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Fuzzy Chance-Constrained Programming with Linear Combination of Possibility Measure and Necessity Measure

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Abstract

Based on the possibility measure and necessity measure, m_λ-measure is presented and some mathematical properties of m_λ-measure are also obtained, including continuity, monotonicity, subadditivity, and so on. Critical values of fuzzy variable with respect to m_λ-measure are introduced and are employed to construct the fuzzy chance-constrained programming models. To solve the models, genetic algorithm based on fuzzy simulation is designed. Finally, two numerical examples are given to show applications of the models and algorithm.

Keywords: Possibility Measure; m_λ-Measure; Chance-Constrained Programming; Fuzzy Simulation; Genetic Algorithm

1 Introduction

Fuzzy programming is an important branch of the operations research. Up to now, a lot of models and algorithms have been presented to solve the optimization problem in fuzzy environment. For instance, Jamison and Lodwick[3] investigated fuzzy linear programming using a penalty method and presented an algorithm for finding the optimal solution. Stanciulescu et.al[11] investigated multi-objective fuzzy linear programming problems with fuzzy decision variables. Inuiguchi et.al[2] investigated interval and fuzzy linear programming and obtained some duality theorems. Liu[4, 5, 6, 8] investigated a class of fuzzy programming models and presented the frameworks of fuzzy chance-constrained programming and dependent-chance programming. Also, Liu and Liu[7] defined the expected value operator for fuzzy variable based on the credibility measure and presented the expected value model for fuzzy programming. Sinha[10] considered a multi-level linear programming problem and applied fuzzy mathematical programming approach to obtain the solution of the system. For the duality theory in fuzzy mathematical programming, interested readers may refer to Wu[12], Zhang et.al[14], and so on.

In this paper, we shall consider fuzzy mathematical programming from other point of view. This paper is organized as follows. On the basis of possibility measure and necessity measure, we firstly present m_λ-measure in Section 2, which is defined as the linear combination of possibility measure and necessity measure to balance optimism and pessimism. And the parameter λ is predetermined by the decision-maker according to his character. In Section 3, mathematical properties of m_λ-measure are explored, including continuity, monotonicity, subadditivity, boundary, and so on. In addition, in order to construct fuzzy programming model, we introduce the notions of the critical values for fuzzy variable with respect to m_λ-measure and then the continuity and monotonicity of critical values are investigated. In Section 4, after constructing chance-constrained programming(CCP) models, we explore some crisp equivalents of objective function and constraints. Sensitivity analysis with respect to parameter is also discussed. Additionally, in order to solve the models, hybrid genetic algorithm is designed to get the approximate optimal solution. At last, two examples are provided to show applications of the models and algorithm.

2 Possibility Measure, Necessity Measure and m_λ-Measure

Fuzzy set theory was initialized by Zadeh[13] in 1965. Up to now, it has been further developed by many researchers such as Nahmias[9], Dubois and Prade[1]. In fuzzy set theory, possibility measure and necessity measure are employed to describe the chance of fuzzy event. Also, they are basic tools to construct mathematical models for fuzzy programming. Let ξ be a fuzzy variable with membership function $\mu_\xi(x)$, and B an arbitrary subset of $\Re$. Then the possibility measure of fuzzy event $\{\xi \in B\}$ is defined as

$$\text{Pos}\{\xi \in B\} = \sup_{x \in B} \mu_\xi(x).$$

The necessity of this fuzzy event is defined as the impossibility of the opposite event. That is,

$$\mathrm{Nec}\{\xi \in B\} = 1 - \mathrm{Pos}\{\xi \in B^c\} = 1 - \sup_{x \in B^c} \mu_\xi(x).$$

Also, we have the relation $\mathrm{Pos}\{\xi \in B\} \geq \mathrm{Nec}\{\xi \in B\}$.

For the possibility measure, it is necessary to point out that a fuzzy event may fail even though its possibility achieves 1. For instance, Let $\xi = (1, 2, 3)$ be a triangular fuzzy number. It is easy to see $\mathrm{Pos}\{\xi < 2\} = 1$. But if the realization value of ξ lies in the interval $[2, 3]$, then it is clear that the fuzzy event does not hold. If the necessity achieves 1, the fuzzy event must hold. For instance, we have $\mathrm{Nec}\{\xi < r\} = 1$ if and only if $r \geq 3$. It is easy to see that all the realization values of ξ satisfy the fuzzy inequality. In other words, the fuzzy event $\{\xi < r\}$ must hold.

In decision-making systems, possibility measure and necessity measure are always employed in literature. Generally speaking, the possibility measure is much suitable for the optimistic decision-maker. If the decision-maker is pessimistic, he may use the necessity measure as a tool to make decision. For instance, a decision-maker wants to seek the best decision to maximize the chance of fuzzy event $\{f(\boldsymbol{x}, \boldsymbol{\xi}) \in B\}$, where $\boldsymbol{x}$ is the decision variables vector and $\boldsymbol{\xi}$ is the fuzzy parameters vector. When the possibility measure is employed as a chance measure, a decision $\boldsymbol{x}^*$ will be recognized as the best decision if it satisfies $\mathrm{Pos}\{f(\boldsymbol{x}^*, \boldsymbol{\xi}) \in B\} = 1$. In fact, a fuzzy event may fail even though its possibility achieves 1. This fact implies that for some realization value $\bar{\boldsymbol{\xi}}$ with $\mu_{\boldsymbol{\xi}}(\bar{\boldsymbol{\xi}}) > 0$, the event $\{f(\boldsymbol{x}^*, \bar{\boldsymbol{\xi}}) \in B\}$ may not appear. So if the decision-maker is optimistic and does not care about the potential risk, he may select the possibility measure for decision-making. For the pessimistic decision-maker, he may choose the necessity measure as a chance measure. In fact, the decision $\boldsymbol{x}^*$ is not necessarily the best decision for the necessity measure since the corresponding objective value is less than or equal to 1. Thus the decision-maker may select a better solution $\bar{\boldsymbol{x}}^*$ as the optimal decision. Actually, if the necessity measure of fuzzy event $\{f(\bar{\boldsymbol{x}}^*, \boldsymbol{\xi}) \in B\}$ achieves 1, then for any realization value $\bar{\boldsymbol{\xi}}$ of $\boldsymbol{\xi}$ with $\mu_{\boldsymbol{\xi}}(\bar{\boldsymbol{\xi}}) > 0$, the event $\{f(\bar{\boldsymbol{x}}^*, \bar{\boldsymbol{\xi}}) \in B\}$ must hold.

In practice, most decision-makers are neither absolutely optimistic nor absolutely pessimistic. If the decision-maker is an eclectic, he will be suggested to use a linear combination of possibility measure and necessity measure to deal with the problem. By using this way, we may produce a balance between the optimism and the pessimism. For the convenience of expression, we use the notation "m_λ" for short, that is,

$$m_\lambda\{\cdot\} = \lambda \mathrm{Pos}\{\cdot\} + (1 - \lambda)\mathrm{Nec}\{\cdot\},$$

where the parameter $\lambda \in [0, 1]$ is predetermined by the decision-maker according to his character. Generally speaking, the larger the parameter λ is, the more optimistic the decision-maker is. We may see that if the parameter $\lambda = 1$, then m_λ-measure degenerates to the possibility measure. If the parameter $\lambda = 0$, then m_λ-measure degenerates to the necessity measure. If the parameter $\lambda = 0.5$, then m_λ-measure degenerates to the credibility measure presented by Liu and Liu[7].

3 Mathematical Properties of m_λ-Measure

Theorem 3.1 *Let ξ be a fuzzy variable. Then we have*
(a) $\mathrm{Nec}\{\xi \in A\} \le m_\lambda\{\xi \in A\} \le \mathrm{Pos}\{\xi \in A\}$ *for any set* $A \subset \Re$*;*
(b) $0 \le m_\lambda\{\xi \in A\} \le 1$ *for any set* $A \subset \Re$*;*
(c) $m_\lambda\{\xi \in A\} \le m_\lambda\{\xi \in B\}$ *whenever* $A \subset B$*;*
(d) if $\mathrm{Pos}\{\xi \in A\} < 1$ *or* $m_\lambda\{\xi \in A\} \le \lambda$*, then* $m_\lambda\{\xi \in A\} = \lambda \mathrm{Pos}\{\xi \in A\}$*;*
(e) For any $A \subset \Re$ *and* $\lambda \in [0, 1]$*, we have* $m_\lambda\{\xi \in A\} + m_{1-\lambda}\{\xi \in A^c\} = 1$*;*
(f) if $\lambda \ge 0.5$*,* m_λ*-measure is subadditive, that is,* $m_\lambda\{\xi \in A \cup B\} \le m_\lambda\{\xi \in A\} + m_\lambda\{\xi \in B\}$ *for any* $A, B \subset \Re$*;*
(g) if $\lambda_1 < \lambda_2$*, then* $m_{\lambda_1}\{\xi \in A\} \le m_{\lambda_2}\{\xi \in A\}$ *for any* $A \subset \Re$*.*

Proof. The parts (a), (b), (c), (d) and (e) may be proven easily by the definition of m_λ-measure. We first prove the part (f) in the following. The proof is divided into the following four cases:

Case 1: $\mathrm{Pos}\{\xi \in A\} = 1$ and $\mathrm{Pos}\{\xi \in B\} = 1$. For this case, we have $m_\lambda\{\xi \in A\} \ge \lambda \ge 0.5$ and $m_\lambda\{\xi \in B\} \ge \lambda \ge 0.5$. Therefore,

$$m_\lambda\{\xi \in A \cup B\} \le 1 \le \lambda + \lambda \le m_\lambda\{\xi \in A\} + m_\lambda\{\xi \in B\}.$$

Case 2: $\mathrm{Pos}\{\xi \in A\} < 1$ and $\mathrm{Pos}\{\xi \in B\} < 1$. For this case, it is clear that $\mathrm{Pos}\{\xi \in A \cup B\} = \mathrm{Pos}\{\xi \in A\} \vee \mathrm{Pos}\{\xi \in B\} < 1$. Thus, by using the part (d), we have

$$\begin{aligned} m_\lambda\{\xi \in A\} + m_\lambda\{\xi \in B\} &= \lambda \mathrm{Pos}\{\xi \in A\} + \lambda \mathrm{Pos}\{\xi \in B\} \\ &\ge \lambda(\mathrm{Pos}\{\xi \in A\} \vee \mathrm{Pos}\{\xi \in B\}) \\ &= \lambda \mathrm{Pos}\{\xi \in A \cup B\} \\ &= m_\lambda\{\xi \in A \cup B\}. \end{aligned}$$

Case 3: $\mathrm{Pos}\{\xi \in A\} = 1$ and $\mathrm{Pos}\{\xi \in B\} < 1$. For this case, it is easy to see that $\mathrm{Pos}\{\xi \in A \cup B\} = \mathrm{Pos}\{\xi \in A\} \vee \mathrm{Pos}\{\xi \in B\} = 1$. Furthermore, we have the following inequality,

$$\begin{aligned} \mathrm{Pos}\{\xi \in A^c\} &= \mathrm{Pos}\{\xi \in A^c \cap B\} \vee \mathrm{Pos}\{\xi \in A^c \cap B^c\} \\ &\le \mathrm{Pos}\{\xi \in A^c \cap B\} + \mathrm{Pos}\{\xi \in A^c \cap B^c\} \\ &\le \mathrm{Pos}\{\xi \in B\} + \mathrm{Pos}\{\xi \in A^c \cap B^c\}. \end{aligned} \tag{1}$$

Using inequality (1) and the part (d), we may obtain

$$\begin{aligned}
m_\lambda\{\xi\in A\}+m_\lambda\{\xi\in B\} &= \lambda+(1-\lambda)\text{Nec}\{\xi\in A\}+\lambda\text{Pos}\{\xi\in B\}\\
&= \lambda+(1-\lambda)(1-\text{Pos}\{\xi\in A^c\})+\lambda\text{Pos}\{\xi\in B\}\\
&= 1-(1-\lambda)\text{Pos}\{\xi\in A^c\}+\lambda\text{Pos}\{\xi\in B\}\\
&\geq 1-(1-\lambda)(\text{Pos}\{\xi\in B\}+\text{Pos}\{\xi\in A^c\cap B^c\})+\lambda\text{Pos}\{\xi\in B\}\\
&= 1-(1-\lambda)\text{Pos}\{\xi\in A^c\cap B^c\}+(2\lambda-1)\text{Pos}\{\xi\in B\}\\
&\geq 1-(1-\lambda)\text{Pos}\{\xi\in A^c\cap B^c\}\\
&= \lambda+(1-\lambda)(1-\text{Pos}\{\xi\in A^c\cap B^c\})\\
&= m_\lambda\{\xi\in A\cup B\}.
\end{aligned}$$

Case 4: $\text{Pos}\{\xi\in A\}<1$ and $\text{Pos}\{\xi\in B\}=1$. This case may be proven by a similar way of Case 3.

Next, we prove the part (g). If $\text{Pos}\{\xi\in A\}<1$, it follows from the part (d) that $m_\lambda\{\xi\in A\}=\lambda\text{Pos}\{\xi\in A\}$. For this case, we have

$$m_{\lambda_2}\{\xi\in A\}-m_{\lambda_1}\{\xi\in A\}=(\lambda_2-\lambda_1)\text{Pos}\{\xi\in A\}\geq 0,$$

which implies $m_{\lambda_2}\{\xi\in A\}\geq m_{\lambda_1}\{\xi\in A\}$. On the other hand, if $\text{Pos}\{\xi\in A\}=1$, then we have

$$\begin{aligned}
m_{\lambda_2}\{\xi\in A\}-m_{\lambda_1}\{\xi\in A\} &= (\lambda_2-\lambda_1)\text{Pos}\{\xi\in A\}+(\lambda_1-\lambda_2)\text{Nec}\{\xi\in A\}\\
&= \lambda_2-\lambda_1+(\lambda_1-\lambda_2)\text{Nec}\{\xi\in A\}\\
&\geq 0.
\end{aligned}$$

That is, $m_{\lambda_2}\{\xi\in A\}\geq m_{\lambda_1}\{\xi\in A\}$. The proof is thus completed.

Theorem 3.2 *Let ξ be a fuzzy variable, and $A, A_1, A_2, \cdots\subset\Re$. Then we have*

$$\lim_{i\to\infty} m_\lambda\{\xi\in A_i\}=m_\lambda\{\xi\in A\}$$

if one of the following conditions is satisfied

$(a) m_\lambda\{\xi\in A\}\leq\lambda$ *and* $A_i\uparrow A$; $(b) \lim_{i\to\infty} m_\lambda\{\xi\in A_i\}<\lambda$ *and* $A_i\uparrow A$;
$(c) m_\lambda\{\xi\in A\}\geq\lambda$ *and* $A_i\downarrow A$; $(d) \lim_{i\to\infty} m_\lambda\{\xi\in A_i\}>\lambda$ *and* $A_i\downarrow A$.

Proof. (a) If $m_\lambda\{\xi\in A\}\leq\lambda$ and $A_i\uparrow A$, it follows from the monotonicity of m_λ-measure that $m_\lambda\{\xi\in A_i\}\leq\lambda$ for all i. Thus $m_\lambda\{\xi\in A\}=\lambda\text{Pos}\{\xi\in A\}$ and $m_\lambda\{\xi\in A_i\}=\lambda\text{Pos}\{\xi\in A_i\}$. Therefore, we have

$$\begin{aligned}
m_\lambda\{\xi\in A\}=\lambda\text{Pos}\left\{\xi\in\bigcup_{i=1}^{\infty}A_i\right\}&=\lambda\bigvee_{i=1}^{\infty}\text{Pos}\{\xi\in A_i\}\\
=\lambda\lim_{i\to\infty}\text{Pos}\{\xi\in A_i\}&=\lim_{i\to\infty}m_\lambda\{\xi\in A_i\}.
\end{aligned}$$

(b) Since $\lim_{i\to\infty} m_\lambda\{\xi \in A_i\} < \lambda$ and $A_i \uparrow A$, we have $m_\lambda\{\xi \in A_i\} < \lambda$ for all i. Thus $\text{Pos}\{\xi \in A_i\} = \frac{1}{\lambda} m_\lambda\{\xi \in A_i\}$ for all i. Therefore,

$$\text{Pos}\{\xi \in A\} = \text{Pos}\left\{\xi \in \bigcup_{i=1}^{\infty} A_i\right\} = \lim_{i\to\infty} \text{Pos}\{\xi \in A_i\} = \lim_{i\to\infty} \frac{1}{\lambda} m_\lambda\{\xi \in A_i\} < 1,$$

which implies that $m_\lambda\{\xi \in A\} < \lambda$. It follows from the part (a) that $\lim_{i\to\infty} m_\lambda\{\xi \in A_i\} = m_\lambda\{\xi \in A\}$.

(c) Since $m_\lambda\{\xi \in A\} \ge \lambda$ and $A_i \downarrow A$, it is obvious that $\text{Pos}\{\xi \in A\} = 1$ and $\text{Pos}\{\xi \in A_i\} = 1$ for all i. Note that $\text{Pos}\{\xi \in A^c\} = \lim_{i\to\infty} \text{Pos}\{\xi \in A_i^c\}$ since $A_i^c \uparrow A^c$. Thus, we may obtain

$$\begin{aligned} m_\lambda\{\xi \in A\} &= \lambda + (1-\lambda)\text{Nec}\{\xi \in A\} \\ &= \lambda + (1-\lambda)(1 - \text{Pos}\{\xi \in A^c\}) \\ &= \lambda + (1-\lambda)(1 - \lim_{i\to\infty} \text{Pos}\{\xi \in A_i^c\}) \\ &= \lim_{i\to\infty} [\lambda + (1-\lambda)(1 - \text{Pos}\{\xi \in A_i^c\})] \\ &= \lim_{i\to\infty} m_\lambda\{\xi \in A_i\}. \end{aligned}$$

(d) Since $\lim_{i\to\infty} m_\lambda\{\xi \in A_i\} > \lambda$ and $A_i \downarrow A$, it follows that $\text{Pos}\{\xi \in A_i\} = 1$ for each i. Then we may obtain

$$\begin{aligned} \text{Nec}\{\xi \in A\} &= 1 - \text{Pos}\{\xi \in A^c\} \\ &= 1 - \lim_{i\to\infty} \text{Pos}\{\xi \in A_i^c\} \\ &= \lim_{i\to\infty} \text{Nec}\{\xi \in A_i\} \\ &= \lim_{i\to\infty} \frac{m_\lambda\{\xi \in A_i\} - \lambda}{1-\lambda} \\ &= \frac{\lim_{i\to\infty} m_\lambda\{\xi \in A_i\} - \lambda}{1-\lambda} \\ &> 0. \end{aligned}$$

Thus we have $\text{Pos}\{\xi \in A\} = 1$, which implies that $m_\lambda\{\xi \in A\} \ge \lambda$. It follows from the part (c) that $m_\lambda\{\xi \in A_i\} \to m_\lambda\{\xi \in A\}$ as $i \to \infty$. The theorem is proved.

Theorem 3.3 *m_λ-measure is a uniformly continuous function with respect to λ.*

Proof. It suffices to prove that for any given $\varepsilon > 0$, there exists $\delta > 0$ such that

$$|m_{\lambda_1}\{\cdot\} - m_{\lambda_2}\{\cdot\}| < \varepsilon$$

for any $\lambda_1, \lambda_2 \in [0,1]$ with $|\lambda_1 - \lambda_2| < \delta$. In fact, since m_λ-measure is the linear combination of possibility measure and necessity measure, that is,

$$m_\lambda\{\cdot\} = \lambda\text{Pos}\{\cdot\} + (1-\lambda)\text{Nec}\{\cdot\},$$

then we have

$$\begin{aligned} |m_{\lambda_1}\{\cdot\} - m_{\lambda_2}\{\cdot\}| &= |(\lambda_1 - \lambda_2)\text{Pos}\{\cdot\} + (\lambda_2 - \lambda_1)\text{Nec}\{\cdot\}| \\ &\le |(\lambda_1 - \lambda_2)\text{Pos}\{\cdot\}| + |(\lambda_2 - \lambda_1)\text{Nec}\{\cdot\}| \\ &\le |\lambda_1 - \lambda_2| + |\lambda_1 - \lambda_2| \\ &= 2|\lambda_1 - \lambda_2|. \end{aligned}$$

Thus letting $\delta = \varepsilon/2$, we can ensure $|m_{\lambda_1}\{\cdot\} - m_{\lambda_2}\{\cdot\}| < \varepsilon$. The proof is completed.

Note 3.1 *We can see from the proof of Theorem 3.3 that if $|\lambda_1 - \lambda_2| < \varepsilon/2$, then for any fuzzy event $\{\cdot\}$, we have the inequality*

$$|m_{\lambda_1}\{\cdot\} - m_{\lambda_2}\{\cdot\}| < \varepsilon.$$

In the following, we present the definitions of critical values of fuzzy variable with respect to m_λ-measure.

Definition 3.1 *Let ξ be a fuzzy variable, and $\alpha \in (0,1]$. Then*

$$\xi_{\inf}(\lambda;\alpha) = \inf\{r|\ m_\lambda\{\xi \le r\} \ge \alpha\}$$

is called the $(\lambda;\alpha)$-pessimistic value of ξ. And

$$\xi_{\sup}(\lambda;\alpha) = \sup\{r|\ m_\lambda\{\xi \ge r\} \ge \alpha\}$$

is called the $(\lambda;\alpha)$-optimistic value of ξ.

Example 3.1 *Let $\xi = [a,b]$ be an interval fuzzy number. Then we have*

$$\xi_{\sup}(\lambda;\alpha) = \begin{cases} b, & \text{if } \alpha \le \lambda \\ a, & \text{if } \alpha > \lambda, \end{cases}$$

$$\xi_{\inf}(\lambda;\alpha) = \begin{cases} a, & \text{if } \alpha \le \lambda \\ b, & \text{if } \alpha > \lambda. \end{cases}$$

Example 3.2 *Let $\xi = (a,b,c,d)$ be a trapezoidal fuzzy number. Then we have*

$$\xi_{\sup}(\lambda;\alpha) = \begin{cases} \dfrac{\alpha(c-d)}{\lambda} + d, & \text{if } \alpha \le \lambda \\[2ex] \dfrac{(1-\alpha)(b-a)}{1-\lambda} + a, & \text{if } \alpha > \lambda, \end{cases}$$

$$\xi_{\inf}(\lambda;\alpha) = \begin{cases} \dfrac{\alpha(b-a)}{\lambda} + a, & \text{if } \alpha \le \lambda \\[2ex] \dfrac{(1-\alpha)(c-d)}{1-\lambda} + d, & \text{if } \alpha > \lambda. \end{cases}$$

Theorem 3.4 *Let $\xi_{\sup}(\lambda;\alpha)$ and $\xi_{\inf}(\lambda;\alpha)$ be the $(\lambda;\alpha)$-optimistic value and $(\lambda;\alpha)$-pessimistic value of fuzzy variable ξ, respectively. Then*
(a) $\xi_{\sup}(\lambda;\alpha)$ is a decreasing and left-continuous function of α for each fixed λ;
(b) $\xi_{\sup}(\lambda;\alpha)$ is an increasing and right-continuous function of λ for each fixed α;
(c) $\xi_{\inf}(\lambda;\alpha)$ is an increasing and left-continuous function of α for each fixed λ;
(d) $\xi_{\inf}(\lambda;\alpha)$ is a decreasing and right-continuous function of λ for each fixed α.

Proof. (a) For any $\alpha_1, \alpha_2 \in (0,1]$ with $\alpha_1 < \alpha_2$, it is easy to prove

$$\{r | m_\lambda\{\xi \geq r\} \geq \alpha_1\} \supset \{r | m_\lambda\{\xi \geq r\} \geq \alpha_2\},$$

which implies $\xi_{\sup}(\lambda;\alpha_1) \geq \xi_{\sup}(\lambda;\alpha_2)$. Thus $\xi_{\sup}(\lambda;\alpha)$ is a decreasing function of α for each fixed λ.

In the following, we prove the left-continuity of $\xi_{\sup}(\lambda;\alpha)$ with respect to α. Let $\{\alpha_i\}$ be an arbitrary sequence of numbers in $(0,1]$ such that $\alpha_i \uparrow \alpha$. We need to prove that $\lim_{i\to\infty} \xi_{\sup}(\lambda;\alpha_i) = \xi_{\sup}(\lambda;\alpha)$. In fact, if this relation does not hold, we have the inequality $\lim_{i\to\infty} \xi_{\sup}(\lambda;\alpha_i) > \xi_{\sup}(\lambda;\alpha)$. Let z^* be a number such that

$$\lim_{i\to\infty} \xi_{\sup}(\lambda;\alpha_i) > z^* > \xi_{\sup}(\lambda;\alpha).$$

Thus we have

$$m_\lambda\{\xi \geq z^*\} \geq \alpha_i, \ i = 1, 2, \cdots$$

Letting $i \to \infty$, we have $m_\lambda\{\xi \geq z^*\} \geq \alpha$, which implies that $\xi_{\sup}(\lambda;\alpha) \geq z^*$. A contradiction proves that $\lim_{i\to\infty} \xi_{\sup}(\lambda;\alpha_i) = \xi_{\sup}(\lambda;\alpha)$.

(b) For any $\lambda_1, \lambda_2 \in [0,1]$ with $\lambda_1 < \lambda_2$, by using Theorem 3.1(g), we can easily prove that

$$\{r | m_{\lambda_1}\{\xi \geq r\} \geq \alpha\} \subset \{r | m_{\lambda_2}\{\xi \geq r\} \geq \alpha\},$$

which implies $\xi_{\sup}(\lambda_1;\alpha) \leq \xi_{\sup}(\lambda_2;\alpha)$. Thus $\xi_{\sup}(\lambda;\alpha)$ is an increasing function of λ for each fixed α.

Next, we prove the right-continuity of $\xi_{\sup}(\lambda;\alpha)$ with respect to λ. Let $\{\lambda_i\}$ be an arbitrary sequence of numbers in $[0,1]$ such that $\lambda_i \downarrow \lambda$. We need to prove that $\lim_{i\to\infty} \xi_{\sup}(\lambda_i;\alpha) = \xi_{\sup}(\lambda;\alpha)$. In fact, if this equality does not hold, we must have the inequality $\lim_{i\to\infty} \xi_{\sup}(\lambda_i;\alpha) > \xi_{\sup}(\lambda;\alpha)$. Letting $z^* = (\lim_{i\to\infty} \xi_{\sup}(\lambda_i;\alpha) + \xi_{\sup}(\lambda;\alpha))/2$, we have

$$\lim_{i\to\infty} \xi_{\sup}(\lambda_i;\alpha) > z^* > \xi_{\sup}(\lambda;\alpha).$$

Thus we have

$$m_{\lambda_i}\{\xi \geq z^*\} \geq \alpha$$

for each i. By using Theorem 3.3, we have $m_\lambda\{\xi \geq z^*\} = \lim_{i\to\infty} m_{\lambda_i}\{\xi \geq z^*\} \geq \alpha$, which implies that $\xi_{\sup}(\lambda;\alpha) \geq z^*$. A contradiction proves that $\lim_{i\to\infty} \xi_{\sup}(\lambda_i;\alpha) = \xi_{\sup}(\lambda;\alpha)$.

The parts (c) and (d) may be proved similarly. The proof is thus completed.

Theorem 3.5 *Let ξ be a fuzzy variable. Then we have*
(a) if $c \geq 0$, then $(c\xi)_{\inf}(\lambda;\alpha) = c\xi_{\inf}(\lambda;\alpha)$ and $(c\xi)_{\sup}(\lambda;\alpha) = c\xi_{\sup}(\lambda;\alpha)$;
(b) if $c < 0$, then $(c\xi)_{\inf}(\lambda;\alpha) = c\xi_{\sup}(\lambda;\alpha)$ and $(c\xi)_{\sup}(\lambda;\alpha) = c\xi_{\inf}(\lambda;\alpha)$.

Proof. (a) If $c = 0$, then the part (a) is obvious. In the case of $c > 0$, we have

$$\begin{aligned}(c\xi)_{\inf}(\lambda;\alpha) &= \inf\{r \mid m_\lambda\{c\xi \leq r\} \geq \alpha\}\\ &= c\inf\{r/c \mid m_\lambda\{\xi \leq r/c\} \geq \alpha\}\\ &= c\xi_{\inf}(\lambda;\alpha).\end{aligned}$$

By using a similar way, we may prove that $(c\xi)_{\sup}(\lambda;\alpha) = c\xi_{\sup}(\lambda;\alpha)$.

In order to prove the part (b), it suffices to prove that $(-\xi)_{\inf}(\lambda;\alpha) = -\xi_{\sup}(\lambda;\alpha)$ and $(-\xi)_{\sup}(\lambda;\alpha) = -\xi_{\inf}(\lambda;\alpha)$. In fact, we have

$$\begin{aligned}(-\xi)_{\inf}(\lambda;\alpha) &= \inf\{r \mid m_\lambda\{-\xi \leq r\} \geq \alpha\}\\ &= -\sup\{-r \mid m_\lambda\{\xi \geq -r\} \geq \alpha\}\\ &= -\xi_{\sup}(\lambda;\alpha).\end{aligned}$$

Similarly, we may prove that $(-\xi)_{\sup}(\lambda;\alpha) = -\xi_{\inf}(\lambda;\alpha)$.

Thus the proof is completed.

Theorem 3.6 *Let ξ be a fuzzy variable with continuous membership function $\mu_\xi(x)$. Then if $\alpha \leq \lambda$, we have $\xi_{\sup}(\lambda;\alpha) \geq \xi_{\inf}(\lambda;\alpha)$; otherwise, we have $\xi_{\sup}(\lambda;\alpha) \leq \xi_{\inf}(\lambda;\alpha)$.*

Proof. Let r_0 be a number such that $\mu_\xi(r_0) = 1$. Since the membership function $\mu_\xi(x)$ of ξ is continuous, it is easy to prove $m_\lambda\{\xi \leq r_0\} = m_\lambda\{\xi \geq r_0\} = \lambda$. If $\alpha \leq \lambda$, we have $m_\lambda\{\xi \leq r_0\} = m_\lambda\{\xi \geq r_0\} \geq \alpha$, which implies that $\xi_{\inf}(\lambda;\alpha) \leq r_0 \leq \xi_{\sup}(\lambda;\alpha)$. On the other hand, if $\alpha > \lambda$, then we have $m_\lambda\{\xi \leq r_0\} = m_\lambda\{\xi \geq r_0\} < \alpha$, which implies that $\xi_{\sup}(\lambda;\alpha) \leq r_0 \leq \xi_{\inf}(\lambda;\alpha)$. The proof is completed.

4 Fuzzy CCP Models and Algorithm

In this section, we shall employ m_λ-measure to construct fuzzy chance-constrained programming models for fuzzy decision problems.

4.1 Chance-Constrained Programming Models

If the decision-maker wants to maximize the optimistic value of the return function subject to some chance constraints, he may construct the following chance-constrained programming model:

$$\begin{cases} \max\limits_{\boldsymbol{x}} \max\limits_{\bar{f}} \bar{f} \\ \text{subject to:} \\ \qquad m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \geq \bar{f} \right\} \geq \alpha \\ \qquad m_{\lambda_j} \left\{ g_j(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0 \right\} \geq \alpha_j, \quad j = 1, 2, \cdots, n \\ \qquad \boldsymbol{x} \in D. \end{cases} \tag{2}$$

where $\boldsymbol{x}$ is the decision variables vector, $\boldsymbol{\xi}$ is the fuzzy parameters vector, $\alpha, \alpha_j, j = 1, 2, \cdots, n$ are predetermined confidence levels with respect to m_λ-measure and D is the crisp feasible domain of decision variables.

Also, if the decision-maker wants to maximize the pessimistic value of the return function subject to some chance constraints, he may construct the following minimax chance-constrained programming model:

$$\begin{cases} \max\limits_{\boldsymbol{x}} \min\limits_{\bar{f}} \bar{f} \\ \text{subject to:} \\ \qquad m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \leq \bar{f} \right\} \geq \alpha \\ \qquad m_{\lambda_j} \left\{ g_j(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0 \right\} \geq \alpha_j, \quad j = 1, 2, \cdots, n \\ \qquad \boldsymbol{x} \in D. \end{cases} \tag{3}$$

In addition, according to the Hurwicz criterion, chance-constrained programming model may be constructed as the following form:

$$\begin{cases} \max\limits_{\boldsymbol{x}} \left(\beta \max\limits_{\bar{f}} \bar{f} + (1 - \beta) \min\limits_{\bar{g}} \bar{g} \right) \\ \text{subject to:} \\ \qquad m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \geq \bar{f} \right\} \geq \alpha \\ \qquad m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \leq \bar{g} \right\} \geq \alpha \\ \qquad m_{\lambda_j} \left\{ g_j(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0 \right\} \geq \alpha_j, \quad j = 1, 2, \cdots, n \\ \qquad \boldsymbol{x} \in D. \end{cases} \tag{4}$$

In this model, the aim is to optimize the linear combination of optimistic and pessimistic values of the return function, where $\beta \in [0, 1]$ is a parameter predetermined by decision-maker. Obviously, this objective function may balance extreme optimism and extreme pessimism by assigning the weights β and $1-\beta$ to optimistic value and pessimistic value, respectively.

Theorem 4.1 *Let $\{\lambda, \lambda_1, \cdots, \lambda_n\}$ and $\{\lambda^*, \lambda_1^*, \cdots, \lambda_n^*\}$ be two sets of parameters, and z, z^* be the corresponding optimal objective values of the model (2). If $\lambda \leq \lambda^*$, $\lambda_j \leq \lambda_j^*$ for each j, we have $z \leq z^*$.*

Proof. Let D_1 and D_2 be the feasible domains of the model (2) with respect to the parameters sets $\{\lambda, \lambda_1, \cdots, \lambda_n\}$ and $\{\lambda^*, \lambda_1^*, \cdots, \lambda_n^*\}$, respectively. Since m_λ-measure is an increasing function of λ, it is easy to prove $D_1 \subset D_2$. Note that $(\lambda; \alpha)$-optimistic value is an increasing function of λ for each fixed α. Thus we have

$$\max_{\boldsymbol{x} \in D_1} f(\boldsymbol{x}, \boldsymbol{\xi})_{\sup}(\lambda; \alpha) \le \max_{\boldsymbol{x} \in D_2} f(\boldsymbol{x}, \boldsymbol{\xi})_{\sup}(\lambda^*; \alpha),$$

which implies $z \le z^*$. The proof is completed.

Theorem 4.2 *Let $\{\alpha, \alpha_1, \cdots, \alpha_n\}$ and $\{\alpha^*, \alpha_1^* \cdots, \alpha_n^*\}$ be two sets of parameters, and z, z^* be the corresponding optimal objective values of the model (2). If $\alpha \ge \alpha^*$, $\alpha_j \ge \alpha_j^*$ for each j, then we have $z \le z^*$.*

Proof. Let D_1 and D_2 be the feasible domains of the model (2) with respect to $\{\alpha, \alpha_1, \cdots, \alpha_n\}$ and $\{\alpha^*, \alpha_1^* \cdots, \alpha_n^*\}$, respectively. It is easy to prove $D_1 \subset D_2$. Since $(\lambda; \alpha)$-optimistic value is a decreasing function of α for each fixed λ, we have

$$\max_{\boldsymbol{x} \in D_1} f(\boldsymbol{x}, \boldsymbol{\xi})_{\sup}(\lambda; \alpha) \le \max_{\boldsymbol{x} \in D_2} f(\boldsymbol{x}, \boldsymbol{\xi})_{\sup}(\lambda; \alpha^*),$$

which implies $z \le z^*$. The proof is completed.

Note 4.1 *The properties with respect to parameters for the models (3) and (4) may be discussed similarly.*

Theorem 4.3 *Suppose that for any feasible solution $\boldsymbol{x}$, the membership function of $f(\boldsymbol{x}, \boldsymbol{\xi})$ is continuous and z_1, z_2 and z_3 are optimal objective values of the models (2), (3) and (4), respectively. Then if $\alpha \le \lambda$, we have $z_1 \ge z_3 \ge z_2$; otherwise, we have $z_1 \le z_3 \le z_2$.*

Proof. By using Theorem 3.6, we can easily prove this theorem.

For the above models, it is not easy to use the analytic method to solve them since we need to compute chance functions such as $m_{\lambda_j}\{g_j(\boldsymbol{x}, \boldsymbol{\xi}) \le 0\}$. Generally, if the fuzzy parameters are complex, the computation of chance function is not easy. But for some special conditions, we may compute by some shortcuts.

Note 4.2 *If the return function $f(\boldsymbol{x}, \boldsymbol{\xi})$ can be represented by trapezoidal fuzzy number or interval fuzzy number, then the $(\lambda; \alpha)$-optimistic value and $(\lambda; \alpha)$-pessimistic value of $f(\boldsymbol{x}, \boldsymbol{\xi})$ may be computed according to Examples 3.1 and 3.2.*

Theorem 4.4 *Let $g_j(\boldsymbol{x}, \boldsymbol{\xi}) = h_j(\boldsymbol{x}) - v_j(\boldsymbol{x}, \boldsymbol{\xi})$, and $v_j(\boldsymbol{x}, \boldsymbol{\xi})$ be a fuzzy number with continuous membership function $\mu(x)$. Then* $\text{Pos}\{g_j(\boldsymbol{x}, \boldsymbol{\xi}) < 0\} \le \alpha_j$ *if and only if $h_j(\boldsymbol{x}) \ge F_{\alpha_j}$, where $F_{\alpha_j} = \sup\{F | F = \mu^{-1}(\alpha_j)\}$.*

Proof. The inequality $\text{Pos}\{g_j(\boldsymbol{x},\boldsymbol{\xi}) < 0\} \leq \alpha_j$ may be rewritten as $\text{Pos}\{h_j(\boldsymbol{x}) < v_j(\boldsymbol{x},\boldsymbol{\xi})\} \leq \alpha_j$. Since $\mu(x)$ is continuous, there exists real number F such that $\mu(F) = \alpha_j$ for any given confidence level α_j. Let $F_{\alpha_j} = \sup\{F|F = \mu^{-1}(\alpha_j)\}$. By using the continuity of $\mu(x)$ and the properties of fuzzy number, we have $\text{Pos}\{F_{\alpha_j} < v_j(\boldsymbol{x},\boldsymbol{\xi})\} = \sup_{y>F_{\alpha_j}} \mu(y) = \mu(F_{\alpha_j}) = \alpha_j$. Note that the possibility $\text{Pos}\{F_{\alpha_j} < v_j(\boldsymbol{x},\boldsymbol{\xi})\}$ will decrease if the number F_{α_j} is replaced with a larger number. Thus the crisp equivalent of $\text{Pos}\{g_j(\boldsymbol{x},\boldsymbol{\xi}) < 0\} \leq \alpha_j$ is $h_j(\boldsymbol{x}) \geq F_{\alpha_j}$. The proof is completed.

Theorem 4.5 *Let $g_j(\boldsymbol{x},\boldsymbol{\xi}) = h_j(\boldsymbol{x}) - v_j(\boldsymbol{x},\boldsymbol{\xi})$, and $v_j(\boldsymbol{x},\boldsymbol{\xi})$ be a fuzzy number with continuous membership function $\mu(x)$. Then for any $\alpha_j \in (0,1]$, the constraint $m_{\lambda_j}\{g_j(\boldsymbol{x},\boldsymbol{\xi}) \geq 0\} \geq \alpha_j$ is equivalent to $h_j(\boldsymbol{x}) \geq F_{\alpha_j}$, where*

$$F_{\alpha_j} = \begin{cases} \inf\{F|F = \mu^{-1}(\alpha_j/\lambda_j)\}, & \text{if } \alpha_j \leq \lambda_j \\ \sup\{F|F = \mu^{-1}((1-\alpha_j)/(1-\lambda_j))\}, & \text{if } \alpha_j > \lambda_j. \end{cases} \tag{5}$$

Proof. Note that $m_{\lambda_j}\{g_j(\boldsymbol{x},\boldsymbol{\xi}) \geq 0\} \geq \alpha_j$ can be rewritten as $m_{\lambda_j}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) \leq h_j(\boldsymbol{x})\} \geq \alpha_j$. We divide the proof into the following two cases.

Case 1: $\alpha_j \leq \lambda_j$. Since $v_j(\boldsymbol{x},\boldsymbol{\xi})$ is a fuzzy number with continuous membership function, there exists real number F such that $\mu(F) = \alpha_j/\lambda_j$. Let $F_{\alpha_j} = \inf\{F|F = \mu^{-1}(\alpha_j/\lambda_j)\}$. By using the continuity of $\mu(x)$ and the properties of fuzzy number, we have $\text{Pos}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) \leq F_{\alpha_j}\} = \sup_{y\leq F_{\alpha_j}} \mu(y) = \mu(F_{\alpha_j}) = \alpha_j/\lambda_j$ and $\text{Pos}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) > F_{\alpha_j}\} = 1$, which implies that $m_{\lambda_j}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) \leq F_{\alpha_j}\} = \alpha_j$. Note that $m_{\lambda_j}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) \leq F_{\alpha_j}\}$ will increase if the number F_{α_j} is replaced with a larger number. Thus the crisp equivalent of $m_{\lambda_j}\{g_j(x,\xi) \geq 0\} \geq \alpha_j$ is $h_j(\boldsymbol{x}) \geq F_{\alpha_j}$.

Case 2: $\alpha_j > \lambda_j$. For this case, we have that $\text{Pos}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) \leq h_j(\boldsymbol{x})\} = 1$. Then $m_{\lambda_j}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) \leq h_j(\boldsymbol{x})\} \geq \alpha_j$ is equivalent to $\text{Nec}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) \leq h_j(\boldsymbol{x})\} \geq (\alpha_j - \lambda_j)/(1-\lambda_j)$. Since possibility measure and necessity measure are dual, $\text{Nec}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) \leq h_j(\boldsymbol{x})\} \geq (\alpha_j - \lambda_j)/(1-\lambda_j)$ is equivalent to $\text{Pos}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) > h_j(\boldsymbol{x})\} \leq (1-\alpha_j)/(1-\lambda_j)$. By using Theorem 4.4, we have that $\text{Pos}\{v_j(\boldsymbol{x},\boldsymbol{\xi}) > h_j(\boldsymbol{x})\} \leq (1-\alpha_j)/(1-\lambda_j)$ is equivalent to $h_j(\boldsymbol{x}) \geq F_{\alpha_j}$, where $F_{\alpha_j} = \sup\{F|F = \mu^{-1}((1-\alpha_j)/(1-\lambda_j))\}$. Thus the proof is completed.

Corollary 4.1 *Let $g_j(\boldsymbol{x},\boldsymbol{\xi}) = h_j(\boldsymbol{x}) - v_j(\boldsymbol{x},\boldsymbol{\xi})$, and $v_j(\boldsymbol{x},\boldsymbol{\xi}) = (v_j^1(\boldsymbol{x}), v_j^2(\boldsymbol{x}), v_j^3(\boldsymbol{x}), v_j^4(\boldsymbol{x}))$ be a trapezoidal fuzzy number. Then for any $\alpha_j \in (0,1]$, the constraint $m_{\lambda_j}\{g_j(\boldsymbol{x},\boldsymbol{\xi}) \geq 0\} \geq \alpha_j$ is equivalent to $h_j(\boldsymbol{x}) \leq F_{\alpha_j}$, where*

$$F_{\alpha_j} = \begin{cases} \dfrac{a_j}{\lambda_j}(v_j^2(\boldsymbol{x}) - v_j^1(\boldsymbol{x})) + v_j^1(\boldsymbol{x}), & \text{if } \alpha_j \leq \lambda_j \\ \dfrac{1-a_j}{1-\lambda_j}(v_j^3(\boldsymbol{x}) - v_j^4(\boldsymbol{x})) + v_j^4(\boldsymbol{x}), & \text{if } \alpha_j > \lambda_j. \end{cases} \tag{6}$$

4.2 Simulation Algorithms and Hybrid Genetic Algorithm

If the functions satisfy the conditions of Note 4.2 or Theorem 4.5, we may convert the objective function or chance constraints into their crisp equivalents. If we cannot check constraints or compute objective value by analytic methods, we may design simulation algorithms to obtain the approximate value.

1. Compute chance measure with the form $m_\lambda\{f(\boldsymbol{x}, \boldsymbol{\xi}) \leq 0\}$.

Step 1. Randomly generate crisp vectors $\boldsymbol{u}_k$ from the ε-level set of fuzzy vector $\boldsymbol{\xi}$, respectively, where ε is a sufficiently small number and $k = 1, 2, \cdots, N$;

Step 2. Let $\nu_k = \mu_{\boldsymbol{\xi}}(\boldsymbol{u}_k)$ for $k = 1, 2, \cdots, N$, where $\mu_{\boldsymbol{\xi}}$ is the membership function of $\boldsymbol{\xi}$;

Step 3. Return the following value

$$L = \lambda \times \max_{1 \leq k \leq N} \{\nu_k | f(\boldsymbol{x}, \boldsymbol{u}_k) \leq 0\} + (1 - \lambda) \times \min_{1 \leq k \leq N} \{1 - \nu_k | f(\boldsymbol{x}, \boldsymbol{u}_k) > 0\}.$$

2. Compute the optimistic value $\sup \left\{ \bar{f} | m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \geq \bar{f} \right\} \geq \alpha \right\}$.

Note that $m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \geq \bar{f} \right\}$ is a decreasing function of $\bar{f}$. We shall employ the bisection algorithm to simulate $(\lambda; \alpha)$-optimistic value.

Step 1. Randomly generate two values $\bar{f}_1$ and $\bar{f}_2$ such that $m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \geq \bar{f}_1 \right\} \geq \alpha$ and $m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \geq \bar{f}_2 \right\} < \alpha$;

Let $\bar{f} = \dfrac{\bar{f}_1 + \bar{f}_2}{2}$;

Step 2. Step 3. If $m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \geq \bar{f} \right\} \geq \alpha$, then let $\bar{f}_1 = \bar{f}$; otherwise, let $\bar{f}_2 = \bar{f}$;

Step 4. If $\left| \bar{f}_1 - \bar{f}_2 \right| < \delta$, return $\bar{f}_1$; otherwise, go to Step 2.

3. Compute the pessimistic value $\inf \left\{ \bar{f} | m_\lambda \{ f(\boldsymbol{x}, \boldsymbol{\xi}) \leq \bar{f} \right\} \geq \alpha \}$.

Note that $m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \leq \bar{f} \right\}$ is an increasing function of $\bar{f}$. The bisection algorithm is also employed to simulate $(\lambda; \alpha)$-pessimistic value.

Step 1. Randomly generate two values $\bar{f}_1$ and $\bar{f}_2$ such that $m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \leq \bar{f}_1 \right\} \geq \alpha$ and $m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \leq \bar{f}_2 \right\} < \alpha$;

Let $\bar{f} = \dfrac{\bar{f}_1 + \bar{f}_2}{2}$;

Step 2. Step 3. If $m_\lambda \left\{ f(\boldsymbol{x}, \boldsymbol{\xi}) \leq \bar{f} \right\} \geq \alpha$, then let $\bar{f}_1 = \bar{f}$; otherwise, let $\bar{f}_2 = \bar{f}$;

Step 4. If $\left|\bar{f}_1 - \bar{f}_2\right| < \delta$, return $\bar{f}_1$; otherwise, go to Step 2.

In order to solve the models (2), (3) and (4), we may design genetic algorithm based on the fuzzy simulation to obtain the approximate optimal solution, in which simulation algorithms are employed to check the feasibility of solutions and compute the objective values if analytic methods are invalid. We list the procedure of the hybrid genetic algorithm as follows:

Step 1. Initialize *popsize* chromosomes in which the simulation algorithm may be employed to check the feasibility of solution if the fuzzy parameters are complex;

Step 2. Calculate the objective values for all chromosomes by simulation algorithm or analytic method if possible;

Step 3 Compute the fitness of each chromosome according to the objective value;

Step 4. Select the chromosomes by spinning the roulette wheel;

Step 5. Update the chromosomes by crossover and mutation operations in which the feasibility of offspring may be checked by simulation algorithm or analytic method;

Step 6. Repeat Step 2 to Step 5 for a given number of cycles;

Step 7. Output the best chromosome as the approximate optimal solution.

5 Numerical Examples

Example 1: Consider the following model:

$$\begin{cases} \max\limits_{\boldsymbol{x}} \max\limits_{\bar{f}} \bar{f} \\ \text{subject to:} \\ \quad m_{0.3}\left\{\xi_1 x_1^2 + \xi_2^2 x_2^2 - \xi_3 x_3^2 \geq \bar{f}\right\} \geq 0.85 \\ \quad m_{0.3}\left\{\xi_1^2 x_1 + \xi_2 x_2^2 - \xi_3 x_3^3 \geq 0\right\} \geq 0.90 \\ \quad m_{0.3}\left\{x_1 + \xi_1 \xi_2 x_2 + \xi_3 x_3 \geq 10\right\} \geq 0.95 \\ \quad x_1^2 + x_2^2 + x_3^2 \leq 100 \\ \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0. \end{cases} \tag{7}$$

In the above model, $\xi_1 = (1, 2, 3)$, $\xi_2 = (2, 4, 6)$ and $\xi_3 = (3, 4, 5)$ are triangular fuzzy numbers. After running hybrid genetic algorithm (2000 cycles in fuzzy simulation) with different parameters, we list the optimal objective values in Table 1, where the notations "P_c" and "P_m" denote crossover probability and mutation probability, respectively, and the error is computed according to the following formula:

$$Error = \frac{\text{the largest optimal objective} \ - \ \text{optimal objective}}{\text{the largest optimal objective}} \times 100\%. \qquad (8)$$

We can see that by using different parameters, the errors of the optimal objective value are no more than 1.2671%. The best objective value is 584.0607, and the corresponding approximate optimal solution is

$$(x_1^*, x_2^*, x_3^*) = (0.3712, 9.9842, 0.0001).$$

For the two chance constraints, we have

$$m_{0.3}\left\{\xi_1^2 x_1^* + \xi_2 x_2^{*2} - \xi_3 x_3^{*3} \geq 0\right\} \approx 0.9743 \geq 0.90,$$

$$m_{0.3}\left\{x_1^* + \xi_1\xi_2 x_2^* + \xi_3 x_3^* \geq 10\right\} \approx 0.9869 \geq 0.95.$$

Table 1: Comparison of the Optimal Objectives

popsize	P_c	P_m	**Generation**	**Optimal Objective**	**Error**
20	0.1	0.1	500	579.2542	0.8229%
20	0.1	0.2	700	579.1016	0.8491%
20	0.2	0.2	800	576.6602	1.2671%
30	0.2	0.3	1000	583.9844	0.0000%
30	0.4	0.4	1500	583.9844	0.0000%
30	0.5	0.4	1500	583.9844	0.0000%
30	0.5	0.6	2000	584.0607	0.0000%

Example 2: Consider the following minimax chance-constrained programming model:

$$\begin{cases} \max\limits_{\boldsymbol{x}} \inf\limits_{\bar{f}} \bar{f} \\ \text{subject to:} \\ \qquad m_{0.2}\left\{\xi_1 x_1^2 + \xi_2 x_2^2 - \xi_3 x_3^2 + x_4 \leq \bar{f}\right\} \geq 0.90 \\ \qquad m_{0.2}\left\{x_1 + \xi_2 x_2^2 - x_3 + \xi_4 x_4^2 \geq 0\right\} \geq 0.90 \\ \qquad m_{0.2}\left\{\xi_1 x_1^2 - x_2 + \xi_3 x_3^2 - x_4 \geq 10\right\} \geq 0.90 \\ \qquad x_1^2 + x_2^2 + x_3^2 + x_4^2 \leq 100. \end{cases} \qquad (9)$$

In the above model, $\xi_1 = (2,3,5,6)$, $\xi_2 = (4,5,6,7)$, $\xi_3 = (3,4,5,6)$ and $\xi_4 = (-4,-3,-2,-1)$ are trapezoidal fuzzy numbers. In fact, we may convert this model into the following form:

$$\begin{cases} \max \quad 5.875x_1^2 + 6.875x_2^2 - 3.125x_3^2 + x_4 \\ \text{subject to:} \\ \qquad x_1 + 4.125x_2^2 - x_3 - 3.875x_4^2 \geq 0 \\ \qquad 2.125x_1^2 - x_2 + 3.125x_3^2 - x_4 \geq 10 \\ \qquad x_1^2 + x_2^2 + x_3^2 + x_4^2 \leq 100. \end{cases} \tag{10}$$

After running genetic algorithm with different parameters, we list the optimal objective values in Table 2, where the error is computed according to the formula (8).

We can see that by using different parameters, the errors of the optimal objective value are no more than 1.4320%. The best objective is 687.5080, and the corresponding solution is

$$(x_1^*, x_2^*, x_3^*, x_4^*) = (0.1603, -9.9986, 0.0016, 0.0532).$$

Table 2: Comparison of the Optimal Objectives

popsize	P_c	P_m	**Generation**	**Optimal Objective**	**Error**
30	0.1	0.1	500	684.8353	0.3888%
30	0.2	0.2	700	677.6629	1.4320%
40	0.2	0.3	800	687.5012	0.0000%
40	0.3	0.3	1000	687.4548	0.0000%
40	0.3	0.4	1200	687.4925	0.0000%
50	0.4	0.4	1500	687.5016	0.0000%
50	0.5	0.6	2000	687.5058	0.0000%
50	0.6	0.6	3000	687.5080	0.0000%

6 Conclusion

The main work of this paper can be summarized as the following three aspect: (i) m_λ-measure was presented and its mathematical properties were investigated; (ii) chance-constrained programming models with respect to m_λ-measure were constructed and some crisp equivalents of objectives and constraints were obtained; (iii) genetic algorithm based on the fuzzy simulation technique was designed to solve the models.

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