MHD Flow and Heat Transfer over a Stretching Sheet

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Abstract

An analysis is made for the steady two-dimensional, laminar boundary layer flow of a viscous, incompressible, electrically conducting fluid near a stagnation point of stretching sheet in the presence of a magnetic field. It is assumed that the sheet is stretched in its own plane with velocity and temperature proportional to the distance from the stagnation point. The governing boundary layer equations are transformed to ordinary differential equations by taking suitable similarity variables. The solutions of momentum and energy equations have been obtained independently by a perturbation technique for a small magnetic parameter. The effects of the various parameters such as velocity parameter, Hartmann number, Prandtl number and Eckert number for velocity and temperature distributions have been discussed in detail with graphical representation.

Keywords: Boundary layer, MHD, Heat transfer, Stretching sheet

1. Introduction

Flow and heat transfer of an incompressible viscous fluid over a stretching sheet has wide important applications in several manufacturing process from industry such as the extrusion of polymers, the cooling of metallic plates, the aerodynamic extrusion of plastic sheets, etc. The study of heat transfer and flow field is necessary for determining the quality of the final products of such processes. Crane (1970) studied the flow over a linearly stretching sheet in an ambient fluid and gave a similarity solution in closed analytical form for the steady two-dimensional problem. Gupta and Gupta (1977), Carragher and Crane (1982), Dutta et al. (1985), Chiam (1994), Magyari and Keller (1999, 2000) and more recently Mahapatra and Gupta (2002,

2004) studied the heat transfer in the steady two-dimensional stagnation-point flow of a viscous, and incompressible Newtonian and viscoelastic fluids over a horizontal stretching sheet considering the case of constant surface temperature.

Based on the above mentioned investigations and applications, this paper is concerned with a steady, two-dimensional stagnation flow of an electrically conducting fluid, over a stretching surface in the presence of magnetic field. Numerical solution obtained for the governing momentum and energy equations using perturbation technique.

2. Formulation of the problem

Consider the two-dimensional steady flow of a viscous incompressible electrically conducting fluid near a stagnation point over a flat sheet such that the sheet is stretched in its own plane with velocity proportional to the distance from the stagnation point in the presence of an externally applied normal magnetic field of constant strength H_0. The stretching surface has uniform temperature T_w and a linear velocity u_w while the velocity of the flow external to the boundary layer is $u_e(x)$. The system of boundary layer equations (which model the figure 1) are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\sigma_e \mu_e^2 H_0^2 (u_e - u)}{\rho} \tag{2}$$

$$\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = \kappa\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y} \right)^2 + \sigma_e \mu_e^2 H_0^2 u^2 \tag{3}$$

where υ is the coefficient of kinematic viscosity , ρ the density, σ_e the electrical conductivity, μ_e the magnetic permeability, C_p the specific heat at constant pressure, κ the thermal conductivity of the fluid under consideration and μ the coefficient of viscosity. The other symbols have their usual meanings.

The boundary conditions are:

$$\begin{aligned} y = 0 : \ u = u_w = cx, \ v = 0; \ T = T_w \\ y = \infty : \ u = u_e(x) = ax; \ T = T_\infty \end{aligned} \tag{4}$$

where c is a proportionality constant of the velocity of stretching sheet, a is a constant proportional to the free stream velocity far away from the sheet and T_∞ is the temperature of the ambient fluid.

3. Analysis

The continuity equation (1) is identically satisfied by stream function $\psi(x, y)$, defined as

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} \tag{5}$$

For the solution of the momentum and energy equation (2) and (3), the following dimensionless variables are defined:

$$\psi(x, y) = x\sqrt{c\upsilon} f(\eta) \tag{6}$$

$$\eta = y\sqrt{\frac{c}{\upsilon}} \tag{7}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

Substituting (5) – (8) into Eqs. (2) and (3) we obtain

$$f''' + ff'' - f'^2 - H_a^2 f' + H_a^2 \lambda + \lambda^2 = 0 \tag{9}$$

$$\frac{1}{\Pr} \theta'' + f\theta' + Ecf''^2 + H_a^2 Ecf'^2 = 0 \tag{10}$$

The corresponding boundary conditions are:

$$\begin{aligned} \eta = 0: \quad & f = 0, \quad f' = 1; \quad \theta = 1 \\ \eta = \infty: \quad & f' = \lambda; \quad \theta = 0 \end{aligned} \tag{11}$$

where, $\lambda = \dfrac{a}{c}$ is the velocity parameter, $H_a = \mu_e H_0 \sqrt{\dfrac{\sigma_e}{\rho c}}$ is the Hartmann number,

$\Pr = \dfrac{\mu C_p}{\kappa}$ is the Prandtl number and $Ec = \dfrac{u_w^2}{C_p(T_w - T_\infty)}$ is the Eckert number.

For numerical solution of the equations (9) and (10), we apply a perturbation technique as:

$$f(\eta) = \sum_{i=0}^{\infty} \left(H_a^2\right)^i f_i(\eta) \tag{12}$$

$$\theta(\eta) = \sum_{j=0}^{\infty} \left(H_a^2\right)^j \theta_j(\eta) \tag{13}$$

substituting equations (12) and (13) and its derivatives in equations (9) and (10) and then equating the coefficients of like powers of H_a^2, we get the following set of equations:

$$f_0''' + f_0 f_0'' - f_0'^2 + \lambda^2 = 0 \tag{14}$$

$$\theta_0'' + \Pr f_0 \theta_0' = -\Pr Ecf_0''^2 \tag{15}$$

$$f_1''' + f_0 f_1'' - 2f_0' f_1' + f_0'' f_1 = f_0' - \lambda \qquad (16)$$

$$\theta_1'' + \Pr f_0 \theta_1' = -\Pr f_1 \theta_0' - \Pr Ec\left(2f_0'' f_1'' + f_0'^2\right) \qquad (17)$$

$$f_2''' + f_0 f_2'' - 2f_0' f_2' + f_0'' f_2 = -f_1 f_1'' + f_1'^2 + f_1' \qquad (18)$$

$$\theta_2'' + \Pr f_0 \theta_2' = -\Pr f_1 \theta_1' - \Pr f_2 \theta_0' - \Pr Ec\left(2f_0'' f_2'' + f_1''^2 + 2f_0' f_1'\right) \qquad (19)$$

with the boundary conditions:

$$\eta = 0: \quad f_i = 0, \quad f_0' = 1, \quad f_j' = 0; \quad \theta_0 = 1, \quad \theta_j = 0;$$
$$\eta = \infty: \quad f_0' = \lambda, \quad f_j' = 0; \quad \theta_i = 0; \qquad\qquad i \geq 0, \quad j > 0 \qquad (20)$$

The equation (14) is that obtained by Mahapatra and Gupta (2002) for the non-magnetic case and the remaining equations are ordinary linear differential equations and have been solved numerically by Runge-Kutta method of fourth order.

4. Results and discussion

The velocity profiles for various values of λ and the Hartmann number H_a are plotted against η in Fig. 2. It is observed from the figure that the thickness of the velocity boundary layer decreases with the increase in λ. Also, it can be seen that the flow has an inverted boundary layer structure when $\lambda < 1$ and velocity distribution decreases with increase in H_a. When $\lambda > 1$ the velocity distribution increases with increasing values of λ and H_a.

The Figure 3 shows that for a fixed value of λ and Ec i.e. $\lambda = 0.1$ and $Ec = 0.0$ the temperature distribution decreases with the increasing values of both, Pr and H_a. Further for fixed values of λ, Pr and Ec, the temperature distribution increase with the increasing values of H_a. Fig. 4 shows the temperature distribution with η for various values of λ and H_a with $\Pr = 0.05$ and $Ec = 0.0$. It is observed from the figure that the temperature distribution decreases with the increase in λ, while it increases with the increase in H_a for $\lambda < 1$, and decreases with the increase in H_a for $\lambda > 1$. Fig. 5 shows that for a fixed value of λ and Pr i.e. $\lambda = 0.1$ and $\Pr = 0.05$ the temperature distribution decreases with the increasing values of Ec while it increases with the increase in H_a. Fig. 6 shows the variation of temperature distribution with η for various values of Ec and H_a when $\lambda = 3.0$ and $\Pr = 0.05$. It can be seen that the temperature distribution increases with decreases in Ec and H_a.

References

[1] B.K. Dutta, P. Roy and A.S.Gupta, Temperature field in flow over a stretching surface with uniform heat flux, Int. Comm. Heat Mass Transfer, 12 (1985), 89-94.

[2] E. Magyari and B. Keller, Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface, J Phys D Appl Phys., 32 (1999), 577-585.

[3] E. Magyari and B. Keller, Exact solutions for selfsimilar boundary-layer flows induced by permeable stretching surfaces, Eur J Mech B-Fluids, 19 (2000), 109-122.

[4] L.J. Crane, Flow past a stretching plate, Zeit Angew Math Phys., 21 (1970), 645-647.

[5] P. Carragher and L.J. Crane, Heat transfer on a continuous stretching sheet, Zeit Angew Math Mech., 62 (1982), 564-565.

[6] P.S. Gupta and A.S. Gupta, Heat and mass transfer on stretching sheet with suction or blowing, Can. J. Chem. Eng., 55 (1977), 744-746.

[7] T.C. Chiam, Stagnation-point flow towards a stretching plate, J Phys Soc Japan, 63 (1994), 2443-2444.

[8] T.R. Mahapatra and A.S. Gupta, Heat transfer in stagnation-point flow towards a stretching sheet, Heat and Mass Transfer, 38 (2002), 517-521.

[9] T.R. Mahapatra and A.S. Gupta, Stagnation-point flow of a viscoelastic fluid towards a stretching surface, Int. J Non-Linear Mech 39 (2004), 811-820.
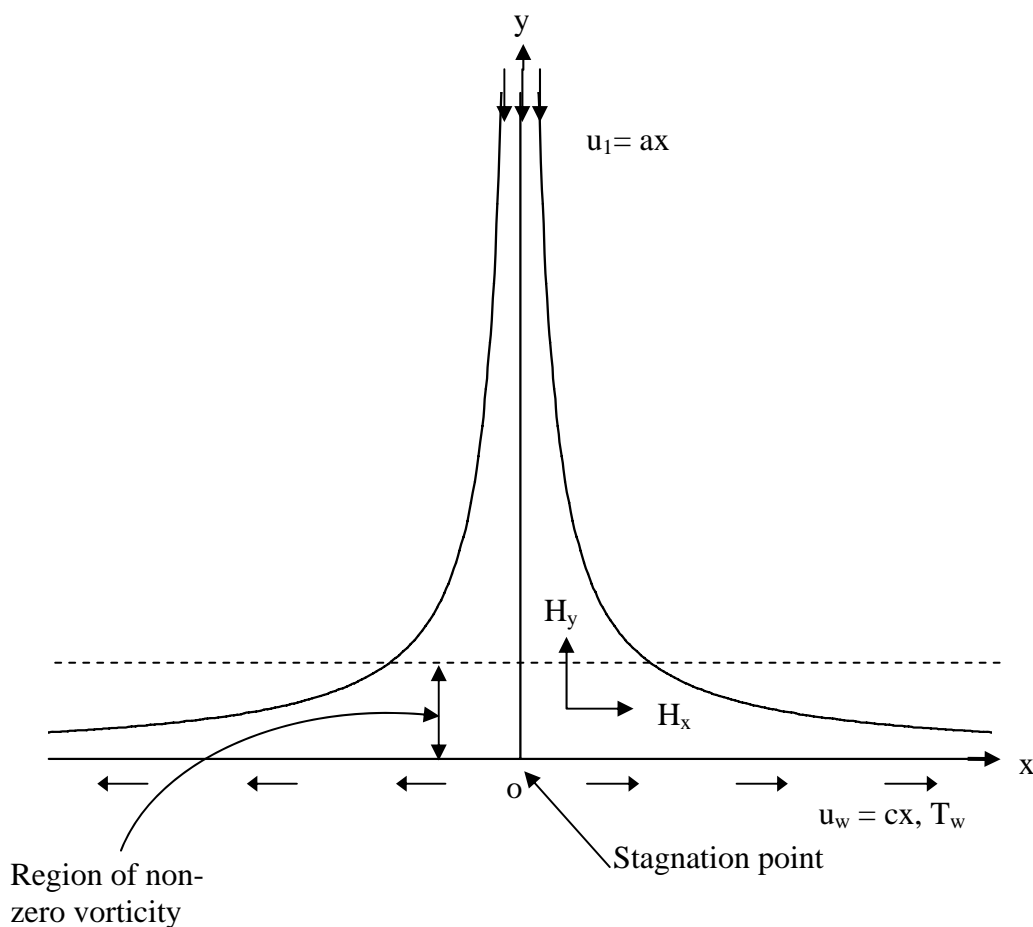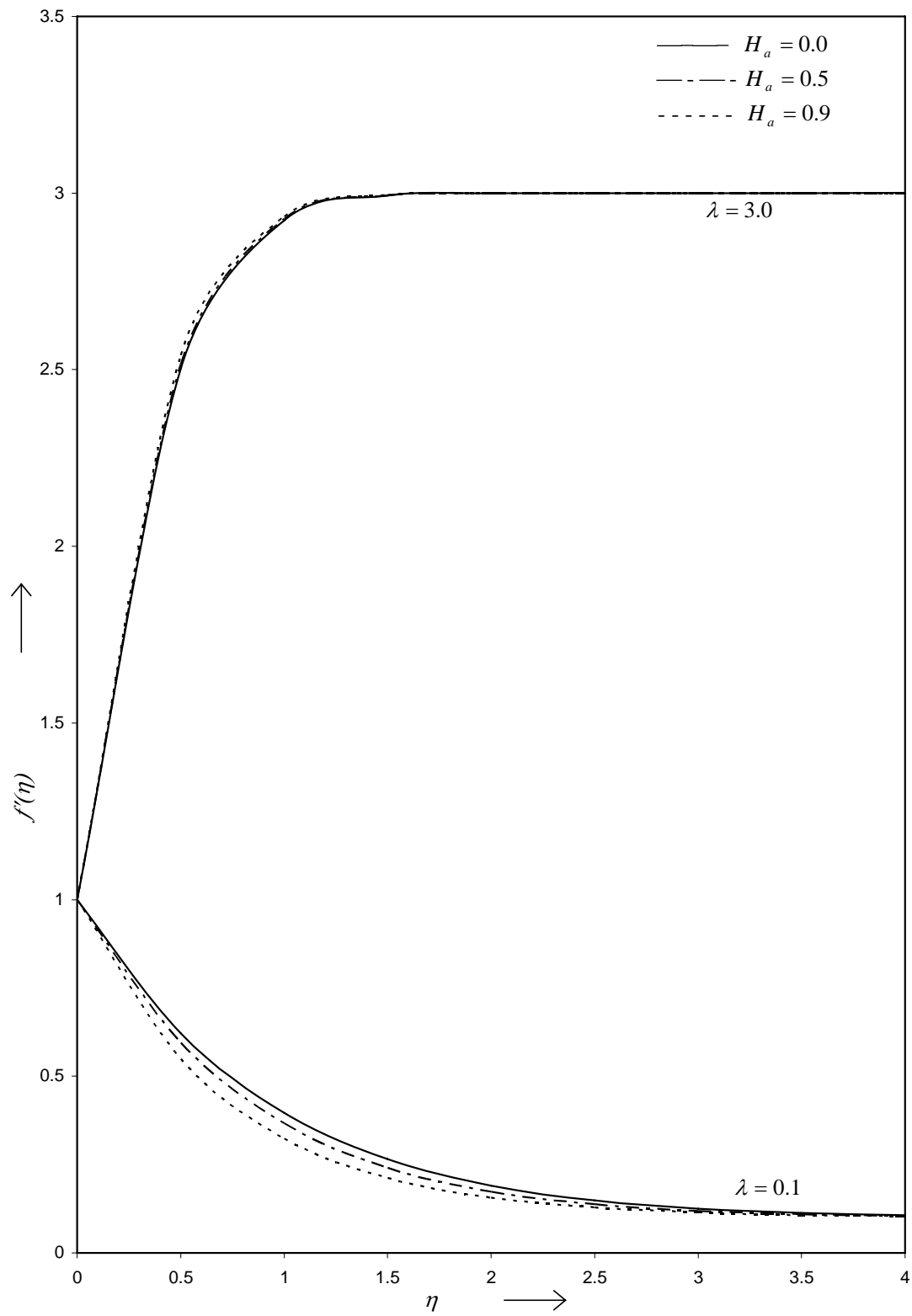

Fig.1. A sketch of the physical problem.

Fig. 2. Velocity distribution against η for different values of λ and H_a.
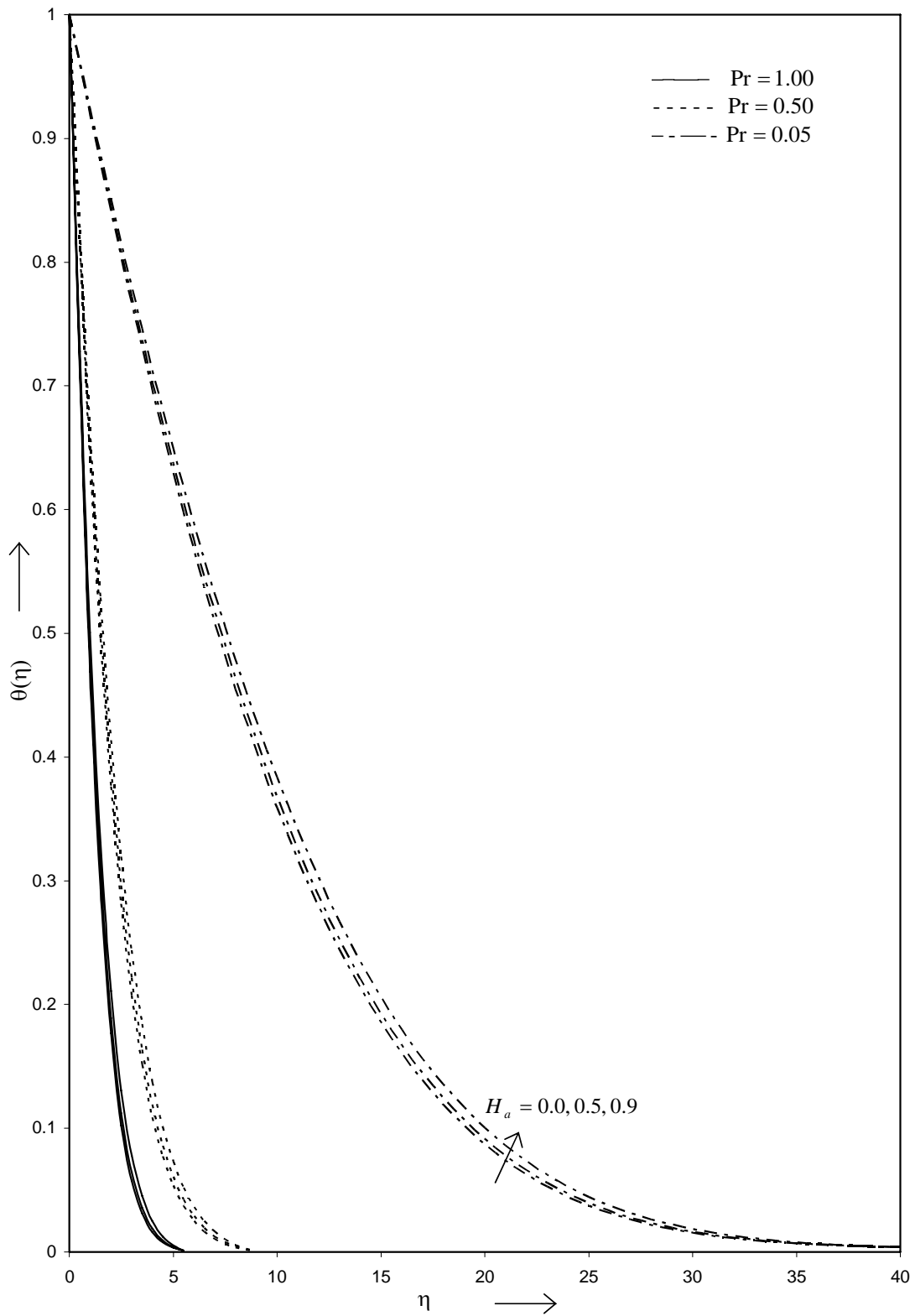
Fig. 3. Temperature distribution against η for various values of H_a and Pr with $\lambda = 0.1$ and $Ec = 0.0$.
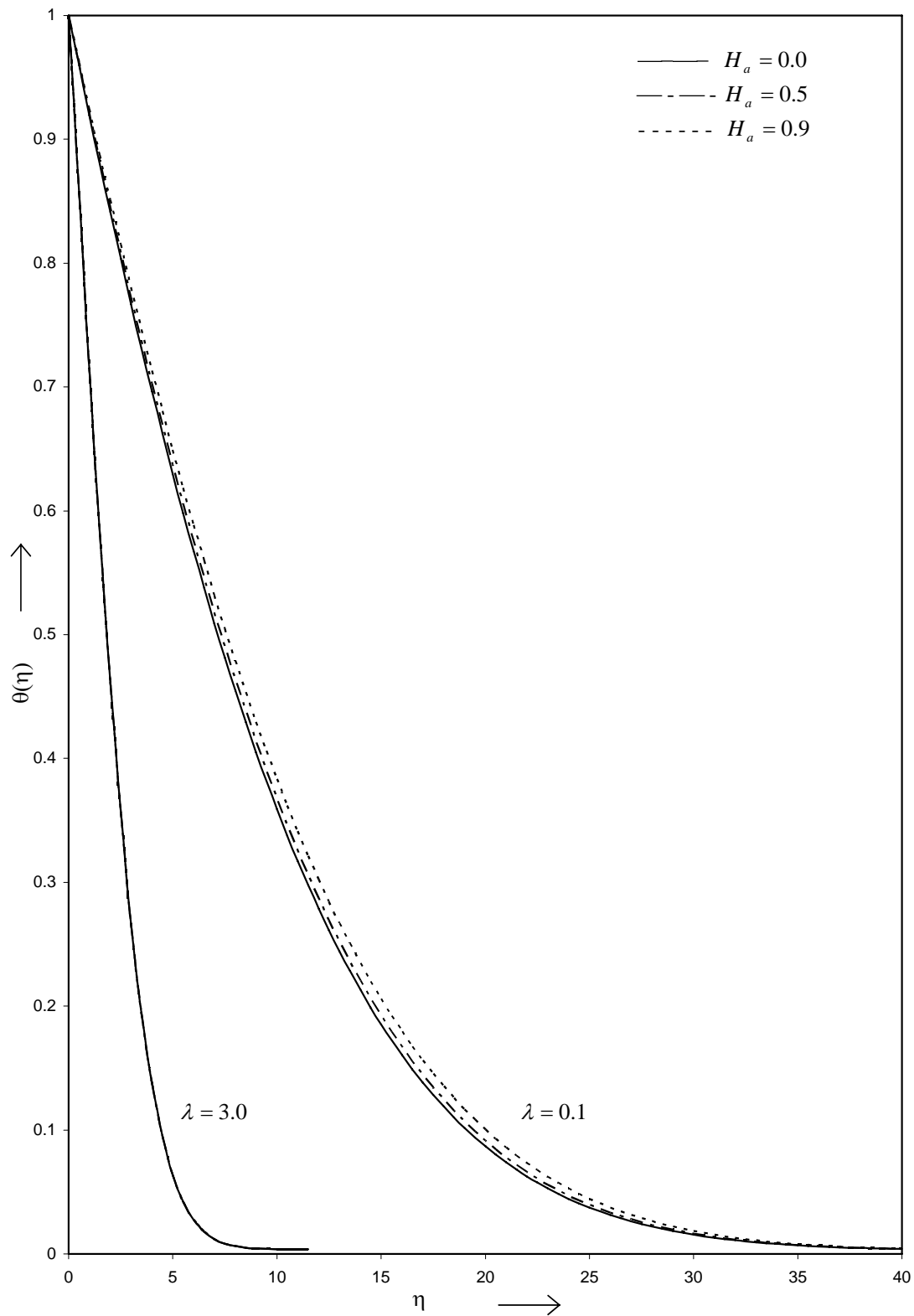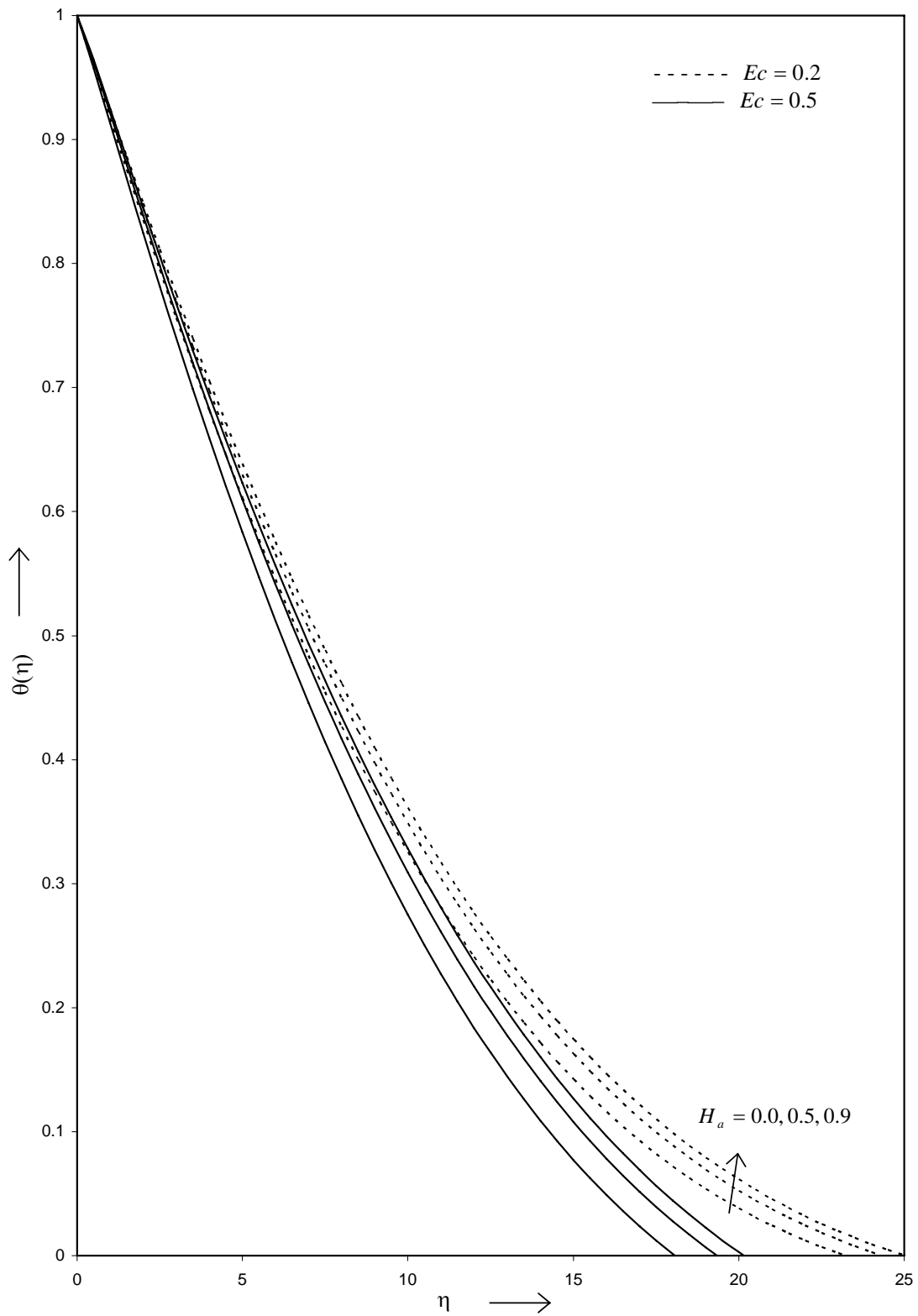
Fig. 4. Temperature distribution against η for various values of H_a and λ with $\Pr = 0.05$ and $Ec = 0.0$.

Fig. 5. Temperature distribution against η for various values of H_a and Ec with $\lambda = 0.1$ and $Pr = 0.05$.
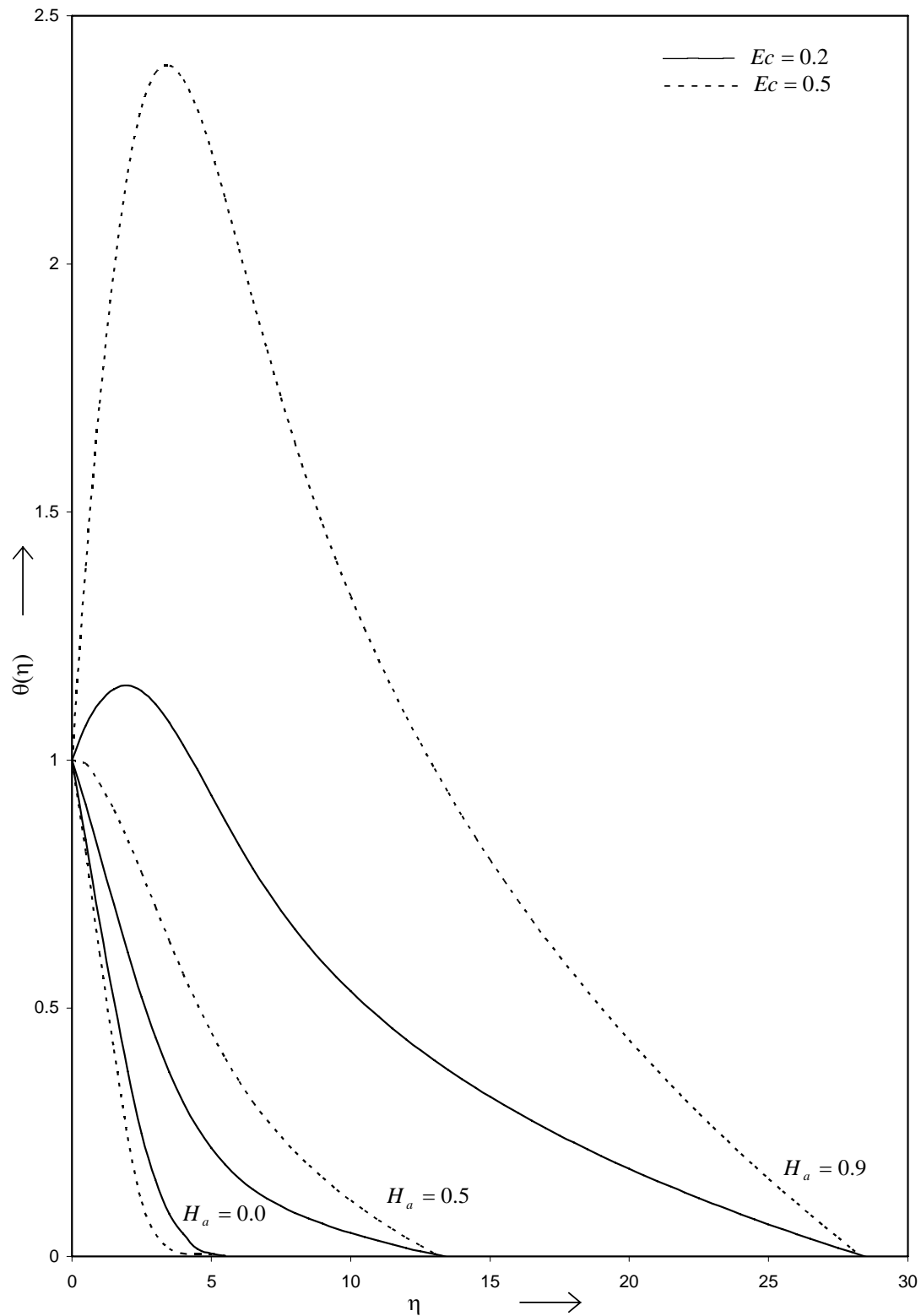
Fig. 6. Temperature distribution against η for various values of H_a and Ec with $\lambda = 3.0$ and $Pr = 0.05$.