u_x u_{xx})\} = 0, \quad (5)$$

where $\psi = y' =$ constant.

This non-linear system describes the coupled longitudinal and vertical displacement's dynamics, and is valid also for slightly sagged horizontal cables if one assumes $T_s \approx H$, rendering $\rho = 1$. It contains quadratic and cubic nonlinear terms due to the cable's axial stretching even in the absence of initial sag (taut string case).

3 Calculation of Infinitesimal Generators

An infinitesimal generator X for the problem is written in the following form

$$X = \alpha(x,t,u,v)\frac{\partial}{\partial x} + \beta(x,t,u,v)\frac{\partial}{\partial t} + \gamma(x,t,u,v)\frac{\partial}{\partial u} + \eta(x,t,u,v)\frac{\partial}{\partial v}. \quad (6)$$

The main aim is to determine all functions α, β, γ, and η that correspond to the one-parameter symmetry groups of Equations (4) and (5). Since the governing equations form a system of second order PDE, the second prolongation of the infinitesimal generator is needed.
The second prolongation of X is given by

$$pr^{(2)}X = X + \gamma^x\frac{\partial}{\partial u_x} + \gamma^t\frac{\partial}{\partial u_t} + \gamma^{xx}\frac{\partial}{\partial u_{xx}} + \gamma^{xt}\frac{\partial}{\partial u_{xt}} + \gamma^{tt}\frac{\partial}{\partial u_{tt}} + \eta^x\frac{\partial}{\partial v_x} + \eta^t\frac{\partial}{\partial v_t} + \eta^{xx}\frac{\partial}{\partial v_{xx}} + \eta^{xt}\frac{\partial}{\partial v_{xt}} + \eta^{tt}\frac{\partial}{\partial v_{tt}}, \quad (7)$$

where

$$\gamma^x = D_x(\gamma - \alpha u_x - \beta u_t) + \alpha u_{xx} + \beta u_{xt}$$

$$\eta^x = D_x(\eta - \alpha v_x - \beta v_t) + \alpha v_{xx} + \beta v_{xt}$$

$$\gamma^{xx} = D_x^2(\gamma - \alpha u_x - \beta u_t) + \alpha u_{xxx} + \beta u_{xxt}$$

$$\eta^{xx} = D_x^2(\eta - \alpha v_x - \beta v_t) + \alpha v_{xxx} + \beta v_{xxt}$$

$$\gamma^t = D_t(\gamma - \alpha u_x - \beta u_t) + \alpha u_{xt} + \beta u_{tt}$$

$$\eta^t = D_t(\eta - \alpha v_x - \beta v_t) + \alpha v_{xt} + \beta v_{tt}$$

$$\gamma^{tt} = D_t^2(\gamma - \alpha u_x - \beta u_t) + \alpha u_{xtt} + \beta u_{ttt}$$

$$\eta^{tt} = D_t^2(\eta - \alpha v_x - \beta v_t) + \alpha v_{xtt} + \beta v_{ttt} \tag{7'}$$

and

$$D_x = \frac{\partial}{\partial x} + u_x \frac{\partial}{\partial u} + u_{xx} \frac{\partial}{\partial u_x} + u_{xt} \frac{\partial}{\partial u_t} + u_{xxx} \frac{\partial}{\partial u_{xx}} + u_{xxt} \frac{\partial}{\partial u_{xt}} + u_{xtt} \frac{\partial}{\partial u_{tt}} + \ldots$$
$$+ v_x \frac{\partial}{\partial v} + v_{xx} \frac{\partial}{\partial v_x} + v_{xt} \frac{\partial}{\partial v_t} + v_{xxx} \frac{\partial}{\partial v_{xx}} + v_{xxt} \frac{\partial}{\partial v_{xt}} + v_{xtt} \frac{\partial}{\partial v_{tt}} + \ldots$$

$$D_t = \frac{\partial}{\partial t} + u_t \frac{\partial}{\partial u} + u_{xt} \frac{\partial}{\partial u_x} + u_{tt} \frac{\partial}{\partial u_t} + \ldots + v_t \frac{\partial}{\partial v} + v_{xt} \frac{\partial}{\partial v_x} + v_{tt} \frac{\partial}{\partial v_t} + \ldots$$

$$D_x^2 = D_x(D_x)$$

$$D_t^2 = D_t(D_t)$$

According to Theorem 2.31 [8],

$$pr^{(2)} X[\rho u_{tt} - \{u_{xx} + \frac{\overline{E}}{\rho^3}(u_{xx} + \psi v_{xx} + 3u_x u_{xx} + \psi u_x v_{xx} + \psi u_{xx} v_x + v_x v_{xx})\}]|_{F=0} = 0, \tag{8}$$

$$pr^{(2)} X[\rho v_{tt} - \{v_{xx} + \frac{\overline{E}}{\rho^3}(\psi u_{xx} + \psi^2 v_{xx} + u_x v_{xx} + u_{xx} v_x + 3\psi v_x v_{xx} + \psi u_x u_{xx})\}]|_{F=0} = 0. \tag{9}$$

where the notation $|_{F=0}$ means that the second prolongation of X is applied to the solutions of Equations (4) and (5). This implies

$$\rho\gamma^{tt} - \{\gamma^{xx} + \frac{\overline{E}}{\rho^3}(\gamma^{xx} + \psi\eta^{xx} + \gamma^x u_{xx} + \gamma^{xx} u_x + \gamma^x \psi v_{xx} + \eta^{xx} \psi u_x + \gamma^{xx} \psi v_x$$

$$+\eta^x\psi u_{xx} + \eta^x v_{xx} + \eta^{xx} v_x)\} = 0, \tag{10}$$

$$\rho\eta^{tt} - \{\eta^{xx} + \frac{\overline{E}}{\rho^3}(y_x^2\eta^{xx} + \psi\gamma^{xx} + \gamma^x v_{xx} + \eta^{xx}u_x + \gamma^{xx}v_x + \eta^x u_{xx} + 2\eta^x\psi v_{xx}$$
$$+2\eta^{xx}\psi v_x + \gamma^x\psi u_{xx} + \gamma^{xx}\psi u_x + \eta^x\psi v_{xx} + \eta^{xx}\psi v_x)\} = 0. \tag{11}$$

Substituting expressions $(7, 7')$ and (4) - (5) into Equations (10) and (11) and then equating the coefficients of all independent monomials to zero, we obtain the determining equations. After solving the determining equations, the functions α, β, γ, and η can be expressed as follows

$$\alpha(x, t, u, v) = k_1 x + k_2, \tag{12}$$

$$\beta(x, t, u, v) = k_1 t + k_3, \tag{13}$$

$$\gamma(x, t, u, v) = k_1 u + k_4 t + k_5, \tag{14}$$

$$\eta(x, t, u, v) = k_1 v + k_6 t + k_7. \tag{15}$$

where $k_1, ..., k_7$ are arbitrary constants.

Thus, the infinitesimal generators have the form

$$X = (k_1x+k_2)\frac{\partial}{\partial x}+(k_1t+k_3)\frac{\partial}{\partial t}+(k_1u+k_4t+k_5)\frac{\partial}{\partial u}+(k_1v+k_6t+k_7)\frac{\partial}{\partial v}, \tag{16}$$

The infinitesimal generator contains seven arbitrary constants. Consequently, the Lie algebra derived from the governing equations is spanned by the following seven linearly independent generators:

$X_1 = x\frac{\partial}{\partial x} + t\frac{\partial}{\partial t} + u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v}$, $X_2 = \frac{\partial}{\partial x}$, $X_3 = \frac{\partial}{\partial t}$,

$X_4 = t\frac{\partial}{\partial u}$, $X_5 = \frac{\partial}{\partial u}$, $X_6 = t\frac{\partial}{\partial v}$, $X_7 = \frac{\partial}{\partial v}$.

From the commutator table (Table 1.), we get by considering the adjoint representation that the following system of one- dimensional subalgebras spanned by the following vectors is optimal:

$X_1 + a_4X_4 + a_6X_6$, (a_4 and a_6 are arbitrary constants)

$X_2 + a_4X_4 + a_6X_6$, ($a_4^2 + a_6^2 > 0$)

Table 1: Commutator table

	X_1	X_2	X_3	X_4	X_5	X_6	X_7
X_1	0	$-X_2$	$-X_3$	0	$-X_5$	0	$-X_7$
X_2	X_2	0	0	0	0	0	0
X_3	X_3	0	0	X_5	0	X_7	0
X_4	0	0	$-X_5$	0	0	0	0
X_5	X_5	0	0	0	0	0	0
X_6	0	0	$-X_7$	0	0	0	0
X_7	X_7	0	0	0	0	0	0

$a_2X_2 + a_5X_5 + a_7X_7,\ \ (a_2^2 + a_5^2 + a_7^2 = 1)$

$a_2X_2 + a_3X_3,\ \ (a_2^2 + a_3^2 = 1)$

$X_2 + a_5X_5 + a_6X_6,\ \ (a_6 \neq 0)$

$X_3 + a_4X_4 + a_6X_6.$ (a_4 and a_6 are arbitrary constants)

4 Invariant Solutions

Given the infinitesimal generators $X = \alpha\frac{\partial}{\partial x} + \beta\frac{\partial}{\partial t} + \gamma\frac{\partial}{\partial u} + \eta\frac{\partial}{\partial v}$ of the one - parameter symmetry groups of Equations (4) and (5), the invariant solutions with respect to the one-parameter group generated by X can be found. Invariants $I(x,t,u,v)$ are calculated by solving the characteristic system

$$\frac{dx}{\alpha} = \frac{dt}{\beta} = \frac{du}{\gamma} = \frac{dv}{\eta}. \tag{17}$$

After expressing independent variables as functions of invariants and substituting them into dependent variables, a system of ordinary differential equation will result. Some cases of generator X which reduce the system of partial differential equations to a system of ordinary differential equations will be considered here:

Case1 For $X = X_2 + a_4X_4 + a_6X_6 = \frac{\partial}{\partial x} + a_4t\frac{\partial}{\partial u} + a_6t\frac{\partial}{\partial v}$,
there are three independent invariants which are obtained by solving the equation

$$XI = \frac{\partial I}{\partial x} + a_4t\frac{\partial I}{\partial u} + a_6t\frac{\partial I}{\partial v} = 0.$$

One of them is $I_1 = t$, while other invariants are found from the characteristic system

$$\frac{dx}{1} = \frac{du}{a_4 t} = \frac{dv}{a_6 t}.$$

By integrating the equation $\frac{dx}{1} = \frac{du}{a_4 t}$, we get $a_4 tx + C_1 = u$. This gives the invariant $I_2 = u - a_4 tx$. Likewise, we get $I_3 = v - a_6 tx$. Let $u - a_4 tx = \phi_1(t)$ and $v - a_6 tx = \phi_2(t)$. It follows that

$$u = \phi_1(t) + a_4 tx, \tag{18}$$

$$v = \phi_2(t) + a_6 tx. \tag{19}$$

Their derivatives are $u_x = a_4 t,\ u_t = \phi_1' + a_4 x,\ u_{xx} = 0,\ u_{tt} = \phi_1'',\ v_x = a_6 t,\ v_t = \phi_2' + a_6 x,\ v_{xx} = 0,\ v_{tt} = \phi_2''$. After substituting these derivatives into Equations (4) and (5), the system of partial differential equations reduces to the system of ordinary differential equations

$$\rho \phi_1'' = 0, \tag{20}$$

$$\rho \phi_2'' = 0. \tag{21}$$

The functions $\phi_1 = at + b$ and $\phi_2 = ct + d$ are solutions to this system, where a, b, c and d are arbitrary constants. By substituting ϕ_1 and ϕ_2 into Equation (18) and Equation (19), we get the exact solutions

$$u = at + b + a_4 tx, \tag{22}$$

$$v = ct + d + a_6 tx. \tag{23}$$

Case2 For $X = a_2 X_2 + a_3 X_3 = a_2 \frac{\partial}{\partial x} + a_3 \frac{\partial}{\partial t}$,
the three independent invariants are $I_1 = u,\ I_2 = v$ and $I_3 = a_3 x - a_2 t$. Let $u = \phi_1(z)$ and $v = \phi_2(z)$, where $z = a_3 x - a_2 t$. This yields the reduced system of ordinary differential equations

$$a_2^2 \rho \phi_1'' - a_3^2 \{\phi_1'' + \frac{\overline{E}}{\rho^3}(\phi_1'' + \psi\phi_2'' + 3a_3\phi_1'\phi_1'' + a_3\psi\phi_1'\phi_2'' + a_3\psi\phi_1''\phi_2' + a_3\phi_2'\phi_2'')\} = 0, \tag{24}$$

$$a_2^2 \rho \phi_2'' - a_3^2 \{\phi_2'' + \frac{\overline{E}}{\rho^3}(\psi\phi_1'' + \psi^2\phi_2'' + a_3\phi_1'\phi_2'' + a_3\phi_1''\phi_2' + 3a_3\psi\phi_2'\phi_2'' + a_3\psi\phi_1'\phi_1'')\} = 0. \tag{25}$$

The functions $\phi_1 = az + b$ and $\phi_2 = cz + d$ are solutions to this system, where a, b, c and d are arbitrary constants. Hence the following are exact solutions to the initial system

$$u = a(a_3x - a_2t) + b, \tag{26}$$

$$v = c(a_3x - a_2t) + d. \tag{27}$$

Case3 For $X = a_2X_2 + a_5X_5 + a_7X_7 = a_2\frac{\partial}{\partial x} + a_5t\frac{\partial}{\partial u} + a_7t\frac{\partial}{\partial v}$,
the three independent invariants are $I_1 = t,\ I_2 = \frac{a_5}{a_2}x - u$ and $I_3 = \frac{a_7}{a_2}x - v$. Let $\frac{a_5}{a_2}x - u = \phi_1(t)$ and $\frac{a_7}{a_2}x - v = \phi_2(t)$. This yields a system of ordinary differential equations

$$\rho\phi_1'' = 0, \tag{28}$$

$$\rho\phi_2'' = 0. \tag{29}$$

The functions $\phi_1 = at + b$ and $\phi_2 = ct + d$ are solutions to this system, where a, b, c and d are arbitrary constants. We get the exact solutions

$$u = \frac{a_5}{a_2}x - (at + b), \tag{30}$$

$$v = \frac{a_7}{a_2}x - (ct + d). \tag{31}$$

Case4 For $X = X_2 + a_5X_5 + a_6X_6 = \frac{\partial}{\partial x} + a_5\frac{\partial}{\partial u} + a_6t\frac{\partial}{\partial v}$,
the three independent invariants are $I_1 = t,\ I_2 = u - a_5x$ and $I_3 = v - a_6tx$. Let $u - a_5x = \phi_1(t)$ and $v - a_6tx = \phi_2(t)$. It follows that $u = \phi_1(t) + a_5x$, $v = \phi_2(t) + a_6tx$ and we obtain a system of ordinary differential equations

$$\rho\phi_1'' = 0, \tag{32}$$

$$\rho\phi_2'' = 0. \tag{33}$$

The functions $\phi_1 = at + b$ and $\phi_2 = ct + d$ are solutions to this system, where a, b, c and d are arbitrary constants. Thus, we get the exact solutions

$$u = at + b + a_5x, \tag{34}$$

$$v = ct + d + a_6tx. \tag{35}$$

Case5 For $X = X_3 + a_4X_4 + a_6X_6 = \frac{\partial}{\partial t} + a_4t\frac{\partial}{\partial u} + a_6t\frac{\partial}{\partial v}$,
the three independent invariants are $I_1 = x,\ I_2 = u - \frac{a_4}{2}t^2$ and $I_3 = v - \frac{a_6}{2}t^2$. Let $u - \frac{a_4}{2}t^2 = \phi_1(x)$ and $v - \frac{a_6}{2}t^2 = \phi_2(x)$. It follows that $u = \phi_1(x) + \frac{a_4}{2}t^2$, $v = \phi_2(x) + \frac{a_6}{2}t^2$. This yields a system of ordinary differential equations

$$a_4\rho - \{\phi_1'' + \frac{\overline{E}}{\rho^3}(\phi_1'' + \psi\phi_2'' + 3\phi_1'\phi_1'' + \psi\phi_1'\phi_2'' + \psi\phi_1''\phi_2' + \phi_2'\phi_2'')\} = 0, \tag{36}$$

$$a_6\rho - \{\phi_2'' + \frac{\overline{E}}{\rho^3}(\psi\phi_1'' + \psi^2\phi_2'' + \phi_1'\phi_2'' + \phi_1''\phi_2' + 3\psi\phi_2'\phi_2'' + \psi\phi_1'\phi_1'')\} = 0. \quad (37)$$

Case6 For $X = X_1 = x\frac{\partial}{\partial x} + t\frac{\partial}{\partial t} + u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v}$,
the three independent invariants are $I_1 = \frac{x}{t}$, $I_2 = \frac{u}{x}$ and $I_3 = \frac{v}{x}$. Let $\frac{u}{x} = \phi_1(z)$ and $\frac{v}{x} = \phi_2(z)$ where $z = \frac{x}{t}$. It follows that $u = x\phi_1(z)$, $v = x\phi_2(z)$. This yields a system of ordinary differential equations

$$\rho(z^2\phi_1'' + z\phi_1') + \{z^2\phi_1'' + 2z\phi_1' + \frac{\overline{E}}{\rho^3}(z^2\phi_1'' + 2z\phi_1' + \psi(z^2\phi_2'' + 2z\phi_2')$$
$$+3(z\phi_1'+\phi_1)(z^2\phi_1''+2z\phi_1')+\psi(z\phi_1'+\phi_1)(z^2\phi_2''+2z\phi_2')+\psi(z^2\phi_1''+2z\phi_1')(z\phi_2'+\phi_2)$$
$$+(z\phi_2' + \phi_2)(z^2\phi_2'' + 2z\phi_2'))\} = 0, \quad (38)$$

$$\rho(z^2\phi_2'' + z\phi_2') + \{z^2\phi_2'' + 2z\phi_2' + \frac{\overline{E}}{\rho^3}(\psi(z^2\phi_1'' + 2z\phi_1') + \psi^2(z^2\phi_2'' + 2z\phi_2')$$
$$+(z\phi_1'+\phi_1)(z^2\phi_2''+2z\phi_2')+(z^2\phi_1''+2z\phi_1')(z\phi_2'+\phi_2)+3\psi(z\phi_2'+\phi_2)(z^2\phi_2''+2z\phi_2')$$
$$+\psi(z\phi_1' + \phi_1)(z^2\phi_1'' + 2z\phi_1'))\} = 0. \quad (39)$$

5 Conclusion

In this paper invariant solutions of the equations of motion of the inclined unsagged cables are investigated. By using the infinitesimal generators, the system of partial differential equations can be reduced to a system of ordinary differential equations. In some cases it is possible to solve these systems analytically.

ACKNOWLEDGEMENTS. The first author would like to express deep gratitude to Development and Promotion for Science and Technology talents project of Thailand (DPST) for the financial support.

References

[1] E. Ozkaya and M. Pakdemirli, Lie Group Theory and Analytical Solutions for the Axially Accelerating String Problem. *J. Sound Vib,* **230**(2000), 729–742.

[2] G. Bluman and S. Kumei, Properties of the Wave Eqution, *J. Math. Phys,* **28** (1987), 307–318.

[3] G. Rega, N. Srinil, W. Lacarbonara and S. Chucheepsakul, Resonant Nonlinear Normal Modes of Inclined Sagged Cables, *J. EUROMECH,* **457** (2004), 7–10.

[4] J.E. Peters and W.F. Ames, Group Properties of the Non-Linear Dynamic Equations of Elastic String, *J. Non-Linear Mech,* **25** (1990), 107–115.

[5] M. Torissi and A. Valenti, Group Properties and Invariant Solutions for Infinitesimal Transformations of a Non-Linear Wave Equation, *J. Non-Linear Mech,* **20** (1985), 135–144.

[6] N. Srinil, G. Rega and S. Chucheepsakul, Three-Dimensional Non-linear Coupling and Dynamic Tension in the Large - Amplitude Free Vibrations of Arbitrarily Sagged Cables, *J. Sound Vib,* **269** (2004), 823–852.

[7] N. Srinil, S. Chucheepsakul and G. Rega, Internally Resonant Nonlinear Free Vibrations of Horrizontal/Inclined Sagged Cables, *in proc. of IDECT'05,* ASME (2005).

[8] P.J. Olver, *Application of Lie Group to Differential Equations,* Springer, New York, 1986.

[9] R.F. Fung, Y.C. Wang and J.W. Wu, Group Properties and Group-Invariant Solutions for Infinitesimal Transformations of the Non-Linearly Traveling String, *J. Int. J. Non-Linear Mech,* **34** (1999), 693–698.

[10] W.F. Ames, R.J. Lohner and E. Adams, Group Properties of $u_{tt} = [f(u)u_x]_x$, *J. Non-Linear Mech,* **16** (1981), 439–447.

Received: February 10, 2008