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Analysis of the Dynamic Contact Inside the Piezoelectric Motor Using RBDO

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Abstract

In this paper, we study some methods in the context of the Reliability-Based Design Optimization analysis (RBDO) of the piezoelectric motor with travelling wave taking into account the dynamic contact between the different components (stator and rotor). The RBDO play a dominant role in the structural optimization problem introducing the reliability concept. Generally, the life of these piezoelectric motor is limited by important abrasion of the different components. So, the notion of random variables and the risk of failure must be integrated in the mechanical analysis to ensure the good working of the system. The numerical treatment of the dynamic contact inside the motor is presented with the good choice of relaxation factors. The efficiency of the hybrid RBDO method has been demonstrated on static and dynamic cases with extension to the variability of the probabilistic model. We propose two methods: the dynamic hybrid method and the frequencies hybrid method as extension of the improved hybrid method presented in further works. The discussion of these different approaches based on the RBDO is given. The numerical results show the efficiency of these two proposed methods and the efficiency of each method is done.

Mathematics Subject Classification: 49Q10, 60K20, 60K10, 74H20,74S05

Keywords: Reliability analysis, Finite element analysis, Reliability-based design Optimization, Dynamic contact

1 Introduction

The modeling of the piezoelectric motors with travelling waves implies a large variety of physical and mechanical phenomena. This variety leads to approaches and models quite as many and varied, which rest mainly on phenomenological and numerical analyses.

The generation of a progressive wave of volume imposes the respect of geometrical constraints and mechanics relating to the periodicity of the motors structure. Under normal conditions of operation, the motors are subjected to:

- An axial static loading of pre-stressing producing axial and radial deformations in stator and rotor,
- A dynamic excitation of the stator, involving deformations of bending out of the plane, which produce by drive, a rigid displacement of the rotor's body,
- Efforts of contact and friction static and dynamic in the contact zone between the stator and the rotor.

The aim of this paper is firstly to propose a numerical modelling by the finite element method of the mechanical behaviour of piezoelectric motor SHINSEI USR 60 pennies dynamic loading taking into account the contact without friction and secondly to introduce the RBDO analysis to study this motor.

Contact problems are treated using an augmented Lagrangian approach to identify the candidate contact surface and contact stresses [1] and the dynamic treatment is solved using a time integration scheme. The time integration parameters are specially selected to ensure that the solution of dynamic contact problem is unconditionally stable and reduce significantly the spurious high-frequency modes, which persist in the traditional Newmark method.

The stochastic role of each parameter of design in the default risk is highlighted. For this reliability analysis of the structure, we propose a coupling direct mechanic-reliability between the augmented Lagrangian method to solve the contact, developed on a computer code by finite elements, and probabilistic method FORM [12].

Deterministic design optimization enhanced by reliability performances and formulated within the probabilistic framework is called Reliability-based Design Optimization (RBDO). Generally, the purpose of the RBDO is to design structures that should be economic and reliable by introducing safety criteria in the optimization procedure and it is often formulated as a minimization of the initial structural cost under constraints imposed on the values of elemental reliability indices corresponding to various limit states [12]. This method is used to perform new design to the studied motor.

We propose in this paper, two methods: the dynamic hybrid method and the frequencies hybrid method as extension of the improved hybrid method presented in further works. The discussion of these different approaches based on the RBDO is given. The numerical results show the efficiency of these two proposed methods and the efficiency of each method is done. In this work, we model the Young's modules of three materials and the external loading by the normal law.

The results obtained then enable us to propose a whole of recommendations to optimize the reliability of piezoelectric motor SHINSEI USR 60.

The paper is organized as follows. In section 2, we present a brief description of our treatment of the dynamic contact. In section 3, The RBDO method is introduced and we discuss the two variant methods in the hybrid contexte: the dynamic hybrid method and the frequencies hybrid method. The section 4 is dedicated to the presentation of the piezoelectric motor SHINSEI USR 60 and the founding numerical results.

2. Brief description of the dynamic contact

2.1 Treatment of the contact

Contact between solids is generally governed by two constraints which can be written as follows:

$$\begin{aligned} u_N - \delta < 0 &\Rightarrow r_N = 0 \\ u_N - \delta = 0 &\Rightarrow r_N \le 0 \end{aligned} \tag{1}$$

where δ represents the gap between the contacting bodies, u_N the normal component of the displacement field and r_N the normal reaction [7].

From the Hamilton principle, the system energy and work of external forces can be written in the following form:

$$\begin{aligned} \Pi(u) = \int_{t_1}^{t_2}\int_V \frac{1}{2}\varepsilon^T D\varepsilon dVdt - \int_{t_1}^{t_2}\int_V \frac{1}{2}\rho\left(\frac{du}{dt}\right)^T\left(\frac{du}{dt}\right)dVdt \\ - \int_{t_1}^{t_2}\int_{S_1} u^T PdSdt - \int_{t_1}^{t_2}\int_{S_2} u^T RdSdt - \int_{t_1}^{t_2}\int_{S_1} u^T R_C dSdt \end{aligned} \tag{2}$$

Enforcement of the zero-penetration condition on contacting boundaries yields: $T^T u - \delta \ge 0, \quad u \in S_C$.

Where, ε is the strain vector; D is the material matrix; ρ is the mass density; u is the displacement vector; P is the external load vector; R is the reaction force vector on prescribed displacement boundary; T is the contact constraint matrix; S_1 is the boundary with prescribed external forces; S_2 is the boundary with prescribed displacements and S_c is the contacting boundary.

The augmented Lagrangian approach relative to the dynamic contact problem is given by the weak form of the equilibrium state:

$$\Pi^*(u, \delta u) = \Pi(u, \delta u) + \int_{t_1}^{t_2}\int_{S_c} \left(T^T \delta u - \delta\right)^T r_N dt \tag{3}$$

where δ is the gap between two contacting bodies.

This nonlinear equation must be solved but it cannot be solved directly, so r_N is considered as known via an iterative process using the Newton-Raphson method. The reaction r_N is written in function of a Lagrange multiplier and a penalty coefficient as follows:

$$r_N = \lambda_n + \varepsilon_n u_n \tag{4}$$

2.1 Integration time algorithm

The finite difference method is employed in the time domain to establish a relationship between acceleration, velocity and displacement fields. The Newmark method, the most frequently used implicit time integration scheme, allows us to approximate velocities and displacements at time t+Δt as follows:

$$^{t+\Delta t}\dot{U} = {}^{t}\dot{U} + [(1-\gamma)^{t}\ddot{U} + \gamma\ {}^{t+\Delta t}\ddot{U}]\Delta t, \tag{5}$$

$$^{t+\Delta t}U = {}^{t}U + {}^{t}\dot{U}\,\Delta t + [(\frac{1}{2}-\beta)^{t}\ddot{U} + \beta\ {}^{t+\Delta t}\ddot{U}]\Delta t^{2}, \tag{6}$$

It has been shown that the use of the trapezoidal rule (γ=0.5 and β=0.25) with a fully implicit treatment of the contact constraints produces oscillations, which can be significant as the time steps and spacial discretization are defined. Recently, the generalised-α method was developed for solving structural dynamics problem with second-order accuracy [14]. In this method, the equation of motion is modified as follows:

$$M^{(t+\Delta t)-\alpha_B}\ddot{U} + C^{(t+\Delta t)-\alpha_H}\dot{U} + K^{(t+\Delta t)-\alpha_H}U = {}^{(t+\Delta t)-\alpha_H}F, \tag{7}$$

where

$$^{(t+\Delta t)-\alpha_H}U = (1-\gamma)^{t+\Delta t}U + \alpha_H\,{}^{t}U, \tag{8}$$

$$^{(t+\Delta t)-\alpha_H}\dot{U} = (1-\alpha_H)^{t+\Delta t}\dot{U} + \alpha_H\,{}^{t}\dot{U}, \tag{9}$$

$$^{(t+\Delta t)-\alpha_B}\ddot{U} = (1-\alpha_B)^{t+\Delta t}\ddot{U} + \alpha_B\,{}^{t}\ddot{U}, \tag{10}$$

$$^{(t+\Delta t)-\alpha_H}F = (1-\alpha_H)^{t+\Delta t}F + \alpha_H\,{}^{t}F, \tag{11}$$

The Newmark time integration scheme is used to solve the above equation of motion. In this study, we used γ=0.5 and β=0.25, with the additional parameters $\alpha_B = 0.5$ and $\alpha_H = 0.5$ for the integration constants. Earlier work by the authors indicates that these parameters result in second-order accuracy, unconditional stability, energy and momentum conservation, and not cause numerical dissipation [2]. For linear problems, both the generalised-α and the Newmark trapezoidal schemes involve the same percentage error elongation and do not cause amplitude decay. Furthermore, the proposed scheme improves convergence in frictional contact problems.

3. Reliability based design optimization

It is strong need to integrate the reliability analysis in the optimization process to control the reliability level and to minimize the structural cost or weight in the non-critical regions of the structure. The ultimate goal of design under uncertainty is to reach an optimum in terms of total cost. In principle, an optimum balance between structural system reliability and other conflicting societal goals must be obtained. This is a difficult task. The integration of reliability analysis into engineering design optimization is termed Reliability-based Design Optimization (RBDO).

Traditionally, the solution of the RBDO model is achieved by alternating reliability and optimization iterations. This approach leads to low numerical efficiency, which is disadvantageous for engineering applications on real structures. In order to avoid this difficulty, the hybrid RBDO methods are proposed [4,5].

3.1 Hybrid RBDO method

To improve the numerical performance, the hybrid approach consists in minimizing a new form of the objective function $F(x,y)$ subject to a limit state and to deterministic and reliability constraints, as:

$$\begin{aligned} \min_{x,y}: \quad & F(x,y) = f(x)\times d_\beta(x,y) \\ subject to: \quad & G(x,y) \le 0 \\ & g_k(x,y) \le 0 \\ and \quad & d_\beta(x,y) \ge \beta_t \end{aligned} \tag{12}$$

Here, $d_\beta(x,y)$ is the distance in the hybrid space between the optimum and the design point, $d_\beta(x,y)=d(u)$. The minimization of the function $F(x,y)$ is carried out in the Hybrid Design space (HDS) of deterministic variables x and random variables y.

In [4], it is shown that this method reduced the computational time almost 80% relative to the classical RBDO approach. Using the hybrid one, the optimization process is carried out in the Hybrid Design space, where all numerical information about the optimization process can be modelled. Furthermore, the classical RBDO approach has weak convergence stability because is carried out in two spaces (physical and normalized space).

The efficiency of the hybrid approach allows extension for solving more complex problems. In the same direction, the dynamic hybrid method is proposed [9]. The efficiency of this method is showed in [10].

3.2 Dynamic Hybrid method

The solution of the above nested problems leads to very large computational time, especially for large-scale structures. In order to improve the numerical performance, the hybrid approach consists in minimizing a new form of the objective function $F(\mathbf{x},\mathbf{y})$ subject to a limit state and to deterministic as well as to reliability constraints:

$$\begin{array}{lll} \min & : f(\mathbf{x}).d_\beta(\mathbf{x},\mathbf{y},t) & \\ \textit{subject to} & : G(\mathbf{x},\mathbf{y},t) \leq 0 & \\ & : d_\beta(\mathbf{x},\mathbf{y},t) \leq \beta_c(t) & \forall t \in [0 \;\; T] \\ \textit{and} & : g_k(\mathbf{x},t) \leq 0 & \end{array} \tag{13}$$

In the case $t = 0$ (static problem), $d_\beta(\mathbf{x},\mathbf{y})$ is the distance in the hybrid space between the optimum and the design point, $d_\beta(\mathbf{x},\mathbf{y}) = d(\mathbf{u})$. The minimization of the function $F(\mathbf{x},\mathbf{y})$ is carried out in the Hybrid Design Space (HDS) of deterministic variables $\mathbf{x}$ and random variables $\mathbf{y}$.

An example of this HDS is given in figure 1, containing design and random variables, where the reliability levels d_β can be represented by ellipses in case of normal distribution, the objective function levels are given by solid curves and the limit state function is represented by dashed level lines except for $G(\mathbf{x},\mathbf{y}) = 0$. We can see two important points: the optimal solution P_x^* and the reliability solution P_y^* (i.e. the design point found on the curves $G(\mathbf{x},\mathbf{y}) = 0$ and $d_\beta = \beta_t$).

3.3 Frequencies Hybrid Method (FHM)

The response of a structure to a dynamic excitation depends, to a large extent, on the first few natural frequencies of the structure. Excessive vibration occurs when the frequency of the dynamic excitation is close to one of the natural frequencies of the structure. In designing most structures, it is often necessary to restrict the fundamental frequency or several of the lower frequencies of the structure to a prescribed range in order to avoid severe vibration. The hybrid formulation will not be able in its traditional formulation to determine the critical region about eigen-frequency.

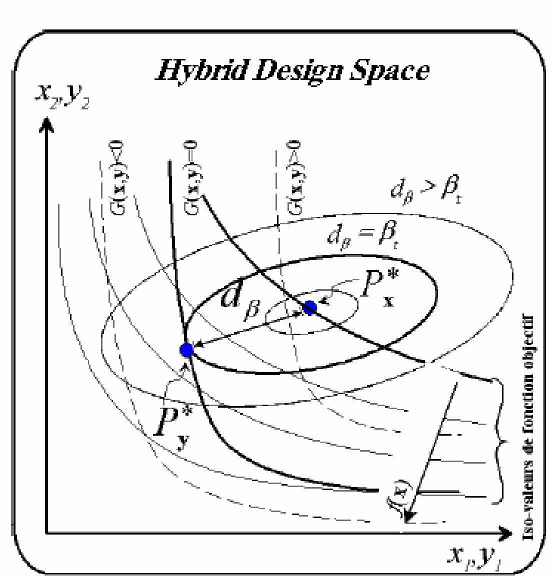

Figure 1. Hybrid Design Space in the case t = 0.

A new formulation was developed within the framework of calculations into dynamic excitation. The goal of this development is to seek the dangerous frequencies bands relative to different eigen-frequencies. The principal idea is to seek more than only one point of design. The frequencies band critical is limited by a lower limit and an upper limit (Figure 2). These two points are sought for each iteration.

Formulation

The new formulation of frequencies problem is:

$$\begin{aligned} &\min_{x,y} && : F(\mathbf{x},\mathbf{y}) = f(\mathbf{x}).d_{\beta_1}(\mathbf{x},\mathbf{y}).d_{\beta_2}(\mathbf{x},\mathbf{y}) \\ &\textit{subject to} && : G(\mathbf{x},\mathbf{y}) \le 0 \\ & && : g_k(\mathbf{x}) \le 0 \\ &\textit{and} && : d_{\beta_1}(\mathbf{x},\mathbf{y}) \ge \beta_t \\ & && : d_{\beta_2}(\mathbf{x},\mathbf{y}) \ge \beta_t \end{aligned} \tag{14}$$

An example of this Hybrid Design Space and Displacement and eigen-frequency is given in the figure 3, in this case, we can see three important points: the optimal solution P_x^* and two point of the reliability solution P_y^* (i.e. the design point found on the curves $G(\mathbf{x},\mathbf{y}) = 0$ and $d_\beta = \beta_t$).

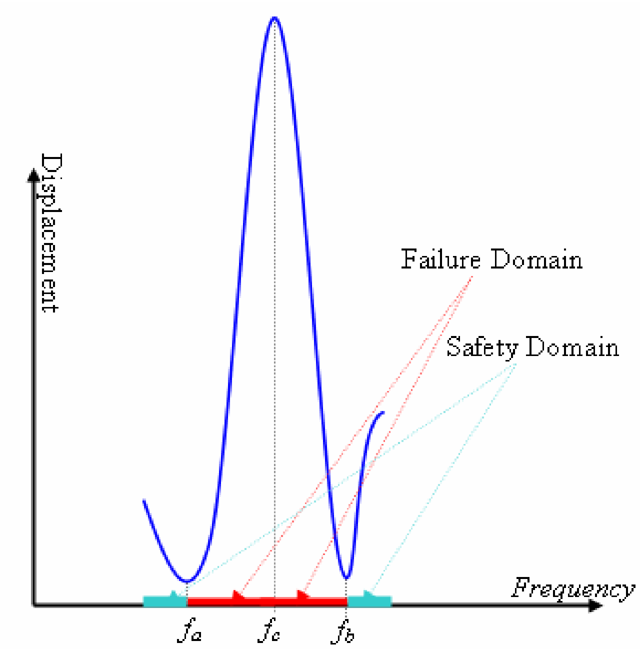

Figure 2. Displacement and eigen-frequency.

Optimality conditions:

For this problem, the Lagrangian is written like follows:

$$L_H(x,y,\lambda) = f(x).d_{\beta_a}(x,y).d_{\beta_b}(x,y) + \lambda_{\beta_a}\left[\beta_t - d_{\beta_a}(x,y)\right] + \lambda_{\beta_b}\left[\beta_t - d_{\beta_b}(x,y)\right] + \lambda_G\, G(x,y) + \sum_k \lambda_k\, g(x) \tag{15}$$

The optimality conditions of this Lagrangian are:

$$\frac{\partial L_H}{\partial x_i} = d_{\beta_a}(x,y)d_{\beta_b}(x,y)\frac{\partial f}{\partial x_i} + \left[f(x)d_{\beta_b}(x,y) - \lambda_{\beta_a}\right]\frac{\partial d_{\beta_a}}{\partial x_i} + \left[f(x)d_{\beta_a}(x,y) - \lambda_{\beta_b}\right]\frac{\partial d_{\beta_b}}{\partial x_i} + \lambda_G\frac{\partial G}{\partial x_i} + \sum_k \lambda_k \frac{\partial g_k}{\partial x_i} = 0 \tag{16}$$

$$\frac{\partial L_H}{\partial y_i} = \left[f(x)d_{\beta_b}(x,y) - \lambda_{\beta_a}\right]\frac{\partial d_{\beta_a}}{\partial y_i} + \left[f(x)d_{\beta_a}(x,y) - \lambda_{\beta_b}\right]\frac{\partial d_{\beta_b}}{\partial y_i} + \lambda_G\frac{\partial G}{\partial y_i} = 0 \tag{17}$$

$$\frac{\partial L_H}{\partial \lambda_{\beta_a}} = \beta_t - d_{\beta_a}(x,y) = 0 \tag{18}$$

$$\frac{\partial L_H}{\partial \lambda_{\beta_b}} = \beta_t - d_{\beta_b}(x,y) = 0 \tag{19}$$

$$\frac{\partial L_H}{\partial \lambda_G} = G(x,y) = 0 \tag{20}$$

$$\frac{\partial L_H}{\partial \lambda_k} = g_k(x) = 0 \tag{21}$$

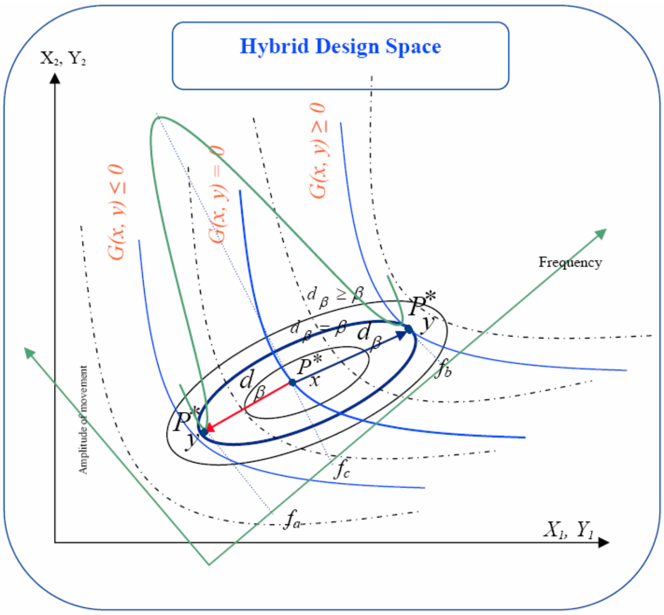

Figure 3. Hybrid Design Space& Displacement and eigen-frequency.

3.4 Extension to multiple failures

In the case of multiple failure modes, we have several limit states that should be considered. Let N_r be the number of failure modes, $G_r(\mathbf{x},\mathbf{y})$, $d_{\beta_{a_r}}$ and $d_{\beta_{b_r}}$ *(r=1…….N_r)* are the limit state functions and the reliability indexes, respectively. Therefore, the hybrid problem can be expressed by:

$$\text{min: } F(x,y) = \quad f(\{x\}) \cdot \sum_{r=1}^{N_r} \left(d_{\beta_{a_r}}(\{x\},\{y\}) . d_{\beta_{b_r}}(\{x\},\{y\}) \right) \tag{22}$$

subject to $\quad G_r(\{x\},(y)) \leq 0 \, , g_k(\{x\}) \leq 0$

and $\quad \beta_{a_r}(\{x\},\{y\}) \geq \beta_t \, , \beta_{b_r}(\{x\},\{y\}) \geq \beta_t$

The optimality condition for this problem can be similarly verified as the single limit state problem.

4. Numerical Results

In this example, we present a study on the Reliability design optimisation on the stator and the rotor of a piezoelectric motor with annular progressive wave SHINSEI USR 60 (figure 4) taking the dynamic contact into account, which will be subjected to the constraint stress [7].

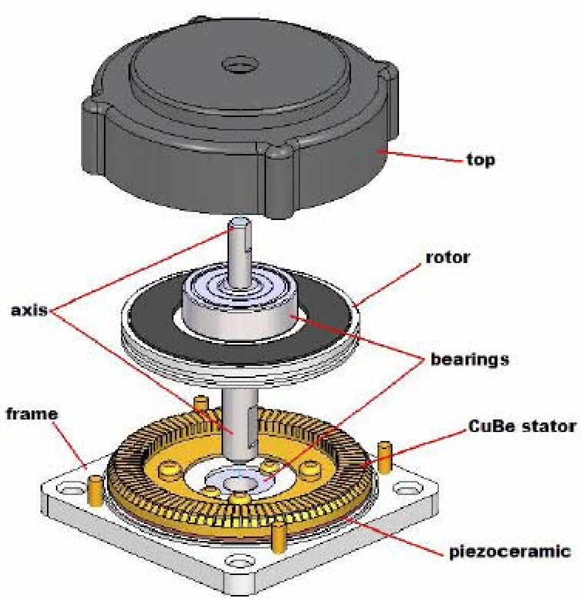

Figure 4. Motor SHINSEI USR 60

4.1 Geometry and operation

The operation principle is described on figure 4. This motor is made of two main parts:

- The stator, which is a beryllium-copper annular plate. At its circumference, teeth are machined to amplify the vibration movement and eliminate the wear particles. At its bottom surface, piezoelectric ceramics are glued to excite the metallic part. The stator is fixed to the frame at its center. To guarantee the free vibration of the stator ring, a decoupling fold is machined between the center and the circumference.
- The rotor, that can be separated in 3 zones: the axis, whose rotating is output of the motor, the friction track in contact with the stator and the spring fold linking the axis to the track and giving the elasticity needed to connect the rotor to the stator.

Other parts are the frame, the top and bearings. The piezoceramic ring is metalize on one face. The cube stator constitutes the ground electrode. Metallization and polarization are designed to excite a particular bending mode (depending of the motor's size). This frequency is generally between 30 and 100 kHz, thus the name of ultrasonic motor (USM). Excitation at a natural frequency creates a travelling wave; each point of the top surface of the stator has an elliptic motion [3]. The stator is in permanent contact with the peaks of the wave which have all the same elementary movement. All these elementary displacements drive the rotor by friction. To have good contact conditions, a thin polymer layer can be sticked on the rotor or the stator [8].

The geometrical cross section of the stator and the rotor shows in figures 5 and 6. The body force F_{ext} on the SHINSEI USR 60 motor, is transmit to the rotor circumferential and punctually done at the ratio $R=21.10^{-3}\ m$ with the value $140\,N$. The mechanical characteristics of different material and geometry are given in tables 1 and 2.

Parameters	H0	H1	H2	H3	H4	H5	H6	H7	R1	R2	R3	R4	R5
(mm)	0.5	0.6	1.2	1	4.2	3.2	1.3	2.1	16.5	22.5	25	29	30

Table 1. Geometry characteristics

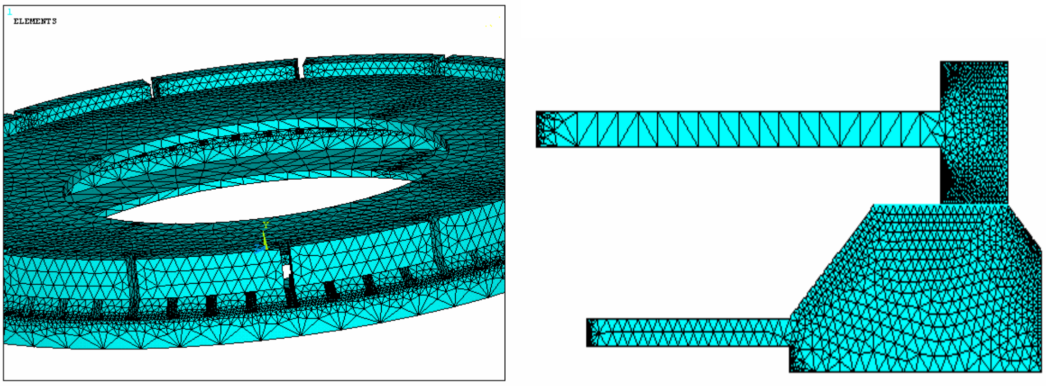

Figure 5. Finite elements modelisation of stator and rotor

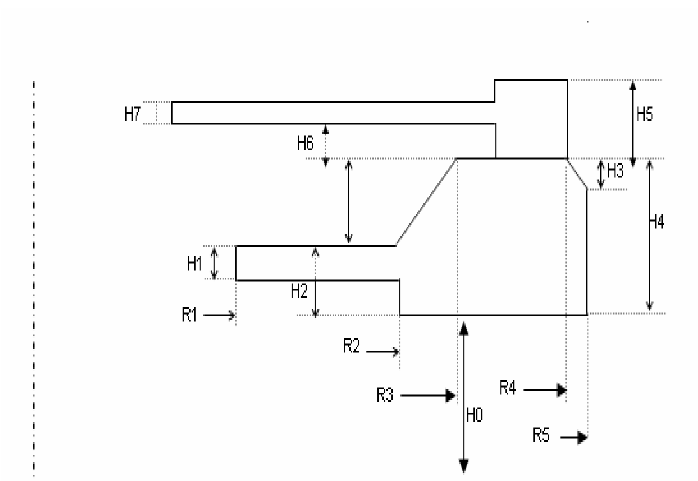

Figure 6. Dimensional parameters of the stator and the rotor.

Rotor		Stator	
E *(MPa)*	ρ *(Kg/m³)*	E *(MPa)*	ρ *(Kg/m³)*
27000	2700	123000	8250

Table 2. Mechanical characteristics

4.2 Result and discussion

The objective of this study is to show the efficiency and the robustness of the dynamic hybrid method and Frequencies hybrid method. We minimize the cost of structure subject to the stress constraint, the target reliability index constraint and the second part subject to eigen-frequency constraint the target reliability index constraint. The system must satisfy the predefined target reliability.

The classical RBDO problem is written as following: the optimization problem is to find the optimum value of the structural volume subject to the maximum stress (transient response). This problem can be expressed as:

$$\begin{aligned} &\min && Area(H0,H2,H3,H5,H6,H7) \\ &\textit{subject to} && \sigma(H0,H2,H3,H5,H6,H7)-\sigma_{ad} \\ & && \beta(\mathbf{x},\mathbf{y}) \geq \beta_t \end{aligned} \tag{23}$$

where

$$\beta(\mathbf{x},\mathbf{y}) = \min\left(dis(\mathbf{u}) = \sqrt{\sum_{i=1}^{6} u_i^2} \right) \tag{24}$$

The classical RBDO approach leads to a weak stability of convergence but the dynamic hybrid method allows the coupling between the reliability analysis and the optimization problem. The dynamic hybrid method problem can expressed as:

$$\begin{aligned} &\min && : S(H0,H2,H3,H5,H6,H7).d_\beta(\mathbf{x},\mathbf{y}) \\ &\textit{subject to} && : \sigma(H0,H2,H3,H5,H6,H7)-\sigma_{ad} = 0 \\ & && : d_\beta(\mathbf{x},\mathbf{y},t) \geq \beta_t \quad , \quad \beta_t = 3.8 \end{aligned} \tag{25}$$

where *H0, H2, H3, H5, H6* and *H7* are grouped in the random vector **y** but to optimize the design, the means m_{H0}, m_{H2} ,m_{H3}, m_{H5}, m_{H6} and m_{H7} are grouped in the deterministic vector **x**, and their standard-deviation equals to $0.1m_x$.

Variables	CRBDO *(safety factor 1.5)*		RHM	
	Design Point	*Optimum Solution*	*Design Point*	*Optimum Solution*
H0	0.52195×10^{-3}	0.53998×10^{-3}	0.57767×10^{-3}	0.71853×10^{-3}
H2	0.11563×10^{-2}	0.12749×10^{-2}	0.11494×10^{-2}	0.13036×10^{-2}
H3	0.66719×10^{-3}	0.99276×10^{-3}	0.87807×10^{-3}	0.11079×10^{-3}
H5	0.32564×10^{-2}	0.24628×10^{-2}	0.25716×10^{-2}	0.23572×10^{-2}
H6	0.12530×10^{-2}	0.13973×10^{-2}	0.80493×10^{-3}	0.87639×10^{-3}
H7	0.20121×10^{-2}	0.20821×10^{-2}	0.14356×10^{-2}	0.13696×10^{-2}
Stress	0.23564×10^{9}	0.23530×10^{9}	0.2358×10^{9}	0.1560×10^{9}
Reliability index	3.6	-------------	3.8	----

Table 3. Results of RBDO in stator and rotor

Table 3 shows the RBDO results using the different distribution laws. After optimizing the structure, the resulting CRBDO can't found the required reliability level but the results of the optimization process in RHM satisfies the reliability constraint.

The objective of the frequencies hybrid method is to minimize the volume subject to eigen-frequency $f_c = 39.5\,KHz$ of functioning mode of the stator (Figure 7) constraint and the system must satisfy predefined target reliability, we consider the target reliability index as $\beta_t = 3.6$.

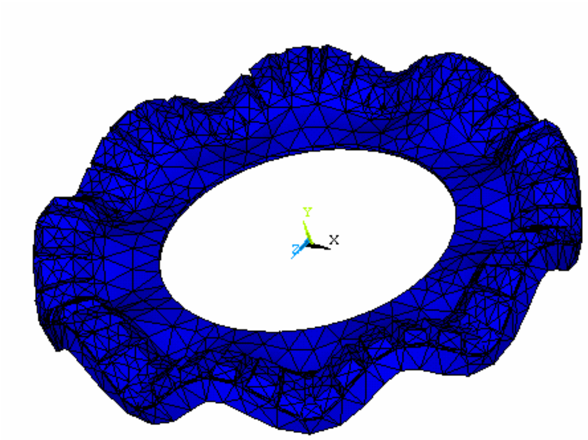

Figure 7: Functioning mode of the stator

Figure 2 shows the relationship between the displacement (the eigen-vector is normal with respect to the mass) and the eigen-frequency f_c. In order to guarantee a required safety level, the performance of the studied structure must be outside of the interval $[f_a, f_b]$. However the good functioning of the stator is on this band of frequency.

The frequencies hybrid method problem can expressed as:

$$\begin{aligned} \min_{x,y} \quad & : F(x,y)=V(R2,R3,R4,H1,H2,H3,H4).d_{\beta_1}(\mathbf{x},\mathbf{y}).d_{\beta_2}(\mathbf{x},\mathbf{y}) \\ \textit{subject to} \quad & : f(R2,R3,R4,H1,H2,H3,H4)-f_c = 0 \\ \textit{and} \quad & : d_{\beta_1}(\mathbf{x},\mathbf{y}) \geq \beta_t \\ & : d_{\beta_2}(\mathbf{x},\mathbf{y}) \geq \beta_t \end{aligned} \qquad (26)$$

where *R2,R3,R4,H1,H2,H3* and *H4* are grouped in the random vector **y** but to optimize the design, the means m_{R2}, m_{R3}, m_{R4}, m_{H1}, m_{H2}, m_{H3}, and m_{H4} are grouped in the deterministic vector **x**, and their standard-deviation equals to $0.1m_x$.

Variables	*Design Point (a)*	*Optimum Solution*	*Design Point (b)*
R2	22.211×10^{-3}	21.214×10^{-3}	20.76×10^{-3}
R3	25.59×10^{-3}	23.959×10^{-3}	25.4×10^{-3}
R4	29.66×10^{-3}	27.925×10^{-3}	28.33×10^{-3}
H1	0.428×10^{-3}	0.562×10^{-3}	0.678×10^{-3}
H2	1.345×10^{-3}	1.483×10^{-3}	1.549×10^{-3}
H3	1.5×10^{-3}	1.318×10^{-3}	1.66×10^{-3}
H4	3.66×10^{-3}	3.435×10^{-3}	3.489×10^{-3}
β	3.65×10^{-3}	-------	3.6
Frequency *[KHz]*	37.2	39.5	41

Table 4. Results of RBDO in stator

Table 4 shows the obtained results using the FHM, the solution redefines a new interval [*37.2*, *41*] kHz of the work of the piezoelectric motor and satisfying a required reliability index (see table 4). Generally, we cannot know if the given interval is large or small, thus depends on the engineering experience, but the FHM is a good tool to control it.

5 Conclusion

The dynamic contact is treated numerically. The contact treatment is performed with an augmented Lagrangian formulation and the dynamic process is treated via an efficient method named a generalized-α method. This combinations gave a good results. The coupling of the two aspects “optimization” and “reliability” in only one formulation through traditional model RBDO, allows a good improvement for the results of the deterministic optimization.

We consider the hybrid formulation of the RBDO. This method includes in its architecture the deterministic and random parameters in only one hybrid space. This space leads on the good control of all parameters of the problem.

The time-variant reliability-based optimization is very interesting to make design for an effective cost, durability and lifetime management of engineering system, it is for that we proposed the Dynamic Hybrid Method (DHM). These advantages motivate us, in order to improve this method, to search the failure critical region about eigen-frequency, it is the Frequencies Hybrid Method (FHM).

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